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THE 3-MODULAR CHARACTERS OF THE RUDVALIS SPORADIC SIMPLE GROUP AND ITS COVERING GROUP

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ABSTRACT. The decomposition numbers of the Rudvalis sporadic simple group and of its double covering group modulo 3 are completely determined. The results were obtained using the computer algebra system MOC developed by Jansen, Lux, Parker, and the author, but the proofs are given in conventional form and can be verified by hand with the ordinary character table.

1. RESULTS

In the present paper we determine the 3-modular decomposition matrices for the sporadic group $G = 2\mathrm{Ru}$, the double cover of the Rudvalis group. We begin by presenting the results. For each nontrivial block in turn we display the table of decomposition numbers followed by a table giving the degrees of the irreducible Brauer characters in the block as well as the factorization of these degrees into prime factors. Projective indecomposable characters are denoted by upper case Φ's and the corresponding irreducible Brauer characters by lower case ϕ's.

1.1. **Block 1.** The principal block

	Φ_1	Φ_2	Φ_3	Φ_4	Φ_5	Φ_6	Φ_7	Φ_8	Φ_9
1	1	.	.	.	.	.	.	.	.
406	.	1	.	.	.	.	.	.	.
21750	.	.	1	1	.	.	.	.	.
34944	.	.	.	.	1	.	.	.	.
34944	.	.	.	.	1	.	.	.	.
45500	.	1	.	.	.	1	.	.	.
52780	.	.	1	1	.	.	1	.	.
63336	.	1	.	.	.	1	.	1	.
65975	1	.	.	.	1	.	1	.	.
75400	.	.	1	.	.	.	1	.	1
76125	1	.	.	.	.	1	1	.	.
102400	.	.	.	1	.	1	1	1	.
105560	.	.	1	1	1	.	1	1	.
118784	.	.	1	1	1	.	1	.	1

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Char.	Degree	Factors
ϕ_1	1	1
ϕ_2	406	$(2)(7)(29)$
ϕ_3	13310	$(2)(5)(11)^3$
ϕ_4	8440	$(2)^3(5)(211)$
ϕ_5	34944	$(2)^7(3)(7)(13)$
ϕ_6	45094	$(2)(7)(3221)$
ϕ_7	31030	$(2)(5)(29)(107)$
ϕ_8	17836	$(2)^2(7)^3(13)$
ϕ_9	31060	$(2)^2(5)(1553)$

1.2. **Block 2.** This is a block of defect 1 containing characters of the simple group.

	Φ_1	Φ_2
3276	1	.
20475	.	1
23751	1	1

Char.	Degree	Factors
ϕ_1	3276	$(2)^2(3)^2(7)(13)$
ϕ_2	20475	$(3)^2(5)^2(7)(13)$

1.3. **Block 3.** This is a block of defect 1 containing characters of the simple group.

	Φ_1	Φ_2
3654	1	.
91350	.	1
95004	1	1

Char.	Degree	Factors
ϕ_1	3654	$(2)(3)^2(7)(29)$
ϕ_2	91350	$(2)(3)^2(5)^2(7)(29)$

1.4. **Block 4.** This is the nonprincipal block of maximal defect.

	Φ_1	Φ_2	Φ_3	Φ_4	Φ_5	Φ_6	Φ_7	Φ_8	Φ_9
28	1	.	.	.	.	.	.	.	.
28	.	1	.	.	.	.	.	.	.
1248	.	.	1	.	.	.	.	.	.
1248	.	.	1	.	.	.	.	.	.
7280	.	.	1	1	.	.	.	.	.
7280	.	.	1	.	1	.	.	.	.
8192	.	.	1	.	.	1	.	.	.
8192	.	.	1	.	.	.	1	.	.
10556	1	.	.	.	.	.	.	1	.
10556	.	1	.	.	.	.	.	.	1
34944	.	.	.	.	.	1	1	1	1
38976	.	.	2	1	1	1	1	1	.
38976	.	.	2	1	1	1	1	.	1
48256	.	.	1	1	1	1	1	1	1

Char.	Degree	Factors
ϕ_1	28	$(2)^2(7)$
ϕ_2	28	$(2)^2(7)$
ϕ_3	1248	$(2)^5(3)(13)$
ϕ_4	6032	$(2)^4(13)(29)$
ϕ_5	6032	$(2)^4(13)(29)$
ϕ_6	6944	$(2)^5(7)(31)$
ϕ_7	6944	$(2)^5(7)(31)$
ϕ_8	10528	$(2)^5(7)(47)$
ϕ_9	10528	$(2)^5(7)(47)$

1.5. **Block 5.** This is a block of defect 1 containing faithful characters.

	Φ_1	Φ_2
3276	1	.
4032	.	1
7308	1	1

Char.	Degree	Factors
ϕ_1	3276	$(2)^2(3)^2(7)(13)$
ϕ_2	4032	$(2)^6(3)^2(7)$

1.6. **Block 6.** This is the dual of Block 5, so the tables are the same.

2. Proofs

The distribution of the characters into blocks is easily determined with the help of the character table. It is also fairly easy to find the number of irreducible Brauer characters in each block. The two blocks of maximal defect each contain 14 ordinary and 9 modular irreducible characters. The blocks containing only three ordinary irreducible characters are blocks of defect 1. Their decomposition matrix is therefore trivially determined.

In the following proofs, PIM stands for projective indecomposable module. We have implicitly assumed that a splitting 3-modular system for 2Ru is given. Ordinary and Brauer characters are sometimes denoted by their degrees.

2.1. **Proof for Block 4.** We start with the proof for the nonprincipal block of maximal defect (Block 4) since it is easier than the one for the principal block and since we are going to use some of these results for the proof of the principal block.

We first consider the set of projective characters displayed in Table 1 (next page). Here, $\Phi = \chi_{15} + \chi_{16} + \chi_{23} + \chi_{33} + \chi_{36}$ is the projective character obtained by inducing the character of degree 3 of the maximal subgroup $(2^2 \times \mathrm{Sz}(8)) : 3$ to the Rudvalis group and restricting it to the principal block.

These projectives show that the decomposition matrix has wedge shape. Therefore, all but Λ_3 are indecomposable. The irreducible Brauer character of degree 1248 is real-valued. Since it is no constituent of 10556_1, it cannot be one of 10556_2 either, the complex conjugate of 10556_1. Therefore, $\Phi_3 := \Lambda_3 - \Lambda_9$ is a projective character. It is clear that it is indecomposable. This completes the proof for Block 4.

TABLE 1. Some projectives for Block 4

	$\chi_{38} \otimes \chi_2$	$\chi_{41} \otimes \chi_2$	$\chi_{38} \otimes \Phi$	$\chi_{46} \otimes \chi_2$	$\chi_{45} \otimes \chi_2$	$\chi_{49} \otimes \chi_2$	$\chi_{50} \otimes \chi_2$	$\chi_{44} \otimes \chi_2$	$\chi_{43} \otimes \chi_2$
	Λ_1	Λ_2	Λ_3	Λ_4	Λ_5	Λ_6	Λ_7	Λ_8	Λ_9
28	1	.	.	.	.	.	.	.	.
28	.	1	.	.	.	.	.	.	.
1248	.	.	1	.	.	.	.	.	.
1248	.	.	1	.	.	.	.	.	.
7280	.	.	1	1	.	.	.	.	.
7280	.	.	1	.	1	.	.	.	.
8192	.	.	1	.	.	1	.	.	.
8192	.	.	1	.	.	.	1	.	.
10556	1	.	.	.	.	.	.	1	.
10556	.	1	1	.	.	.	.	.	1
34944	.	.	1	.	.	1	1	1	1
38976	.	.	2	1	1	1	1	1	.
38976	.	.	3	1	1	1	1	.	1
48256	.	.	2	1	1	1	1	1	1

2.2. **Proof for the principal block.** We begin with the set of projective characters displayed in Table 2. The origin of these characters is documented below.

Char.	Origin
Λ_1 :	$\chi_3 \otimes \chi_2$
Λ_2 :	$\chi_5 \otimes \chi_2$
Λ_3 :	$\chi_{37} \otimes \Phi_{7,4}$
Λ_4 :	$\chi_{37} \otimes \chi_{58}$
Λ_5 :	$\mathrm{Ind}_{M_1}(\theta_{24})$
Λ_6 :	$\mathrm{Ind}_{M_1}(\theta_{15})$
Λ_7 :	$\mathrm{Ind}_{M_3}(\theta_4)$
Λ_8 :	$\chi_6 \otimes \chi_2$
Λ_9 :	$\chi_7 \otimes \chi_2$
Λ_{10} :	$\mathrm{Ind}_{M_1}(\theta_{14})$

In this table the following notations are used. M_i denotes the ith maximal subgroup of the Rudvalis group, where the subgroups are sorted in decreasing order. We shall only need to induce characters for the proof of the principal block, and so we only need to consider the maximal subgroups of the Rudvalis group rather than its covering group. Only the three largest maximal subgroups will be used. In ATLAS [2] notation these are: $M_1 = {}^2F_4(2)$, $M_2 = (2^6\colon U_3(3))\colon 2$ and $M_3 = (2^2 \times \mathrm{Sz}(8))\colon 3$. The character tables of these maximal subgroups are available in the CAS system (see [3]). The numbering of the characters of

TABLE 2. Some projectives for the principal block

	Λ_1	Λ_2	Λ_3	Λ_4	Λ_5	Λ_6	Λ_7	Λ_8	Λ_9	Λ_{10}
1	1	.	.	.	.	.	.	.	.	.
406	.	1	.	.	.	.	.	.	.	1
21750	.	.	1	1	.	.	.	.	.	.
34944	.	.	.	1	1	1	1	.	.	1
34944	.	.	.	1	1	1	1	.	.	1
45500	.	1	1	1	1	.	.	1	.	2
52780	.	.	2	1	1	.	.	.	1	.
63336	.	1	2	2	1	1	.	1	.	2
65975	1	.	1	1	2	1	1	.	1	1
75400	.	.	4	1	1	.	.	1	1	.
76125	1	.	2	1	2	.	.	1	1	1
102400	.	.	4	2	2	1	.	1	1	1
105560	.	.	3	3	2	2	1	.	1	1
118784	.	.	5	2	2	1	1	1	1	1

the maximal subgroups follows the numbering in the CAS tables. Character θ_{24} of M_1 is of degree 675, θ_{15} and θ_{14} have degree 351 and character θ_4 of M_3 has degree 3. All these characters are therefore of defect 0, and hence induce to projective characters.

The character named $\Phi_{7,4}$ is the projective indecomposable character Φ_7 of Block 4. A similar notation will be used later on.

Of course, the above induced characters and tensor products may have some constituents outside the principal block. The complete decomposition of these characters is given in the appendix.

Now observe that $\chi_{37} \otimes \chi_{15} = \chi_{40} + \chi_{57} + \chi_{58} + \chi_{59} + 2\chi_{60} + \chi_{61}$, and that all characters but χ_{40} occurring in this tensor product are of defect 0. If we denote the characters by their degrees, we can write

$$28 \otimes 34944 = 1248 + \text{defect } 0 \text{ characters}.$$

Since 1248 is irreducible as Brauer character, it follows that 34944 also is an irreducible Brauer character.

We claim that $\{\Lambda_1, \dots, \Lambda_9\}$ is a basic set of projective characters. That is to say, every projective indecomposable character of the principal block is a $\mathbb{Z}$-linear combination of $\{\Lambda_1, \dots, \Lambda_9\}$. This follows from the fact that the matrix of scalar products of $\{\Lambda_1, \dots, \Lambda_9\}$ with the ordinary characters of degrees 1, 406, 21750, 34944_1, 45500, 52780, 63336, 75400, and 102400 is unimodular, which can easily be checked from Table 2.

Since Λ_7 has inner product 1 with 34944 and inner product 0 with all the other characters listed above, it follows that Λ_7 is the projective indecomposable corresponding to the irreducible 34944.

TABLE 3. New basic set of projectives for the principal block

	Λ_1	Λ_2	Λ_3	Λ_{11}	Λ_7	Λ_8	Λ_{12}	Λ_9	Λ_{13}
1	1	.	.	.	.	.	.	.	.
406	.	1	.	.	.	.	.	.	.
21750	.	.	1	1	.	.	.	.	.
34944	.	.	.	.	1	.	.	.	.
34944	.	.	.	.	1	.	.	.	.
45500	.	1	1	1	.	1	1	.	.
52780	.	.	2	1	.	.	.	1	.
63336	.	1	2	2	.	1	1	.	1
65975	1	.	1	.	1	.	.	1	.
75400	.	.	4	1	.	1	.	1	.
76125	1	.	2	1	.	1	1	1	.
102400	.	.	4	2	.	1	1	1	1
105560	.	.	3	2	1	.	.	1	1
118784	.	.	5	1	1	1	.	1	.

It is clear the Λ_1 and Λ_2 are PIM's (since they have only 3 ordinary constituents). Also, 406 is irreducible, since it has inner product 1 with Λ_2 and inner product 0 with each of $\{\Lambda_3, \dots, \Lambda_9\}$ and Λ_1.

Each of Λ_2 and Λ_7 can therefore be subtracted from any other projective as many times as 406 respectively 34944 is contained in that projective. If we do so, we obtain the new basic set of projectives displayed in Table 3.

The new projectives were obtained as follows.

Char.	Origin
Λ_{11} :	$\Lambda_4 - \Lambda_7$
Λ_{12} :	$\Lambda_{10} - \Lambda_2 - \Lambda_7$
Λ_{13} :	$\Lambda_6 - \Lambda_7$

It is easy to check (consider the character degrees modulo 9), that of the new projectives Λ_{12} and Λ_{13} are indecomposable. To proceed, we consider some more projectives which are displayed in Table 4. The origin of these characters is documented in the following table.

Char.	Origin
Λ_{14} :	$\mathrm{Ind}_{M_2}(\theta_{13})$
Λ_{15} :	$\mathrm{Ind}_{M_1}(\theta_{23})$
Λ_{16} :	$\chi_{37} \otimes \Phi_{5,4}$
Λ_{17} :	$\mathrm{Ind}_{M_2}(\theta_{12})$
Λ_{18} :	$\chi_{37} \otimes \Phi_{3,4}$

TABLE 4. More projectives for the principal block

	Λ_{14}	Λ_{15}	Λ_{16}	Λ_{17}	Λ_{18}
1	.	.	.	.	.
406	.	.	.	.	.
21750	.	1	1	1	2
34944	1	1	.	3	1
34944	1	1	.	3	1
45500	1	2	.	3	.
52780	1	1	2	1	3
63336	3	2	2	3	3
65975	2	1	1	3	2
75400	3	1	3	.	5
76125	2	2	1	3	1
102400	4	2	3	4	5
105560	4	2	4	4	7
118784	4	2	3	4	7

The characters θ_{13} and θ_{12} of M_2 are of degree 27 and θ_{23} of M_1 is of degree 675, thus all three are of defect 0. The projectives, denoted $\Phi_{5,4}$ and $\Phi_{3,4}$, are the projective indecomposables Φ_5, respectively Φ_3, of Block 4.

TABLE 5. Projectives of Table 4 improved

	Λ_{19}	Λ_{20}	Λ_{21}	Λ_{22}	Λ_{23}
1	.	.	.	.	.
406	.	.	.	.	.
21750	.	1	1	1	2
34944	.	.	.	.	.
34944	.	.	.	.	.
45500	1	2	.	3	.
52780	1	1	2	1	3
63336	3	2	2	3	3
65975	1	.	1	.	1
75400	3	1	3	.	5
76125	2	2	1	3	1
102400	4	2	3	4	5
105560	3	1	4	1	6
118784	3	1	3	1	6

TABLE 6. New basic set of projectives for the principal block

	Λ_1	Λ_2	Λ_{24}	Λ_{25}	Λ_7	Λ_{12}	Λ_9	Λ_{13}	Λ_{26}
1	1	.	.	.	.	.	.	.	.
406	.	1	.	.	.	.	.	.	.
21750	.	.	1	1	.	.	.	.	.
34944	.	.	.	.	1	.	.	.	.
34944	.	.	.	.	1	.	.	.	.
45500	.	1	.	.	.	1	.	.	.
52780	.	.	2	1	.	.	1	.	.
63336	.	1	.	.	.	1	.	1	.
65975	1	.	1	.	1	.	1	.	.
75400	.	.	4	1	.	.	1	.	1
76125	1	.	1	.	.	1	1	.	.
102400	.	.	2	.	.	1	1	1	.
105560	.	.	2	1	1	.	1	1	.
118784	.	.	5	1	1	.	1	.	1

We again subtract off Λ_7 as often as possible and obtain the projectives of Table 5. In terms of the basic set of Table 3, the characters can be written as follows.

	Λ_1	Λ_2	Λ_3	Λ_{11}	Λ_7	Λ_8	Λ_{12}	Λ_9	Λ_{13}
Λ_{19}	.	.	.	.	.	2	-1	1	2
Λ_{20}	.	.	.	1	.	.	1	.	-1
Λ_{21}	.	.	.	1	.	1	-2	1	1
Λ_{22}	.	.	1	.	.	-3	5	-1	-1
Λ_{23}	.	.	1	1	.	.	-2	.	1

This gives us the new basic set of projectives of Table 6. The new projectives were obtained as follows.

Char.	Origin
Λ_{24} :	$\Lambda_3 - \Lambda_{12} - \Lambda_{13}$
Λ_{25} :	$\Lambda_{11} - \Lambda_{12} - \Lambda_{13}$
Λ_{26} :	$\Lambda_8 - \Lambda_{12}$

That these characters are indeed projective can easily be deduced from the projectives of Table 5. Consider, for example, the projective Λ_{19}. Its expression in terms of the basic set implies $\Lambda_{19} + \Lambda_{12} = 2\Lambda_8 + \Lambda_9 + 2\Lambda_{13}$. Since Λ_{12} is indecomposable, this implies that Λ_{12} is contained in Λ_8. Similarly, Λ_{20} shows that Λ_{13} is contained in Λ_{11}.

Of the new projective characters, Λ_{26} is indecomposable. Now $\Lambda_{24} = \Lambda_{22} - 3\Lambda_{12} + \Lambda_9 + 3\Lambda_{26}$. This implies that $\Lambda_{27} := \Lambda_{22} - 3\Lambda_{12}$ is a projective character.

TABLE 7. Hypothetical decomposition matrix

	Φ'_1	Φ'_2	Φ'_3	Φ'_4	Φ'_5	Φ'_6	Φ'_7	Φ'_8	Φ'_9
1	1	.	.	.	.	.	.	.	.
406	.	1	.	.	.	.	.	.	.
21750	.	.	1	.	.	.	.	.	.
34944	.	.	.	1	.	.	.	.	.
34944	.	.	.	1	.	.	.	.	.
45500	.	1	.	.	1	.	.	.	.
52780	.	.	1	.	.	1	.	.	.
63336	.	1	.	.	1	.	1	.	.
65975	1	.	.	1	.	1	.	.	.
75400	.	.	.	.	.	.	.	1	.
76125	1	.	.	.	1	1	.	.	.
102400	.	.	.	.	1	1	1	.	1
105560	.	.	1	1	.	1	1	.	.
118784	.	.	.	1	.	.	.	1	1

Substituting Λ_{24} by Λ_{27}, and renaming the characters, we obtain the matrix displayed in §1.1. We proceed to show that this matrix already is the matrix of decomposition numbers. To do so, we only need to show that Φ_3, Φ_4 and Φ_7 are characters of indecomposable modules.

For this purpose we consider the maximal subgroup $M_2 = (2^6\colon U_3(3))\colon 2$ of the Rudvalis group. Let $\bar{1}$ denote the linear character (and the corresponding module) of M_2 which takes value -1 outside $2^6\colon U_3(3)$. We obtain

$$\mathrm{Ind}_{M_2}(\bar{1}) = \chi_9 + \chi_{25} + \chi_{28}.$$

In terms of degrees, this is

$$\mathrm{Ind}_{M_2}(\bar{1}) = 21750 + 75400 + 91350.$$

Let X denote the component in the principal block of the corresponding induced module. Then the Brauer character of X is $21750 + 75400$. Since $\bar{1}$ is a trivial source module, so is X.

Now suppose for the moment that X is not indecomposable. Then it decomposes as $X = X_1 \oplus X_2$, where X_1 has Brauer character 21750, say, and X_2 has Brauer character 75400. Each of X_1 and X_2 has trivial source, and so a 1-dimensional endomorphism ring. Since all characters in the principal block are real-valued, this implies that X_1 and X_2 are simple.

Now Φ_9 is indecomposable with inner product 1 with the irreducible 75400, so Φ_9 is the character of the projective cover of 75400. Thus, $\Phi_3 - \Phi_9$ and $\Phi_7 - \Phi_9$ are projective indecomposable characters. Moreover, $\Phi_3 - \Phi_9$ has inner product 1 with 21750 and can therefore be subtracted from Φ_4. This gives all the projective indecomposable characters and therefore the hypothetical decomposition matrix displayed in Table 7. The second table below gives the degrees of the irreducible Brauer characters.

Char.	Degree
ϕ_1'	1
ϕ_2'	406
ϕ_3'	21750
ϕ_4'	34944
ϕ_5'	45094
ϕ_6'	31030
ϕ_7'	17836
ϕ_8'	75400
ϕ_9'	8440

Consider $\Phi_9' = 102400 + 118748$, the PIM corresponding to the irreducible Brauer character of degree 8440. As Brauer character, it is $\Phi_9' = 17836 + 34944 + 45094 + 31030 + 75400 + 2 \times 8440$. Since every simple module in the principal block is self-dual, this implies that the radical modulo the socle of the projective indecomposable module corresponding to Φ_9' is semisimple. It therefore has five direct summands, contradicting a result of Webb (see [1, 2.31.5]).

Thus X is indecomposable. Now X is self-dual and has a 2-dimensional endomorphism ring. Again, since every irreducible module in the block is self-dual, it follows that the head and the socle of X are simple and isomorphic. By the projective characters we now know that neither 21750 nor 75400 has repeated composition factors. Therefore, 21750 and 75400 have a common composition factor. This implies that there is a PIM containing 21750 and 75400, and so Φ_3 is indecomposable (since there is just one possible way Φ_3 can split into a sum of two projectives).

Suppose now that $\Phi_4 = \Phi_4' + \Phi_4''$ is the sum of two projectives. Then, as is easily checked by looking at the character degrees modulo 9, each of Φ_4' and Φ_4'' is a PIM and we may assume that $\Phi_4'' = 102400 + 118748$. But then $\Phi_3 = \Phi_4' + \Phi_9$, which is absurd since the projective indecomposable characters are linearly independent. Thus Φ_4 is indecomposable.

We have now found 8 out of 9 PIM's. Hence, either Φ_7 is indecomposable or it contains Φ_9 as a component. This is equivalent to saying that 75400 either has 3 or 2 composition factors.

To decide this question, we again consider the maximal subgroup $M_3 = (2^2 \times \mathrm{Sz}(8)) : 3$. The principal block consists of the three 1-dimensional characters 1, ω and $\bar{\omega}$ corresponding to the factor group of order 3. In the principal block of 2Ru we have

$$\mathrm{Ind}_{M_3}(1) = \chi_1 + \chi_{15} + \chi_{16} + \chi_{21} + \chi_{23} + \chi_{25} + \chi_{32}$$

and

$$\mathrm{Ind}_{M_3}(\omega) = \mathrm{Ind}_{M_3}(\bar{\omega}) = \chi_{23} + \chi_{26} + \chi_{33} + \chi_{36}.$$

This shows that the restriction of $\chi_9 = 21750$ to M_3 has no component in the principal block. Then the same is true for each modular constituent of 21750.

Suppose for a contradiction that $\chi_{25} = 75400$ has just 2 modular constituents. Let ϕ be that one which is not in 21750. Then the restriction of ϕ to the principal block of M_3 is the trivial character.

Then, by Frobenius reciprocity, ϕ is contained in the head and in the socle of $\mathrm{Ind}_{M_3}(1)$. On the other hand, by the decomposition numbers we now know that the multiplicity of ϕ as a composition factor of $\mathrm{Ind}_{M_3}(1)$ is just 1. This shows that ϕ is a direct summand of this permutation module. But then ϕ is liftable to an ordinary character, as is well known. This contradiction shows that 75400 has 3 composition factors and hence completes the proof.

Appendix. Induced characters and tensor products

The first table gives the decomposition of some induced characters and some tensor products.

Θ:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	.	.	.	.	.	.	.	1	.	.	.	1	.	.	.	.	.	.	.
378	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.
378	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
406	1	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.
783	.	.	.	.	.	1	.	.	.	.	1	1	.	.	.	.	.	.	.
3276	1	1	1	.	.	2	.	1	.	.	.	.	.	.	.	.	.	.	.
3654	1	.	.	.	.	1	.	.	.	.	.	.	.	.	1	.	.	.	.
20475	.	.	.	.	.	.	1	1	.	.	.	.	.	.	1	.	.	.	.
21750	.	.	1	.	1	1	.	.	.	.	.	.	.	1	1	.	.	1	2
23751	1	1	1	.	.	2	1	.	1	1	.	.	.	.	1	.	.	.	.
27000	1	1	1	1	.	3	.	.	.	.	1	.	.	.	1	.	.	.	.
27000	1	1	1	1	.	3	.	.	.	.	1	.	.	.	1	.	.	.	.
27000	1	1	1	1	.	3	.	.	.	.	1	.	.	.	1	.	.	.	.
27405	1	2	1	1	.	4	.	1	1	1	1	.	.	.	.	.	1	.	.
34944	1	1	1	1	.	3	1	1	.	.	1	.	.	.	1	.	.	.	1
34944	1	1	1	1	.	3	1	1	.	.	1	.	.	.	1	.	.	.	1
43848	1	1	1	1	.	2	2	.	.	.	1	.	.	.	1	.	.	.	.
43848	1	1	1	1	.	2	2	.	.	.	1	.	.	.	1	.	.	.	.
43848	1	1	1	1	.	2	2	.	.	.	1	.	.	.	1	.	.	.	.
45500	2	.	2	1	.	3	1	.	.	.	.	.	1	1	1	1	.	.	.
52780	.	.	1	1	.	1	1	1	.	.	.	.	.	2	1	.	1	2	3
63336	2	1	2	1	.	3	3	.	.	.	.	.	1	2	2	1	.	2	3
65975	1	1	1	2	.	3	2	1	1	1	1	1	.	1	1	.	1	1	2
71253	.	.	1	1	.	1	2	.	.	.	1	.	.	3	.	1	1	2	3
75400	.	.	1	1	1	.	3	1	.	.	.	.	.	4	1	1	1	3	5
76125	1	.	2	2	.	3	2	.	1	1	.	1	.	2	1	1	1	1	1
81432	1	1	1	2	.	3	3	.	.	.	2	.	.	3	1	1	1	2	3
91350	.	.	2	1	1	1	4	.	.	.	.	.	1	3	1	1	1	2	3
95004	1	.	2	1	.	2	4	.	.	.	.	.	1	3	2	1	1	2	3
98280	.	1	1	2	.	3	3	.	.	.	1	.	.	3	2	1	1	3	6
98280	.	1	1	2	.	3	3	.	.	.	1	.	.	3	2	1	1	4	6
102400	1	1	2	2	.	4	4	1	.	.	.	.	.	4	2	1	1	3	5
105560	1	2	2	2	.	4	4	.	1	1	1	.	.	3	3	.	1	4	7
110592	1	1	2	2	.	4	4	.	.	.	1	.	.	5	2	1	1	3	6
110592	1	1	2	2	.	4	4	.	.	.	1	.	.	4	2	1	1	3	6
118784	1	1	2	2	.	4	4	.	1	1	1	.	.	5	2	1	1	3	7

The next table gives the tensor products which decompose entirely in faithful characters.

Θ:	20	21	22	23	24	25	26	27	28	29
28	1	.	.	.	.	.	.	.	.	.
28	.	1	.	.	.	.	.	.	.	.
1248	.	.	1	.	.	.	.	.	.	.
1248	.	.	1	.	.	.	.	.	.	1
3276	.	.	.	.	.	.	.	.	.	.
3276	.	.	.	.	.	.	.	.	.	.
4032	.	.	.	.	.	.	.	1	.	.
4032	.	.	.	.	.	.	.	.	1	.
7280	.	.	1	1	.	.	.	.	.	.
7280	.	.	1	.	1	.	.	.	.	.
7308	.	.	.	.	.	.	.	1	.	.
7308	.	.	.	.	.	.	.	.	1	.
8192	.	.	1	.	.	1	.	.	.	.
8192	.	.	1	.	.	.	1	.	.	.
10556	1	.	.	.	.	.	.	1	.	.
10556	.	1	1	.	.	.	.	.	1	.
34944	.	.	1	.	.	1	1	1	1	.
38976	.	.	2	1	1	1	1	1	.	.
38976	.	.	3	1	1	1	1	.	1	.
48256	.	.	2	1	1	1	1	1	1	.
87696	.	1	6	1	1	2	2	1	1	1
98280	.	1	8	2	2	2	2	1	.	1
98280	.	1	9	2	2	2	2	.	1	1
221184	.	2	17	4	4	5	5	2	2	2
250560	.	2	15	5	5	5	5	3	3	1

The following tables document the origin of the projectives, where $\Phi = \chi_{15} + \chi_{16} + \chi_{23} + \chi_{33} + \chi_{36}$ and $\Phi_{i,4}$ denotes the projective indecomposable character Φ_i of Block 4.

Char.	Origin
Θ_1 :	$\mathrm{Ind}_{M_1}(\theta_{14})$
Θ_2 :	$\mathrm{Ind}_{M_1}(\theta_{15})$
Θ_3 :	$\mathrm{Ind}_{M_1}(\theta_{23})$
Θ_4 :	$\mathrm{Ind}_{M_1}(\theta_{24})$
Θ_5 :	$\mathrm{Ind}_{M_2}(\theta_2)$
Θ_6 :	$\mathrm{Ind}_{M_2}(\theta_{12})$
Θ_7 :	$\mathrm{Ind}_{M_2}(\theta_{13})$
Θ_8 :	$\mathrm{Ind}_{M_3}(\theta_1)$
Θ_9 :	$\mathrm{Ind}_{M_3}(\theta_2)$
Θ_{10} :	$\mathrm{Ind}_{M_3}(\theta_3)$
Θ_{11} :	$\mathrm{Ind}_{M_3}(\theta_4)$
Θ_{12} :	$\chi_3 \otimes \chi_2$
Θ_{13} :	$\chi_5 \otimes \chi_2$
Θ_{14} :	$\chi_{37} \otimes \Phi_{7,4}$
Θ_{15} :	$\chi_{37} \otimes \chi_{58}$

Char.	Origin
Θ_{16} :	$\chi_6 \otimes \chi_2$
Θ_{17} :	$\chi_7 \otimes \chi_2$
Θ_{18} :	$\chi_{37} \otimes \Phi_{5,4}$
Θ_{19} :	$\chi_{37} \otimes \Phi_{3,4}$
Θ_{20} :	$\chi_{38} \otimes \chi_2$
Θ_{21} :	$\chi_{41} \otimes \chi_2$
Θ_{22} :	$\chi_{38} \otimes \Phi$
Θ_{23} :	$\chi_{46} \otimes \chi_2$
Θ_{24} :	$\chi_{45} \otimes \chi_2$
Θ_{25} :	$\chi_{49} \otimes \chi_2$
Θ_{26} :	$\chi_{50} \otimes \chi_2$
Θ_{27} :	$\chi_{44} \otimes \chi_2$
Θ_{28} :	$\chi_{43} \otimes \chi_2$
Θ_{29} :	$\chi_{37} \otimes \chi_{15}$

BIBLIOGRAPHY

1. D. Benson, *Modular representation theory: New trends and methods*, Lecture Notes in Math., vol. 1081, Springer-Verlag, 1984.
2. J. H. Conway, R. T. Curtis, S. P. Norton, R. A. Parker, and R. A. Wilson, *Atlas of finite groups*, Oxford Univ. Press, London, 1974.
3. J. Neubüser, H. Pahlings, and W. Plesken, *CAS*; *Design and use of a system for the handling of characters of finite groups*, Computational Group Theory, Academic Press, 1984, pp. 195–247.

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