The book is divided into four parts. In the first part the author reviews (without proofs) the basic theory of univariate and bivariate splines, with special emphasis on computational aspects. Part 2 is devoted to curve fitting. This includes methods for least squares fitting with both fixed and free knots, smoothing splines with and without end-point constraints, periodic smoothing, and shape-preserving approximation (with emphasis on convexity).

The fitting of surfaces using tensor-product splines is the subject of Part 3. This part includes methods for fitting both gridded and scattered data using least squares and smoothing criteria. The questions of how to choose knots, and how to deal with large data sets and with incomplete data are also addressed. In addition, the author discusses methods for data on a unit disk, for data on the surface of a sphere, and for reconstructing surfaces from planar contours.

Part 4 of the book explains the organization of FITPACK and the main parameters in the various routines. Numerical results are presented to illustrate the performance of the methods, often on real-life problems. The book contains around 50 figures, and about 150 references. The reader should be aware that it is not intended to cover all applications of splines, and that many aspects of curve and surface fitting are not treated here.

I found the book to be well written in a clear and understandable fashion. It should be useful to a wide audience of potential users of splines.

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23[49M30, 65K10].—ELDON HANSEN, *Global Optimization Using Interval Analysis*, Monographs and Textbooks in Pure and Appl. Math., Vol. 165, Dekker, New York, 1992, xvi + 230 pp., $23\frac{1}{2}$ cm. Price \$110.00.

The book under review presents techniques for the solution of global optimization problems (with several suspected local minima) and of nonlinear systems of equations with several solutions.

The systematic search for methods which solve the global optimization problem probably began in 1975 with Dixon and Szegö [4]. Ten years ago, the general consensus of researchers in the optimization community was that the complete solution of smooth, nonconvex global optimization problems was beyond tractability. (For the related problem of finding all zeros of nonlinear equations, this was explicitly expressed, e.g., in §2.1 of the book by Dennis and Schnabel [3].)

Therefore, research on global optimization has concentrated on heuristic or stochastic methods for finding good local (and hopefully global) minima, and some of these methods could be proved to produce a global solution with probability $1 - \varepsilon$ in finite time, growing rapidly with decreasing ε. See, e.g., Kirkpatrick et al. [7], Levy et al. [8, 9], Byrd et al. [2], and the recent book by Törn and Žilinskas [13].

The situation turned out to be somewhat better for nonlinear systems, where homotopy methods were quite successful in locating several zeros, and for 'generic' polynomial systems it could even be proved that all zeros are found. Excellent treatments of homotopy methods can be found in Zangwill and Garcia [14] and Allgower and Georg [1]. However, for general systems, homotopy methods may fail, too.

In the meantime, two powerful approaches for solving global optimization problems have been developed, which are deterministic in the sense that (assuming exact arithmetic and enough storage) they are guaranteed not to miss any global minimizer. One approach is based on dc-functions, i.e., differences of convex functions, and exploits convexity properties; see the recent books by Horst and Tuy [6] and Pardalos and Rosen [11]. The latter book reports on the successful solution of global quadratic programs with up to 330 variables and 50 constraints.

The other approach is based on interval analysis, and was pioneered by the author of the present book [5]. Theory and algorithms for solving nonlinear systems of equations with several solutions are treated in the reviewer's book [10], and large-scale results for chemical engineering problems in dimensions 18 to 177 were recently reported by Schnepper and Stadtherr [12]. Applications of interval analysis to global approximation so far appeared only in journals and conference proceedings; the book under review is therefore a welcome addition to the literature.

Since interval methods are hardly known in the optimization community, the first part of the book gives a thorough introduction to interval analysis. The main advantage of interval techniques lies in their capacity to provide global information about functions over large regions (box-shaped), e.g., strict bounds on function values, Lipschitz constants, higher derivatives, and thus allows one to estimate the effects of using linear or quadratic approximations to a function. While the bounds are sometimes very wide, they can be proved to be always realistic when the boxes are narrow enough. Therefore, in the applications, interval techniques are combined with branch and bound techniques, which split boxes adaptively until the overestimation problems become insignificant.

The techniques developed are then applied, in turn, to the solution of nonlinear systems, global unconstrained optimization, and global constrained optimization, with an increase in complexity in the three problems. In order that the methods are efficient, Jacobians of nonlinear systems, and hence Hessians for optimization problems, must be provided explicitly (though the methods also work, slowly, with first-order or even zeroth-order information only). Ultimately, everything is reduced to the solution of systems of linear interval equations. For realistic bounds on the solutions of these systems, it is necessary to precondition the system by an explicit inverse matrix, which limits the dimension of the problems which can be solved to several hundreds.

Everything is well motivated, and many small examples are provided which can be checked with pencil and paper. For proofs of more difficult theoretical results references are given to the literature. The algorithmic aspect is treated in detail, so that it is fairly easy to write programs based on the descriptions given. It is emphasized that when the interval arithmetic is implemented with outward directed rounding, then the results of the algorithms are mathematically correct in spite of the rounding errors made by the computer, a fact which might be relevant in some applications.

The test problems discussed by the author have dimension ≤ 10 only, apart from a quadratic problem and a single example in 27 dimensions; and the constrained problems treated have only toy dimensions (the constrained Example 13.9, with 18 dimensions, is reduced to 5 dimensions, unconstrained). This is annoying, since it gives a wrong impression on the range of problems solvable by interval methods. The presentation of the test results is also rather careless; for some of the problems in §9.15, the initial box is not specified, and on p. 148 it is admitted that the results were obtained by an old implementation less efficient than later versions (which probably incorporate all tricks mentioned in the chapter).

It is surprising that hardly any use is made of classical optimization theory or algorithms; the only tools from optimization used are the Fritz John first-order optimality conditions. This is an indication that one can expect much further improvements by combining the methods treated with more of the traditional methods, and in particular with dc-methods. The future of the subject, once firmly rooted in the established tradition, looks very bright.

Altogether, I think that this book is an excellent introduction to interval techniques for the solution of global optimization problems and of nonlinear systems of equations with several solutions, though it takes only the first step towards good general-purpose global optimization software.

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1. E. L. Allgower and K. Georg, *Numerical continuation methods: an introduction*, Springer, Berlin, 1990.

2. R. H. Byrd, C. L. Dert, A. H. G. Rinnooy Kan, and R. B. Schnabel, *Concurrent stochastic methods for global optimization*, Math. Programming **46** (1990), 1–29.

3. J. E. Dennis and R. B. Schnabel, *Numerical methods for unconstrained optimization and nonlinear equations*, Prentice-Hall, Englewood Cliffs, NJ, 1983.

4. L. C. W. Dixon and G. P. Szegö, *Towards global optimization*, Elsevier, New York, 1975.

5. E. R. Hansen, *Global optimization using interval analysis—the multidimensional case*, Numer. Math. **34** (1980), 247–270.

6. R. Horst and H. Tuy, *Global optimization: deterministic approaches*, Springer, Berlin, 1990.

7. S. Kirkpatrick, C. D. Gelatt, and M. P. Vecchi, *Optimization by simulated annealing*, Science **220** (1983), 671–680.

8. A. V. Levy and S. Gomez, *The tunneling method applied to global optimization*, Numerical Optimization 1984 (P. T. Boggs et al., eds.), SIAM Publications, Philadelphia, PA, 1984, pp. 213–244.

9. A. V. Levy and A. Montalvo, *The tunneling algorithm for the global minimization of functions*, SIAM J. Sci. Statist. Comput. **6** (1985), 15–29.

10. A. Neumaier, *Interval methods for systems of equations*, Cambridge Univ. Press, Cambridge, 1990.

11. P. M. Pardalos and J. B. Rosen, *Constrained global optimization: algorithms and applications*, Lecture Notes in Comput. Sci., vol. 268, Springer, Berlin, 1987.

12. C. A. Schnepper and M. A. Stadtherr, *Application of a parallel interval Newton/generalized bisection algorithm to equation-based chemical process flowsheeting*, Lecture at the conference *Numerical Analysis with Automatic Result Verification*, Lafayette, LA, Feb. 1993.

13. A. Törn and A. Žilinskas, *Global optimization*, Lecture Notes in Comput. Sci., vol. 350, Springer, Berlin, 1989.

14. W. I. Zangwill and C. B. Garcia, *Pathways to solutions, fixed points, and equilibria*, Prentice-Hall, Englewood Cliffs, NJ, 1981.

24[90B05, 90C35].—James R. Evans & Edward Minieka, *Optimization Algorithms for Networks and Graphs*, 2nd ed., Dekker, New York, 1992, x + 470 pp., $23\frac{1}{2}$ cm. Price \$59.75.

This book gives a treatment of network optimization that manages to be both inviting and mathematically rigorous. Though network optimization courses vary according to individual interests and preference, the topics covered in this book are likely close to the right ones for the majority of the courses of that title taught in mathematics, engineering, and possibly even computer science departments:

- Basic Graph Theory
- Basic Data Structures and Complexity
- Minimum Spanning Trees and Branchings
- Shortest Paths
- Minimum-Cost Flows
- Maximum Flows
- Matchings (including maximum-weight matchings in arbitrary graphs)
- Chinese Postman
- Traveling Salesman Problem
- Location
- PERT

A major strength of the book is its gentle approach, making it very readable. Each chapter begins with several motivating applications handled by the models in the chapter. The algorithms are stated nicely and rigorously proved correct. Details of the algorithms applied to examples are included. The exercises include algorithm execution, theory, applications, with emphasis on the first of the three.

A second strength of the book is the accompanying software package, for use on DOS machines. I know of none more extensive for use in such a course. The software is easy to use and well documented. Unfortunately, it provides no graphics.

The book is appropriate for an introductory course at the undergraduate or even the graduate level. It should not be taken as a complete research reference, however. Treatment of network optimization advances of the last decade is scant. (One obvious omission is the mention of strongly polynomial methods for minimum-cost flows.)

The authors' preface includes the following apt description:

> ... The focus is on an intuitive approach to the inner workings, interdependencies, and applications of the algorithms. Their place in the hierarchy of advanced mathematics and the details of their computer coding are not stressed....