REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1991 Mathematics Subject Classification can be found in the annual subject index of *Mathematical Reviews* starting with the December 1990 issue.

26[41-02, 65-02, 65Dxx, 65Lxx, 65Mxx, 65Rxx, 41A80].—FRANK STENGER, *Numerical Methods Based on Sinc and Analytic Functions,* Springer Series in Computational Mathematics, Vol. 20, Springer-Verlag, New York, 1993, xvi+565 pp., 24 cm. Price $69.00.

With the Sinc function defined by

$$S(k, h)(x) := \frac{\sin[\pi(x - kh)/h]}{\pi(x - kh)/h},$$

where h is a positive number and k an integer, the *Cardinal function* or *Cardinal series* of a function f bounded on $\mathbb{R}$ is given by

$$C(f, h)(x) := \sum_{k=-\infty}^{\infty} f(kh)S(k, h)(x).$$

This is an entire function of exponential type that reproduces the values of f at the points kh $(k = 0, \pm 1, \pm 2, \dots)$. In other words, $C(f, h)$ is a Lagrange interpolant of f in the space of entire functions of exponential type.

Remarkable work on the Cardinal function is due to Whittaker, 1915. Later, the Cardinal function was rediscovered by electrical engineers, who have used it for the reconstruction of bandlimited signals from samples. When Frank Stenger turned to this subject some 23 years ago, he did not restrict himself to this most natural application. Manipulating the Cardinal function in various ways, he made connection with numerous problems in numerical analysis. In the meantime, his *Sinc methods* have gained a field of applications about as rich as that of polynomials, trigonometric functions, splines, and rational functions.

In essence, Sinc methods consist in the approximation of a function holomorphic in a strip by interpolating entire functions of exponential type (namely the associated Cardinal functions) and methods which arise by transforming the whole situation from the strip to other simply connected domains by a conformal mapping. These methods excel for problems on the whole real line or on semi-infinite and finite intervals having singularities at the endpoints. A typical rate of convergence of a Sinc approximation with a truncated Cardinal series of n terms is $O(e^{-cn^{1/2}})$. Scattered in numerous papers and reports, details of these facts have been known only to specialists so far. It is therefore a great

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service to the mathematical community to present a comprehensive treatise on Sinc methods.

The first chapter contains the theoretical background needed for a thorough mathematical analysis of Sinc methods. Apart from elementary analytic function theory, it introduces to special topics such as Fourier, Laplace and Hilbert transforms, Riemann-Hilbert problems, Fourier series, conformal mapping, spaces of analytic functions, Paley-Wiener theory and, of course, to the Cardinal function.

The second chapter presents some selected material on interpolation and approximation by polynomials, with sections on Chebyshev polynomials, discrete Fourier polynomials, Lagrange interpolation polynomials, and Faber polynomials.

After these preliminaries, Sinc methods with their various aspects and applications are developed in Chapters 3 to 7.

Chapter 3 deals with functions holomorphic in a strip. They are the most natural objects of Sinc methods since in this situation the error $f - C(f, h)$ can be represented by a contour integral. Simple error bounds can be deduced, which are sharp in appropriately defined normed linear spaces. Moreover, integration or differentiation of the Cardinal series, and truncation, leads to efficient algorithms for the approximation of definite and indefinite integrals on $\mathbb{R}$, or derivatives. In addition, approximations of Fourier transforms by Fourier series are obtained.

In Chapter 4, analogous results are deduced for functions holomorphic in a simply connected domain $\mathscr{D}$ by employing the transformed Cardinal function $C(f, h) \circ \phi$, where ϕ is a conformal mapping from $\mathscr{D}$ onto a strip. Now problems on finite and semi-infinite intervals, or on an arc Γ, become accessible. At least for all types of intervals, an appropriate ϕ and its inverse can be explicitly given in terms of simple elementary functions. Mapped by ϕ^{-1}, the originally equidistant points kh on $\mathbb{R}$ lead to nodes on an interval or on Γ, with a special distribution which proves to be powerful even in certain cases of singularities at the endpoints.

Chapter 5, which the author himself terms *the unfinished chapter*, presents various special approximation methods related to Sinc methods. As examples, we mention interpolation and approximation by elliptic functions, and rational approximations to the characteristic, the Heaviside, and the delta function.

Chapter 6 discusses various ways to treat integral equations by Sinc methods. For example, explicit integral representations of a solution may be evaluated approximately by employing the Sinc formulae for indefinite integrals. Sinc functions may be used as basis functions in Galerkin's method or for collocation. Alternatively, a discretization of the integral equation may be achieved by using the quadrature formulae from Chapter 4. Again, certain singularities of the kernels are admissible.

The seventh and last chapter presents the more recent work of Frank Stenger and his students, namely the application of Sinc methods to the approximate solution of ordinary and partial differential equations for initial as well as boundary value problems. Standard methods, such as Galerkin, finite element, spectral, and collocation methods are developed for Sinc functions, and error bounds are given. Examples include the Poisson, heat, and wave equation.

The book is well written. It uses the notions of modern analysis but gives always a brief introduction so that a reader not so familiar with these concepts should not feel lost. In particular, the modern theory of integral equations is nicely outlined. Each section ends with a long list of problems. Their purpose is often to complete details skipped in previous proofs. By this arrangement, the leading ideas become more conspicuous in the main text.

Sinc methods have the potential to become the method of choice for many. The book can therefore be warmly recommended to scientists and engineers. It can also be used for advanced courses in numerical analysis. Even researchers may find the book stimulating since there is still enough room for further developments.

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27[68-02, 68-04, 65G10, 65D10, 65Y10, 65Y15, 68M15, 68Q10].—E. Adams & U. Kulisch (Editors), *Scientific Computing with Automatic Result Verification*, Mathematics in Science and Engineering, Vol. 189, Academic Press, Boston, 1993, x+612 pp., $23\frac{1}{2}$ cm. Price $59.95.

This text consists of a collection of recent papers on development and application of numerical algorithms with automatic result verification. The majority of the papers represent selected material taken from doctoral theses which were written at the Institute for Applied Mathematics at the University of Karlsruhe. The following three areas are presented:

1. The development of computer languages and programming environments that support automatic result verification in scientific computation;
2. The corresponding software for differentiation or integration problems, or for differential and integral equation problems; and
3. Specific examples of applications in the engineering sciences.

The book has a table of contents, a preface, and a well-written introduction that helps the reader to better understand other parts of the book. It has an excellent bibliography of publications on computations with result verifications, and finally, it also has a helpful index.

The papers are subdivided under three chapter subdivisions.

I. *Language and Programming Support for Verified Scientific Computation.* This chapter consists of papers describing existing languages and programming environments that support result verification: PASCAL-XSC, ACRITH-XSC (a Fortran-like language), and C-XSC (a C + + language), and it also proposes methods for accurate floating-point vector arithmetic.

II. *Enclosure Methods and Algorithms with Automatic Result Verification.* In this chapter, correct algorithmic execution procedures are presented for automatic differentiation, numerical quadrature by extrapolation, numerical integration in two dimensions, numerical solution of linear