and nonlinear integral equations, numerical solution of ordinary differential equation initial value problems, and interval arithmetic in staggered correction format.

III. *Applications in Engineering Sciences.* In this chapter one finds various examples of computations with result verification. Included are multiple-precision computations, asymptotic stability computations with applications to control theory, magneto-hydrodynamic flow calculations, computations of discretizations of evolution problems, scattering calculations using the KKR (Koringa, Kohn, and Rostaker) method, and a description of a hardware floating-point unit which extends the standard (scalar) IEEE procedure, for performing vectorized scientific and engineering calculations.

In the reviewer's view, this text is a worthwhile endeavor, especially for the world of parallel and vector computation, for which automatic error control is absolutely essential.

F.S.

28[00A69, 34A55, 45B05, 65R30].—CHARLES W. GROETSCH, *Inverse Problems in the Mathematical Sciences*, Vieweg, Braunschweig/Wiesbaden, 1993, vi+152 pp., 23 cm. Price $30.00.

When we study the sciences, we generally learn the mathematical models that predict the outcome of the experiments. The practice of science, however, frequently proceeds in the opposite direction: given the experimental results, what is the mathematical model? At least in the cases where the general form of the model is known, these problems are called inverse problems. Because of their importance in the practice of science, inverse problems deserve a more prominent place in the curriculum. And now, a book on inverse problems has been written that can even be used as an undergraduate text!

As is appropriate for this level, the recent book by Charles Groetsch deals with one-dimensional inverse problems. It begins (Chapter 2) with a wealth of examples, all physically motivated, that involve first-kind integral equations. These linear examples, many of which are explicitly solvable, illustrate the ill-posedness typical of inverse problems, and motivate the discussion of first- and second-kind integral equations at the end of the chapter.

The third chapter gives examples of inverse problems that involve ordinary differential equations. Again, each problem is physically motivated. Many are nonlinear and none have explicit solutions. For approaches to solving them, the reader must wait until Chapter 5. First, however, comes Chapter 4, which summarizes the necessary functional analysis. Undergraduates may find this chapter rather difficult, but more knowledgeable readers will find it a useful compilation of generally familiar material.

The climax of the book is Chapter 5, which explains eight practical techniques that apply to a wide variety of inverse problems. Included are methods for dealing with ill-posedness and for incorporating prior information about the solution.

There are exercises interspersed throughout the text. At the end is a useful annotated bibliography.

This book is certainly suitable for its stated purpose, which is to introduce college faculty to inverse and ill-posed problems. Not only could it be used as a text for a course on inverse problems, but its many examples could also provide motivational material to be incorporated into other courses. Its reasonable price makes it suitable as a supplementary text. For scientists and engineers faced with inverse problems, the list of techniques in Chapter 5 and the annotated bibliography will both be useful.

MARGARET CHENEY

Department of Mathematical Sciences
Rensselaer Polytechnic Institute
Troy, NY 12180

29[65-02, 65D07, 65D17].—RONALD N. GOLDMAN & TOM LYCHE (Editors), *Knot Insertion and Deletion Algorithms for B-Spline Curves and Surfaces*, Geometric Design Publications, SIAM, Philadelphia, PA, 1993, xiv+197 pp., 25 cm. Price: Softcover $43.50.

This book contains three chapters by Barry and Goldman, two chapters by Lyche and Mørken (one assisted by Strøm), and a final chapter by Banks, Cohen, and Mueller. As this indicates, the book serves largely as a forum for the work of the editors. However, since the most intense existing studies of knot insertion and removal have been made by Lyche and Goldman, a book with this title could not be anything but a forum for the work of these two.

This book is concerned with splines as linear combinations of basis functions, which in turn are composed of piecewise polynomials satisfying certain continuities at the joints. The knot insertion operation derives a containing linear space composed of polynomials having more pieces (and/or relaxed continuities), and explicitly provides the basis conversion operation from the original space to its containing space. The knot deletion operation, conversely, provides the projection from a given space into a subspace having fewer pieces (and/or more strict continuities). Knot insertion can be used to increase the degrees of freedom in spline approximation problems, to change representations from one spline basis to another, and as a means of evaluating splines. Knot deletion has been used as an efficient way of approximating data by beginning with an interpolating spline and passing to a smoothly approximating spline.

The theoretical tools applied by the two editor/authors have been quite different. The chapters contributed by Lyche and Mørken, one on knot deletion and one on how knot insertion influences the size of B-spline coefficients, are based upon discrete splines, the matrices that embody knot insertion, and on least squares problems derived from these matrices. This material is oriented toward the use of B-splines as basis splines. The chapters by Barry and Goldman use multiaffine and multilinear polar forms ("blossoms"). A beginning chapter by Barry provides an overview of the basics of blossoms, but each of the other chapters reproduces the basics again for its own purposes. In all, the blossom material is still too brief (being mainly definitions and statements of a few results) for those who have not encountered the concept before. The references to Ramshaw and Seidel at the end of the first chapter constitute a necessary background. The material by Barry and Goldman covers a wide class of spline bases, some of which also serve as familiar polynomial bases. The class constitutes