

wavelets, especially the modification due to Coifman and Meyer that allows the wavelets to have windows of variable lengths. An adaptive algorithm for finding the optimal Malvar basis is then described. Chapter 7 concentrates on wavelet packets and splitting algorithms. These algorithms are useful in choosing an optimal basis formed by wavelet packets. Borrowing the words of Ville (1947), the author emphasizes the following points in Chapters 6 and 7: In the approach of Malvar's wavelets, we "cut the signal into slices (in time) with a switch; then pass these different slices through a system of filters to analyze them." In the approach of wavelet packets, we "first filter different frequency bands; then cut these bands into slices (in time) to study their energy variations."

The last four chapters introduce some fascinating and promising applications of wavelets. The first of these is Marr's analysis of the processing of luminous information by retinal cells. In particular, Marr's conjecture and a more precise version of it due to Mallat are discussed. Marr's conjecture concerns the reconstruction of a two-dimensional image from zero-crossings of a function obtained by properly filtering the image. In Chapter 9 it is shown that, for some signals, wavelet analysis can reveal a multifractal structure that is not disclosed by Fourier analysis. To this end, two famous examples, the Weierstrass and Riemann functions (which show that a continuous function need not have a derivative anywhere), are examined from the viewpoint of wavelet theory. Chapters 10 and 11 describe how wavelets can shed new light on the multifractal structure of turbulence and on the hierarchical organization of distant galaxies.

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31[41-01, 41A15, 41A63, 68U07].—J. J. RISLER, *Mathematical Methods for CAD*, Cambridge Univ. Press, Cambridge, 1992, 196 pp., 23½ cm. Price \$69.95.

This is a welcome addition to the literature, providing in very readable and efficient form the mathematical underpinnings of Computer-Aided-Design (CAD). A summary of the chapters follows.

Chapter 1 concerns the B-splines, including the case of multiple knots. Variation-diminishing properties, the Schoenberg operator, and the Hermite-Genocchi formula are all discussed. In Chapter 2, Bernstein polynomials and Bézier curves are introduced; they are formulated in terms of B-splines. Knot insertion and subdivision algorithms (such as the "Oslo" algorithm) are given. Chapter 3 is devoted to interpolation of functions from \mathbb{R} to \mathbb{R}^s , in other words, finding curves passing through specified points in \mathbb{R}^s . This task is performed with B-splines and rational spline curves. In Chapter 4, surfaces make their appearance, approximated first by tensor products of splines. Here are Coon's patches and Boolean sums of operators. Triangular patches are then taken up at length. The B-splines are generalized to s dimensions as "polyhedral splines". Box splines are a special case. Chapter 5 is on triangulations and algorithms for obtaining them, such as the Voronoi-Delaunay method. Optimality and complexity questions are addressed. Much of the discussion is valid for arbitrary

dimensions. In Chapter 6, some notions from algebraic geometry are discussed, such as Sturm sequences, resultants, and discriminants.

Most of the results are proved in full, and a praiseworthy effort has been made to avoid excessive generality and complexity. The bibliography is much too brief, however. Furthermore, the publisher is to be seriously faulted for poor typesetting, copyediting, and proofreading. Some Gallicisms should have been excised in the editorial process, for example, “inferior” and “superior” triangular matrices and “application” (for “mapping”).

E. W. C.

32[65-02, 65D17, 68U05].—J.-C. FIOROT & P. JEANNIN, *Rational Curves and Surfaces: Applications to CAD* (translated from the French by M. C. Harrison), Wiley, Chichester, 1992, xiv+322 pp., 23½ cm. Price \$69.95.

If a curve (similarly for surface) is written in Bernstein-Bézier (BB) form

$$p(t) = \sum_{i=0}^n P_i B_i^n(t),$$

then the position and shape of the curve (surface) is completely controlled by the vectors P_i (called control points). When a rational curve (similarly for surface) is written in BB form

$$p(t) = \frac{\sum_i \beta_i P_i B_i^n(t)}{\sum_i \beta_i B_i^n(t)},$$

then the curve (surface) can be controlled by the vectors (P_i, β_i) (called weighted control points) if $\beta_i \neq 0$. However, if some $\beta_i = 0$, then controlling the curve geometrically is a problem as some P_i become infinite.

This book presents a method for describing rational curves as well as surfaces based on projective geometry. The rational curve or surface are successfully controlled and determined by “massic vectors”, a concept introduced by the authors. The “massic vector” is a weighted control point (P_i, β_i) if $\beta_i \neq 0$ or \bar{U}_i if $\beta_i = 0$. By using the “massic vectors” the rational curve above can be rewritten as

$$p(t) = \frac{\sum_{i \in I} \beta_i B_i^n(t) P_i}{\sum_{i \in I} \beta_i B_i^n(t)} + \frac{\sum_{i \in \bar{I}} B_i^n(t) \bar{U}_i}{\sum_{i \in \bar{I}} \beta_i B_i^n(t)},$$

where $I \cup \bar{I} = \{0, 1, \dots, n\}$ and $\beta_i \neq 0$ for $i \in I$. Now “massic vectors” are $\{(P_i, \beta_i)\}_{i \in I} \cup \{\bar{U}_i\}_{i \in \bar{I}}$. By using the “massic vectors”, the simplicity of several algorithms for polynomial BB forms are transferred to rational functions in BB form. Most importantly, the de Casteljau algorithm.

On the downside, one should mention that the projective geometric approach taken by the authors is overly complicated. There are far too many concepts and notations causing some propositions to become merely definitions. This complexity will definitely undermine the usage of the book, especially among students and engineering researchers in the areas of CAD, CAGD, and CAM. It should, however, be of keen interest to CAD mathematicians.