

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1991 Mathematics Subject Classification can be found in the annual subject index of *Mathematical Reviews* starting with the December 1990 issue.

1[00A22, 33-00, 44-00, 65-00].—I. S. GRADSHTEYN & I. M. RYZHIK, *Table of Integrals, Series, and Products*, 5th ed. (Alan Jeffrey, ed.) (translated from the Russian by Scripta Technica, Inc.), Academic Press, Boston, 1994, xlvii + 1204 pp., 24 cm. Price \$54.95.

This volume represents the fifth edition of a well-known work. The fourth edition was reviewed in [1], and an errata notice was published in [2]. Since then, further errata notices have been published in [3]–[6].

Although other comprehensive collections of formulas for integrals and series have been published in the meantime, the present volume contains such a wealth of information that a new edition was to be welcomed, provided it had been prepared with the care merited by its contents and by taking into account the remarks and criticisms made in the reviews of earlier editions. Unfortunately, this is often not the case.

As mentioned in the preface, the entire volume has been reset, presumably using a modern text-processing system on a computer, though nothing is said about this. Some formulas and a few short sections have been added, in particular for some special functions and in the tables for integral transforms, but not to the extent one would expect from reading the preface. Some references, in particular to tables of integral transforms, have been added. Nevertheless, a number of important references are missing.

The contents of this table is well known and has been described in earlier reviews. In particular, Chapters 10 to 17, which constitute somewhat foreign material in this volume, were criticized rather severely in [1]. Therefore, it seems more appropriate to concentrate on aspects of editing and presentation. (An errata notice will be given separately.)

There are quite a number of remarks and suggestions in previous reviews or errata notices which have been neglected or taken into account in a rather careless fashion in preparing this edition. To begin with, although the translation has been criticized as too literal, many verbose and clumsy sections remain unchanged. It is surprising to see that the term *degenerate* (qualified “ridiculous” in [1]) instead of *confluent* for this hypergeometric function has been changed only in §9.2, but not in §7.6 and other places. The Index of Special Functions (pp. xli–xliv) and the Notations (p. xlv) are presented with little care.

In the formula part of the table, errors given in the errata notices have been corrected, but in some cases new misprints have been introduced by doing this.

Suggested new expressions which were intended to simplify certain results have often not been considered at all, or only formally, disregarding the table environment. A minimum of care should have been shown in introducing new formulas, e.g., the letter θ instead of ϑ for denoting the theta functions in 8.199(1)–(3) is to be deplored. Having defined symbols like $[x]$, $(\alpha)_n$, etc., and the value of empty sums and products in the Notations (p. xlv) should have eliminated the need for footnotes or explanations later. The convention (p. 264) of not indicating principal values should have been applied throughout.

The short tables for the Laplace, Fourier, and Mellin transforms in Chapter 17 merit special criticism. Their heterogeneity in layout and notation would suggest careless and superficial editing. Although they are announced in the preface as having been revised, the number of misprints and errors in the tables for the Laplace and Fourier transforms is larger than would usually be considered acceptable in a mathematical reference work.

There are other points of principle. Since the appearance of the fourth edition, expressions for certain integrals, for example of rational functions and powers of logarithms, have been published. These formulas contain a number of parameters and unify simple special cases given in the volume, for instance in 4.23–27, 4.33–35. Other newly evaluated integrals are certainly scattered in the literature, and one would have hoped that some of these might have been incorporated.

Further, it is difficult to understand why curiosities like the Gudermannian $\text{gd}(x)$ or the Lobachevski $\Pi(x)$ are presented in detail, while the polylogarithm $\text{Li}_n(x)$ and its generalization $S_{n,p}(x)$ due to Nielsen, both defined by integrals, with many functional relations, related integrals, and of importance in modern physics, continue to be ignored.

More specifically, infinite series should be replaced by their values whenever this is possible. For example, several of the series in 3.411 and 4.261 can be expressed in terms of $\zeta(n)$, as it has been done in some cases. Obviously superfluous or divergent integrals (the latter with the possible exception of cases which are difficult to recognize as such) should be deleted, keeping an empty number if necessary. Useless cross-references, e.g., in 3.727 and 3.735, and trivial numerical values should be suppressed.

As a rule, definite integrals should be given only when the corresponding indefinite integral cannot be expressed in closed form. For example, all the integrals in 3.351 can be obtained from the indefinite integrals in 2.32. By adding in 2.3 the formula [7, equation 7.4.32] for $\int \exp[-(ax^2 + 2bx + c)] dx$, which is important in applications, and which is curiously missing, the presentation in 3.321–23 could be simplified.

The restrictions should be reconsidered in a number of sections. For uniformity, they should always be enclosed in square brackets, and symbols like \mathbb{R} , \mathbb{Z} , \mathbb{N}_+ , $\in \notin$, etc. should be used, as has been done on rare occasions.

Some remarks about the notation used in this volume seem appropriate. Although it has survived several editions, the function symbol $\bar{\text{Ei}}$ defined in 8.2113. is unnecessary. Its use creates confusion and leads to superfluous comments, in particular in 3.35. It should be replaced by Ei , on the understanding that principal values are not indicated. The letter N for the Bessel function of the second kind or Neumann function is rarely used in the English literature;

Y should be used instead. The Gaussian hypergeometric function ${}_2F_1$ should be written in this form or simply as F , but not in both ways even on the same page. For the probability integral $\Phi(x)$, the term "error function $\operatorname{erf} x$ " is more appropriate, especially as it already occurs in several places and the letter Φ is used for two other functions. In particular, by using the complementary error function $\operatorname{erfc} x = 1 - \Phi(x)$, a number of formulas can be simplified. The inconsistent notation for the theta functions in 6.16 and 8.18–19 should not have survived five editions. At the very least, the notation for elementary functions should be used consistently.

The excessive and arbitrary use of the possessive in function names is annoying. It is also unusual to extend the term Riemann zeta function to the generalized zeta function $\zeta(z, q)$.

In general, the formulas are given in a readable form; although, considering the ease with which typesetting can now be handled, a more skillful layout seems often possible. For example, a number of exponentials have an exponent that reaches down to the main line, e.g., in 7.386. This is unusual and unprofessional. There are also a number of typographical inconsistencies. Last but not least, a more economical, legible, and visually satisfying presentation of many formulas, illustrated by the simple example $\Gamma(\frac{1}{2} + \nu)$ instead of $\Gamma\left(\frac{1}{2} + \nu\right)$, should be adopted in a next, hopefully more carefully prepared, edition.

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1. Y. Luke, *Review* **5**, *Math. Comp.* **36** (1981), 310–312.
2. Errata, *Math. Comp.* **36** (1981), 312–314.
3. Table Errata **577**, *Math. Comp.* **36** (1981), 317–318.
4. Table Errata **589**, *Math. Comp.* **39** (1982), 747–757.
5. Table Errata **601**, *Math. Comp.* **41** (1983), 780–783.
6. Table Errata **607**, *Math. Comp.* **47** (1983), 768.
7. M. Abramowitz and I. A. Stegun, eds., *Handbook of mathematical functions with formulas, graphs, and mathematical tables*, 9th printing with corrections, Dover, New York, 1972.

2[60–02, 60H05, 60H10, 65C05, 81S40].—A. D. EGOROV, P. I. SOBOLEVSKY & L. A. YANOVICH, *Functional Integrals: Approximate Evaluation and Applications*, *Mathematics and Its Applications*, Vol. 249, Kluwer, Dordrecht, 1993, x + 418 pp., 24½ cm. Price \$172.00/Dfl.295.00.

There exists a large family of problems, deterministic or probabilistic in nature, which require the evaluation of an integral, with respect to the law of a stochastic process, of a functional on the space of the trajectories of the process. The book under review provides a good sampling of examples coming from physics, like Feynman integrals; some other less classical examples are: in