3[47–02, 47H05, 47H15, 47H17, 65–02, 65J15, 65J10, 46N40, 46N20].—WOLODYMYR V. PETRYSHYN, *Approximation-Solvability of Nonlinear Functional and Differential Equations*, Monographs and Textbooks in Pure and Appl. Math., Vol. 171, Dekker, New York, 1993, xii + 372 pp., $23\frac{1}{2}$ cm. Price \$115.00.

The purpose of this research monograph is to present a comprehensive theory of mappings of A-*proper* type and its application to differential equations in the context of constructive (abstract numerical) functional analysis. To put the book in perspective, it is useful to trace the roots of the subject, which evolved in response to the following question posed by the author 27 years ago (see, e.g., [5]): "For what type of linear and nonlinear mapping T is it possible to construct a solution to a given operator equation (1) $Tx = f$ as a strong limit of solutions x_n of finite-dimensional approximating equations (2) $T_n(x_n) = f_n$?" In a series of papers in the 1960s (see [7] for references and perspective summary) the author studied this problem, and the notion that evolved from these investigations in 1967 is that of an A-proper mapping [6].

Let X, Y be Banach spaces, D a subset of X and $T: D \subseteq X \to Y$ a possibly nonlinear mapping. Let $\{X_n\}$ and $\{E_n\}$ be sequences of oriented finite-dimensional spaces with $X_n \subset X$, V_n an injective map of X_n into X, and W_n a continuous linear map of Y onto E_n for each positive integer n. An *approximation scheme* $\Gamma = \{X_n, V_n; E_n, W_n\}$ for the equation (1) is said to be *admissible* provided that $\dim X_n = \dim E_n$ for all n, the distance between x and X_n tends to zero as $n \to \infty$ for each x in X, and $\{W_n\}$ is uniformly bounded. Note that the spaces E_n are not required to be subspaces of Y and the sequence $\{X_n\}$ is not assumed to be nested. Hence, for example, the finite element method can be used in the construction of admissible schemes. The existence of an admissible scheme for (X, Y) implies that X is separable, $\bigcup_n X_n$ is dense in X, but Y need not be separable. Examples of simple admissible schemes include various projection and Galerkin-type methods. If X is separable with dual X^*, then (X, X^*) always has an admissible scheme. Also there is an admissible scheme for (X, X) whenever X has a Schauder basis.

The concepts of *approximation properness* (*A-properness*) of $T: D \subseteq X \to Y$ and of the *approximation solvability* of equation (1) are defined in terms of a given admissible scheme for (X, Y). Equation (1) is said to be *strongly* (respectively, *feebly*) *approximation solvable* with respect to the admissible scheme $\Gamma = \{X_n, V_n; E_n, W_n\}$ if for all sufficiently large n, the equation (2), where $f_n := W_n f$, has a solution $x_n \in D_n := D \cap X_n$ such that $x_n \to x_0$ in D (respectively some subsequence of $\{x_n\}$ converges to x_0) and $T(x_0) = f$. The equation (1) is uniquely approximation solvable if the approximate solutions x_n and the limit solution x_0 are unique. Clearly, approximation solvability implies solvability, but the converse need not be true. A mapping $T: D \subseteq X \to Y$ is said to be A-*proper* with respect to Γ if and only if the restriction T_n of $W_n T$ to D_n, $T_n: D_n \subseteq X_n \to E_n$, is continuous for each n and the following condition holds: If $\{x_{nj} \mid x_{nj} \in D_{nj}\}$ is any bounded sequence such that $\|T_{nj}(x_{nj}) - W_{nj}(g)\| \to 0$ as $j \to \infty$ for some g in Y, there exists a subsequence $\{x_{nj(k)}\}$ of $\{x_{nj}\}$ and $x \in D$ such that $x_{nj(k)} \to x$ as $k \to \infty$ and $T(x) = g$.

The motivation for the terminology "A-proper" resides in the connection with the well-known notion of a *proper* map. We recall that a map T is *proper* if the inverse image of each compact set in Y is compact in X. One can show that if $D \subset X$ is open and $T: \overline{D} \to Y$ is continuous and A-proper, then the restriction of T to every bounded closed subset of $\overline{D}$ is a proper map.

The book is organized into five chapters. Chapter I provides an introduction to the general theory of A-proper and pseudo-A-proper maps, including examples and applications to constructive solvability of some 2nd-order differential equations. Chapter II develops the linear theory of A-proper maps and its application to the variational solvability of linear elliptic PDEs of even order. The author shows that the abstract results in this chapter are best possible. Chapter III establishes fixed-point and surjectivity (solvability) theorems for various important classes of A-proper-type maps. These results unify and extend earlier results on monotone and accretive maps. The author also shows how the Friedrichs linear extension theory can be generalized to the extensions of densely defined nonlinear operators in a Hilbert space, and provides applications of the theory to ODEs and PDEs. Chapter IV presents the generalized topological degree theory of A-proper maps developed by Browder and Petryshyn. The degree theory is applied to local bifurcation and asymptotic bifurcation problems. Finally, in Chapter V the author applies the abstract results to the solvability of boundary value problems of ODEs and PDEs and to bifurcation problems. The bibliography consists of 360 references to books and research papers.

The fields of *nonlinear* functional analysis and *numerical* functional analysis (or numerical analysis in abstract spaces) have developed extensively in the last thirty years. Moreover, the two fields have benefited immensely from their interactions. Some historical perspectives on landmarks in these fields are given in the book reviews [2] and [3], and in the books [1] and [4]. As the author states, the main thrust of the development of nonlinear functional analysis and its applications during the last 25 years has been in the direction of breaking out of the classical framework into a much wider field of noncompact operators, such as operators of monotone and accretive type, operators of set and ball-condensing type, and operators of approximation-proper type. There exist extensive treatments of the theories and applications of the first two classes of operators. The present monograph complements the existing literature by providing the first comprehensive study of approximation-solvability of equations involving A-proper type operators. The author is the recipient of the M. Krylov Award (1992) of the Ukrainian Academy of Sciences for originating and developing the theory of A-proper mappings and its manifold applications, and was elected by the Ukrainian Academy in 1992 as a foreign full member. The contributions of Krylov to numerical mathematics in particular are well known to readers of *Mathematics of Computation.*

The book should be of interest to researchers in the theory and applications of nonlinear functional analysis and also to abstract numerical analysts. Among the references, the journals which have the most citations (in terms of the number of cited references published in these journals) include the *Journal of Mathematical Analysis and Applications*, the *Bulletin of the American Mathematical Society, Transactions of the AMS, Journal of Functional Analysis, Nonlinear Analysis: TMA*, and *Soviet Mathematics Doklady.* The perspective of an analyst dominates the book. There are no references in the book to papers in

Mathematics of Computation. But this does not diminish the relevance of this monograph to *numerical* functional analysis. One is reminded of a statement by the late A. S. Householder, in his review of the book by Collatz [1] in *Mathematical Reviews* (MR **29**, #2931): "It seems strange that this book should be the first of its kind, since it hardly needs to be said that 'numerical mathematics' must draw heavily from functional analysis." It has been thirty years since Householder's review. Time has only reinforced the relation between these two fields, which must continue to enrich each other by drawing heavily from each other.

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4[65-06, 65C20, 76-06, 76C05].—J. T. BEALE, G.-H. COTTET & S. HUBERSON (Editors), *Vortex Flows and Related Numerical Methods*, NATO ASI Series, Series C: Mathematical and Physical Sciences, Vol. 395, Kluwer, Dordrecht, 1993, viii + 387 pp., $24\frac{1}{2}$ cm. Price $155.00/Dfl.265.00.

This book consists of a series of twenty-seven papers based on lectures given at the NATO Advanced Research Workshop held in Grenoble, France, in June 1992. A typical conference proceedings volume, it provides an overview of the most current research in the study of vortex flows. The emphasis is on both up-to-date mathematical models as well as numerical methods used to study the particular features of these flows.

The articles are not meant to provide an introduction to the specific subjects, but are rather a summary of recent results in the area. Extensive references serve as a useful guide to related literature. Intended readers are researchers and graduate students with interests in computational fluid dynamics, numerical analysis or applied mathematics in general.

The papers in the first part of the volume cover most recent developments in mathematical and numerical modeling for incompressible vortex flows. The second part treats mainly vorticity generation problems, related boundary layer and wake models in two dimensions. The third part concentrates on vortex