

Supplement to THE FABER POLYNOMIALS FOR ANNULAR SECTORS

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Appendix

Table 2. Faber polynomials up to degree 15

$$\text{Polynomials } \phi_{n-1} = \sum_{j=0}^{n-1} p_j z^{n-j-1} \text{ for } 1 \leq n \leq 15.$$

Degree of Faber Polynomial: 1
p(0): U
Degree of Faber Polynomial: 2
p(0): 2*U
p(1): (S^2+2*S*U-U^2-4)/2
Degree of Faber Polynomial: 3
p(0): 3*U
p(1): (3*(S^2+2*S*U+U^2-4))/4
p(2): (2*S^3+S^2*U-2*S*U^2-8*S+U^3+2*U)/2
Degree of Faber Polynomial: 4
p(0): 4*U
p(1): S^2+2*S*U+3*U^2-4
p(2): (4*S^3+5*S^2*U+2*S*U^2-16*S+U^3-8*U)/3
p(3): (11*S^4-4*S^3*U-6*S^2*U^2-48*S^2+12*S*U^3+32*S*U-5*U^4-16*U^2+16)/8
Degree of Faber Polynomial: 5
p(0): 5*U
p(1): (5*(S^2+2*S*U+5*U^2-4))/4
p(2): (5*(S^3+2*S^2*U+2*S*U^2-4*S+U^3-5*U))/3
p(3): (5*(9*S^4+8*S^3*U+4*S^2*U^2-42*S^2+4*S*U^3-20*S*U-U^4-10*U^2+24))/24
p(4): (12*S^5-13*S^4*U+8*S^3*U^2-56*S^3+14*S^2*U^3+60*S^2*U-20*S*U^4-72*S*U^2+32*S+7*U^5+28*U^3+8*U)/8
Degree of Faber Polynomial: 6
p(0): 6*U
p(1): (3*(S^2+2*S*U+7*U^2-4))/2
p(2): (4*S^3+11*S^2*U+14*S*U^2-16*S+11*U^3-32*U)/2
p(3): (39*S^4+76*S^3*U+74*S^2*U^2-192*S^2+44*S*U^3-256*S*U+7*U^4-128*U^2+144)/16
p(4): (46*S^5+31*S^4*U+34*S^3*U^2-248*S^3+22*S^2*U^3-120*S^2*U-20*S*U^4-136*S*U^2+256*S+7*U^5+24*U^3+104*U)/20
p(5): (21*S^6-34*S^5*U+65*S^4*U^2-100*S^4-20*S^3*U^3+112*S^3*U-65*S^2*U^4-312*S^2*U^2+72*S^2+70*S*U^5+304*S*U^3+144*S*U-21*U^6-100*U^4-72*U^2-32)/16

Degree of Faber Polynomial: 7	
p(0):	$7U$
p(1):	$(7(S^2+2S+U+9U^2-2-4))/4$
p(2):	$(7(12S^3+7S^2+2U+10S^2U-8S^2U^2-8S^2U^3-22U^2U))/6$
p(3):	$(7(17S^4+20S^3+2U+26S^2U^2-36S^2U^3-20S^2U^4))/6$
p(4):	$(7(112S^5+217S^4+52U+28S^3U^2-32U^2))/6$
p(5):	$(83S^6+2U+3-96U+20S^2U^2-60U^2+4-672S^2U^2+112U^3+9U^4+60U^5+112U^6+60U^7)/6$
p(6):	$(432S^7+5U^3+77U^4+15U^5+22U^6+184U^7-224U^8+224U^9+2880U^{10})/2880$
p(7):	$(144S^8+19U^4+6U^5+102S^6U^2-64S^6U^3-145S^6U^4+50S^6U^5+80S^6U^6+32U^3S^3U^2-64S^3U^3+147S^2U^5+78U^2S^2U^5+528S^2U^6-124S^2U^6+640S^2U^7-576S^2U^8-128S^2U^9+182U^9+5208U^9+316U^{10})/16$
Degree of Faber Polynomial: 8	
p(0):	$8U$
p(1):	$2(S^2+2S^2U+11U^2-4)$
p(2):	$(2(4S^3+17S^2+2U+26S^2U^2-16S^2U^3-56U^3))/3$
p(3):	$(65S^4+164S^3+20S^2+2U^2-240S^2U^2+244S^2U^3-60U^3)/3$
p(4):	$(64S^5+129S^4+4U+176S^3U^2-272S^3U^3-138S^3U^4-60U^4+5U^5+544S^2U^2+1384S^2U^3+119U^5-184U^3+415U^4)/10$
p(5):	$(1475S^6+22978S^5+34303S^4+4U^2+10060U^2+1652U^3+3U^3+1624S^3+2281S^4+13320S^4+2U^2+1808U^2+4832U^2+5760U^3+20288U^3+78U^4+6+908U^4)$
p(6):	$(6532S^7+6541S^6+12214S^5+U^2+4664S^5+9853S^4+U^3-5408U^3+316S^3U^4-4304U^4-975536S^3+11123S^2+U^5+57408S^2U^3+120352S^2U+9002S^2U^6-47776S^2U^4-33344S^2U^5-66048S^2U^6+2269U^7+12892U^5+17056U^3+14592U^4)/2520$
p(7):	$(517S^8+184S^7+U+532S^6+U^2+256S^6+2296S^5+U^3-3072S^5+2450S^4+U^4+1920S^4+U^2+960S^4+392S^3+U^5+7680S^3+U^3+8960S^3+U^2+2604S^2+U^6+15360S^2+U^4-17280S^2+U^2-3072S^2+1848S^2+U^7+10752S^2+U^5+14080S^2+2048S^2+42240S^2+2688U^6-4160U^4+1024U^2+2561/128$
Degree of Faber Polynomial: 9	
p(0):	$9U$
p(1):	$(9(S^2+2S^2U+13U^2-2-4))/4$
p(2):	$(9(16S^3+5U^3+24S^2U+14S^2U^2+5S^2U^3+17U^3))/4$
p(3):	$(9(61S^4+2S^3+1049S^3+49S^2U^2-33S^2U^3+36U^3))/4$
p(4):	$(9(38S^5+143S^4+54S^3U^2+242S^3U^2-244S^3U^3+236S^2U^3-690S^2U^4+140S^2U^5-788S^2U^6+41U^5-378U^3+532U^4))/20$
p(5):	$(3(1160S^6+486S^5+U+691S^4+U^2+1155S^4+544S^3+U^3-1788S^3+U^2+262S^2U^2-4S^2U^3+2260S^2U^2+90S^2U^5-1316S^2U^3+3496S^2U^4+1444U^2-800))/80$
p(6):	$(2751S^7+6103S^6+6U+7054S^5+5U^2+21612S^5+2288S^4+U^3-4113S^4+U^4+1171S^3U^4-32516S^3U^2+50928S^3+1463S^2U^5+4506S^2U^3+22256S^2U^6+896S^2U^6-7240S^2U^4+22240S^2U^3+33984S^2U^2+1169U^5+5776U^3-16416U^4)/560$
p(7):	$(11627S^8+20412S^7+U+8094S^6+U^2+97314S^6-39552S^6+5U^3+180092S^5+U+40712S^5+U^3+41730U^3+27586S^4+49228S^4+U^3+3442+27636S^4+U^2+300608S^2-44534S^2+U^6+2708U^3+221636S^2U^2+300608S^2+30072S^2U^7+181068S^2U^5+259904S^2U^3+73856S^2U^6+6739U^8$
Degree of Faber Polynomial: 10	
p(0):	$10U$
p(1):	$(5(S^2+2S^2U+15U^2-4))/2$
p(2):	$(5(4S^3+23S^2+2U+38S^2U^2-16S^2U^3-80U^3))/6$
p(3):	$(5(17S^4+484S^3+3U+182S^2U^2-96S^2U^3-212S^2U^4-320S^2U^4+480U^2+112))/16$
p(4):	$(43S^5+193S^4+U+387S^3U^2-284S^3U^3+446S^3U^4-960S^3U^2+320S^2U^4-1308S^2U^4+448S^2U^5-808U^3+812U^4)/6$
p(5):	$(595S^6+2314S^5+U+4079S^4+U^2+4460S^4+4156S^3+U^3-13712S^4+2693S^3+U^4-17640S^3+29220S^2+10590S^2U^5-11344S^2U^3-18184S^2U^4+193U^6-3244U^4+10396U^2+25600)/72$
p(6):	$(4106S^7+4068S^6+U^4+10392S^6+23652S^5+1506S^4+U^3-9456S^4+1968S^3U^4+10392S^3U^2+2652S^3U^5-5365S^2U^4+17116S^2U^4+14S^2U^6-2652S^2U^4+10392S^2U^2-6464S^2U^2+168U^5+2524U^3+4736U^4)/56$
p(7):	$(46543S^8+116424S^7+U+120004S^6+U^2+418656S^6+23624S^5+U^3-90340S^5+U+59314S^4+U^4-607584S^4+U^2+1233624S^4+415800S^3+U^5-131072S^3+U^3+1995488S^3+U-38892S^2+U^6-311072S^2+U^4+596032S^2+U^2+1265152S^2+25592S^2U^8-37408U^2+45608U^4-209920U^2+201600)/8064$
p(8):	$(99134S^9+258177S^8+U+8U+46062S^7+U^2+396504S^7-300212S^6+U^3-2225376S^6+U+916530S^6+U^4+1666776S^6+U^2+13610464S^5+665814S^4+U^3+486248S^3+U^4+7118S^3+U^6-3520104S^3+U^4+882080S^2+U^7+1852U+660732S+959334U^2-258840U^3+131614U^4+851666U^3+302976U^4)/36288$
p(9):	$(1278210+1654U^2+498S^8+U^2-2492S^8+3768S^7+U^3-10720S^7+U^2+244U^5+U^2+53976S^6+U^2+15344S^6-31500S^5+U+144U^5+U^2+4704S^4+18186S^4+U^6+13720S^4+U^4+U^4+U^2+30400S^4+13464S^3+U^7+132832S^3+U^5+U^3+83200S^3+U+24453S^2+U^8-176400S^2+U^6-341040S^2+U^4+144000S^2+U^2+3200S^2+12870S^2U^9+93984S^2U^7+196896S^2U^5+108800S^2U^3+6400S^2U^3+2431U^10-18876U^8-44016U^6-30400U^4+3200U^2-512)/256$
Degree of Faber Polynomial: 11	
p(0):	$11U$
p(1):	$(11(S^2+2S^2U+17U^2-4))/4$
p(2):	$(11(12S^3+13S^2+2U+22S^2U^2-8S^2U^3-46U^3))/6$
p(3):	$(11(127S^4+4148S^3+3U+362S^2U^2-156S^2U^3+452S^2U^4-566S^2U+451U^3+1004U^2+324U^4+432S^3+998S^2+U^3-1164S^3+5620S^2U+2664S^2U^2+704S^2U^3+5393U^5-1984U^3+16396U^4)/80$
p(4):	$(11(571S^5+62650S^4+U+5549S^4+U^2+4400S^4+6796S^3+U^3-2650S^3+U+5333S^3+U^2+2491S^2+U^2+9472S^2+1660S^2U^5-19840S^2U^3+22528S^2U+643U^6-7264U^4-16384U^2-40321)/576$
p(5):	$(11(12646S^6+10643S^6+U+19298S^5+U^2-22692S^5-112S^5+24317U^6+17U^2+4)/4$
p(6):	$(11(12646S^6+10643S^6+U+19298S^5+U^2-22692S^5-112S^5+24317U^6+17U^2+4)/4$

- 20317*S^4*U^3-74590*S^4*U^0*U1162*S^3*U^4-103828*S^3*U^2
- 59568*S^3*6601*S^2*U^5-77496*S^2*U^3*U319616*S^2*U
- 5810*S^0*U^6-33104*S^0*U^4+116624*S^0*U^2+44544*S^0*U^0
- 5810*S^0*U^6-36992*U^3+43156*U^0)/2520
- P(7): (11*(4109*S^8+13720*S^7+107696*S^6+9040054*S^5+400119316*S^4+U^2
- 15432*S^5*U^3-176956*S^5*U^2+22533728*S^3*U^2-2295286*S^3+20508*S^2*U^7
- 188272*S^4+3720*S^3+34864*S^2*U^5+72448*S^3+3034786*S^2+217816*S^1+64*S^0
- 196*S^2*U^6-33864*S^2*U^5+74240*S^2*U^4+176944*S^2*U^3+439816*S^2*U^2+44800
- 1400*U^0*U^3+360*U^0*U^2+455652*U^0*U^1+480248*S^0*U^2-2294232*S^1
- 11*(1224672*S^9-574117932*S^8+1171768*S^7+5656872*S^6+U^4
- 311584*S^5+U^2+18119392*S^5-262758*S^4+U^5
- 1848234*S^4+U^3+1357472*S^4+U^1-163976*S^3+U^6
- 2185080*S^4+U^4+4042560*S^3+U^2-11138432*S^3
- 362196*S^2*U^2+2423976*S^3+U^5+6414688*S^2*U^3
- 9714432*S^2*U+200616*S^0*U^6-1404864*S^0*U^6-2330784*S^0*U^4
- ...-311484928*U^0)/362880
- P(8): (147*S^10+5629694*S^9+U^0-6999537*S^8*U^2
- 178*S^8+5268744*S^7+U^3-45889632*S^7*U^2
- 1*S^6*U^4+71485128*S^6*U^2+105034752*S^6
- 14*S^5*U^5-126671232*S^5*U^2+37767680*S^5*U
- 4*S^4*U^6+1912160*S^4*U^4+18132576*S^4*U^2
- 12*S^4*U^6+19091432*S^3+U^7-1851868276*S^4*U^8
- 58521856*S^3+42148352*S^2+U^9+196781124*S^7*U^2
- 2070231531814400*S^2+U^3+34903296*S^7
- 102300988*U^8-56512896*U^6-41162816*U^4
- 4*S^2+72576000)/7257600
- P(9): (15*943*S^10*U^0*U^2-5416*S^9*U^9+U^2-5416*S^9*U^9+U^2-5416*S^9*U^9+U^2
- 6180*S^7*U^4+52128*S^7*U^2+24556*S^7
- 6*U^5+201096*S^6*U^3-77056*S^6*U^3+77028*S^5*U^6
- 15*U^5+49728*S^5*U^2-43304*S^5-30630*S^4*U^7
- 1*U^5+344960*S^4*U^3+182560*S^4*U+38610*S^3*U^8
- 13*U^5-902720*S^3*U^4-482800*S^3*U^2-3840*S^3
- 2*U^9+412632*S^2*U^9+95928*S^2*U^5
- 4*60*S^2*U^3+556800*S^2*U+24310*S^2*U-495624*S^0*U^8
- 84*S^0*U^6-380800*S^4*U+576000*S^4*U^2-3072*S^4+399*U^11
- 10*U^9+97152*U^7+90272*U^5+15840*U^3+6236*U^0)/236
- Degree of Fcber Polynomial: 12
- P(10): 34*(S^2+2*S*U+U^2+U)
- P(11): 457*S^2+24*U^0*S^2+U^2-8960/780
- P(12): 157*S^4+340*S^3+U^2-336*S^2+1220*S*U^3
- P(13): 1312*S^4+4147*U^2+2673*U^2+4321/8
- P(14): (212*S^5+1217*U^4+3128*S^3*U^2-1456*S^3+4634*S^2*U^3
- 6360*S^2*U+4220*S*U-11072*S^2+24232*S^2+2429*U^5
- 9912*U^3+6208*U^0)/20
- P(15): (1115*S^6+59338*S^5+U^2+1283054*U^2-8780*S^4+40212*S^3*U^3
- 37104*S^3*U+18361*S^2*U^2+66120*S^2*U^2-19520*S^2
- +10730*S^4+561808*S^3*U^3+53888*S^3+281*U^6
- 27788*U^4+47552*U^2-8960/780
- P(16): (57376*S^7+25727*S^6+U^55682*S^5*U^2-47328*S^5
- +17153*S^7+25727*S^6+U^55682*S^5*U^2-47328*S^5
- +128660*U^3+33985*S^4+185200*S^4+U^60140*S^3*U^4-11008*S^3*U^2
- +11858*U^5+150872*S^2*U^4+175295*S^2-101378*S^1+991*U^7
- 35500*U^0+5159848*U^0+12424201321140*S^6*U^2-4081728*S^6
- 19653316*S^6+3116138750*S^4+12880352*S^4+1294056*S^3*U^5
- 20748992*S^4*U^3+12880352*S^4+1294056*S^3*U^5
- 77181952*S^3*U^3+34007532*S^3+U^389732*S^2*U^6
- 8095921*S^2*U^4+34054592*S^2*U^2+14350336*S^2
- 630320*S^2+23465344*S^0*U^3+2180332*S^0*U^3-21803008*S^0*U
- 28224000)/26880
- P(18): (42896*S^9+148871*S^8+U^0+207232*S^7*U^2-449936*S^7
- 17240*S^6*U^3-1309744*S^6*U+127828*S^5*U^4
- 1456232*S^4*U^3+3649248*S^4*U+1264*S^3*U^6
- 1041232*S^4*U^3+3649248*S^4*U+1264*S^3*U^6
- 528772*S^5*U^4+2963328*S^3*U^2-2295286*S^3+20508*S^2*U^7
- 20784*S^2*U^5+1866448*S^2*U^3-32159648*S^2*U-9720*S^0*U^8
- 87168*U^6+169888*U^4-1709056*S^0*U^2+905216*S^0+1887*U^9
- +1392*U^7+463072*S^5+262400*U^3+505984*U^0)/3360
- P(19): (4351359*S^9+10713797398*S^9+U^533921*S^8*U^2-56455164*S^8
- 4924486*S^7*U^3+...-179191680*S^6*U^4
- 5034244*S^5*U^2+...-8424584*S^6
- 10726154*S^5*U^4+...-720848*S^3*U^6
- 9250160*S^5*U^4+...-86576*S^2*U^6
- 9579596*S^5*U^4+...-18001024*S^2
- 4189504*S^5*U^4+...-148*S^2+1
- 7485920*S^4+U^6+646864*S^4*U^2+125
- 4895280*S^4+34389112*S^4+U^125
- 22599072*S^4+U^125+119251786*S^3*U^2
- (27310220*S^3+151600727*S^3+U^10-10336476*U^8
- 644089600*S^2+293635543*S^2+U^5-30262632*S^8*U
- 6445816*S^7*U^4+745973808*S^7*U^2+1586798400*S^7
- 67238610*S^6*U^5-3075995872*S^6*U^3+736284480*S^6*U
- 1042603660*S^5*U^6+3607711248*S^5*U^4+957926400*S^5*U^2
- 2903936000*S^5-367369354*S^4*U^7+341917680*S^4*U^5
- +574419104*S^4*U^3+1425471488*S^4*U+52942708*S^3*U^8
- 1618188800*S^3+662849211*S^2*U^9+551018592*S^2*U^7
- 1332378664*S^2*U^5+148368896*S^2*U^3+63319232*S^2*U
- 509697460*S^0*U^4+60395312*U^4+126540344*U^7
- 31583584*U^5+283073152*U^3+59397120*U^0)/6652800
- P(111): (2399*S^12-4828*S^11*U-20746*S^10*U^2-32560*S^10
- 137060*S^9*U^3+123040*S^9*U+240735*S^8*U^4-92880*S^8*U^2
- 137232*S^8-1848*S^7*U^5-900480*S^7*U^3-713088*S^7*U
- 557172*S^6*U^6+2745120*S^6*U^4+1907136*S^6*U^2
- 143360*S^6-723096*S^5*U^7-2751856*S^5*U^5-862848*S^5*U^3
- 917504*S^5*U^4+173745*S^4*U^8-635040*S^4*U^6
- 5070240*S^4*U^4+4372480*S^4*U^2-138880*S^4
- 408800*S^3*U^9+3991680*S^3*U^7+11136384*S^3*U^5
- 8888320*S^3*U^3+1091120*S^3*U+424442*S^3*U^0
- 3809560*S^2*U^2-52285632*S^2*U^0-69175040*S^2*U^4
- 4650624*S^0*U^7+4759552*S^0*U^5+1508160*S^0*U^3+618680*U^9
- 23939*U^12-27222*U^10-844272*U^8-989184*U^6-353920*U^4
- 18432*U^2+2046)/1024
- Degree of Fcber Polynomial: 13
- P(1): 13*(S^2+2*S*U+U^2+U^2-4)/4
- P(2): (13*(S^2+2*S*U+U^2+U^2-4)*S^0*U^2-4*S^4+3*S^3-29*U^0)/3
- P(3): (13*(S^2+2*S*U+U^2+U^2-4)*S^6+2*U^2-30*S^2+132*S^0*U^3
- 124*S^0*U^175*U^4-286*U^2+2401/8
- P(4): (13*(116*S^5+5731*S^4+U^2064*S^3*U^2-808*S^3+3382*S^2*U^3
- 3900*S^2*U+3340*S^0*U^4-7416*S^0*U^2-1376*S^2+2247*U^5
- 7676*U^3+4024*U^0)/120
- P(5): (49316*S^6+11474*S^5*U-73192*S^3*U+50503*S^2*U^4-147060*S^2*U

