MATHEMATICS OF COMPUTATION
VOLUME 64, NUMBER 210
APRIL 1995, PAGES 889-890

# ON THE PRIMALITY OF $n! \pm 1$ AND $2 \cdot 3 \cdot 5 \cdots p \pm 1$

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ABSTRACT. For each prime $p$ let $p\#$ be the product of the primes less than or equal to $p$. Using a new type of microcomputer coprocessor, we have found five new primes of the form $n! - 1$, two new primes of the form $p\# + 1$, seven new primes of the form $p\# - 1$, and greatly extended the search bounds for primes of the form $n! \pm 1$ and $p\# \pm 1$.

For each prime $p$ let $p\#$ be the product of the primes less than or equal to $p$. Since Euclid's proof of the infinitude of primes, the numbers $n! \pm 1$ and $p\# \pm 1$ have been frequently checked for primality, most recently in [1, 2, 4, 5, and 7] using the classic $N^2 - 1$ primality tests of [3]. Using a new type of microcomputer coprocessor board designed for doing huge integer arithmetic, we have greatly expanded the range over which these numbers have been tested and found the fourteen new primes marked by an asterisk in Table 1.

TABLE 1. Factorial and primorial primes

| form | values for which the form is prime | search limit |
|---|---|---|
| $n! + 1$ | 1, 2, 3, 11, 27, 37, 41, 73, 77, 116, 154, 320, 340, 399, 427, 872 and 1477 (4042 digits) | 4580 |
| $n! - 1$ | 3, 4, 6, 7, 12, 14, 30, 32, 33, 38, 94, 166, 324, 379 and 469, **546***, **974***, **1963***, **3507*** and **3610*** (11277 digits) | 4580 |
| $p\# + 1$ | 2, 3, 5, 7, 11, 31, 379, 1019, 1021, 2657, 3229, 4547, 4787, 11549, 13649, 18523, **23801*** and **24029*** (10387 digits) | 35000 |
| $p\# - 1$ | 3, 5, 11, 41, 89, 317, 337, 991, 1873, 2053, **2377***, **4093***, **4297***, **4583***, **6569***, **13033*** and **15877*** (6845 digits) | 35000 |

*Newly shown prime. Here $p\#$ is the product of primes $\leq p$.

The coprocessor is based on LSI Logic's L64240 MFIR chip, originally designed for real-time digital signal processing. An early version of this coprocessor, also built by Robert and Harvey Dubner, is described in [6]. When used in an Intel 80486 based microcomputer, the coprocessor can increase the speed of the basic arithmetic by a factor of over fifty. For example, the complete primality proof for the 11272-digit number $3610! - 1$ took 2.9 hours. Unfortunately, because of the very large number of divisors of $p\#$ at which tests must be conducted (even using the methods of [2]), the numbers of the form $p\# \pm 1$ are the slowest numbers to prove prime using the classic tests. The proof for

Received by the editor December 17, 1992 and, in revised form, April 8, 1994.
1991 *Mathematics Subject Classification*. Primary 11A41; Secondary 11A51.
*Key words and phrases*. Prime numbers, factorial primes.

the 6845-digit number $15877\# - 1$ took 1.86 CPU days, and the proof for the 10387-digit number $24029\# + 1$ took 3.73 CPU days.

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