initiated a research program on parallel methods for ordinary differential equations. This tract is a compilation of six papers [1, 2, 3, 4, 5, 6] coming out of this research, together with an extended introduction by B. P. Sommeijer. The introduction is more than a preamble to the rest of the monograph, and is, in essence, a further paper. The aspect of parallelism considered here, "across the method", makes sense only for multistage methods. Explicit Runge-Kutta methods, in their traditional formulation, have little to offer, but there seem to be reasonable prospects for iterated Runge-Kutta methods and for the type of multivalue multistage methods described here as block Runge-Kutta methods. For stiff problems, implicit Runge-Kutta methods can sometimes be effectively implemented using parallel iterations. To all these options, methods generalizing predictor-corrector methods can be added. The papers included here discuss many aspects of these many method types, such as order, convergence of iterations, stability, local error estimation and stepsize control. The theoretical work is supported by insightful numerical evaluations and comparisons. For both nonstiff and stiff problems, and for the related differential-algebraic equations, parallelism across the method is still a vital area of research and the work presented in this tract is a valuable introduction to this topic.

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1. P. J. van der Houwen and B. P. Sommeijer, *Parallel iteration of high-order Runge-Kutta methods with stepsize control*, J. Comput. Appl. Math. **29** (1990), 111–127.

2. ——, *Block Runge-Kutta methods on parallel computers*, Z. Angew. Math. Mech. **72** (1) (1992), 3–18.

3. B. P. Sommeijer, W. Couzy, and P. J. van der Houwen, *A-stable parallel block methods for ordinary and integro-differential equations*, Appl. Numer. Math. **9** (1992), 267–281.

4. P. J. van der Houwen, B. P. Sommeijer, and W. Couzy, *Embedded diagonally implicit Runge-Kutta algorithms on parallel computers*, Math. Comp. **58** (1992), 135–159.

5. P. J. van der Houwen and B. P. Sommeijer, *Iterated Runge-Kutta methods on parallel computers*, SIAM J. Sci. Statist. Comput. **12** (1991), 1000–1028.

6. ——, *Analysis of parallel diagonally implicit iteration of Runge-Kutta methods*, Appl. Numer. Math. **11** (1993), 169–188.

10[65L15, 65Y15, 34B24].—JOHN D. PRYCE, *Numerical Solution of Sturm-Liouville Problems*, Monographs on Numerical Analysis, Oxford Univ. Press, Oxford, 1993, xiv + 322 pp., 24 cm. Price $56.50.

The past few years have seen a remarkable production of mathematical software for Sturm-Liouville problems; all of these codes treat various fairly wide classes of Sturm-Liouville problems (both regular and singular) with automatic error control of some kind. The timely text of Pryce discusses all of these codes (as well as other numerical methods) and, more importantly, provides considerable detail on the underlying mathematics. In the words of the author "It

[Sturm-Liouville software] is an area, like the numerical analysis of partial differential equations, of which it can be considered a small part, where a pervasive role is played by deep classical and functional analysis." The sophistication of these modern software packages is such that a thorough understanding of the related mathematics is crucial if one wants to fully understand them.

The initial chapter of the book contains introductory material on the mathematical notation and terminology to be used along with some examples of how Sturm-Liouville problems can arise in applications. The second chapter provides background on the mathematical theory needed to study these problems, including standard transformations and asymptotic developments that are used to study the qualitative behavior of eigenvalues and eigenfunctions. Besides giving insight into the theory, much of this material is (or should be) incorporated into the various software packages. The third and fourth chapters introduce the classical numerical techniques of finite differences and variational methods. While these have not been successfully incorporated into mathematical software (for a variety of reasons discussed in the text, including inefficiencies at large eigenvalue indices and in estimating global errors), they do shed light on some of the numerical problems associated with Sturm-Liouville calculations. Also, some research effort has been given to overcoming some of their shortcomings. Chapters 5 and 6 address the two methods underlying the recently released software packages: shooting (after a Prüfer transformation) and coefficient approximation (which the author graciously calls Pruess methods, though, as the author makes clear, only the theory, not the algorithm, originated with this reviewer). Great care is taken with the mathematics underlying the codes. Chapter 7 presents the relevant theory for singular problems, the most likely to be encountered in applications. How the numerical methods are adapted to these singular cases is the focus of Chapter 8. Chapter 9 discusses the computation of Sturm-Liouville eigenfunctions whose efficient and accurate calculation is still not as well developed as that for eigenvalues. Chapter 10 is devoted to the computation of physical resonances, another area where the author has made significant contributions. There is an assortment of topics in Chapter 11, including the use of interval arithmetic, multipoint and vector problems, and the calculation of the spectral density function. The final chapter contains a dozen challenges for future research, both theoretical and computational. A discussion of benchmarking and a long list of problems are contained in a second appendix.

To this reviewer, the major strength of the book is the thoroughness with which the author presents both theoretical and numerical material related to Sturm-Liouville calculations. Of course, one is always at some risk describing "current" software, as there are always updates and upgrades occurring, which quickly render any printed description out of date. For instance, there are several places in the text regarding SLEDGE where the author's comments, while valid for the version he had, no longer hold. This may be true for his own NAG codes as well! Another strength is the numerous problem sets throughout the text, some of which are challenging; a graduate course on this specialized material could be self-contained if this text were used. Finally, the text is a must for any current or prospective researcher who is interested in Sturm-Liouville calculations, and it is strongly recommended for anyone interested in developing serious mathematical software to see the interplay between theory and implementation.

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11[65-02, 68-02, 68U07].—JOSEF HOSCHEK & DIETER LASSER, *Fundamentals of Computer Aided Geometric Design* (translated from the German by Larry L. Schumaker), A K Peters, Wellesley, MA, 1993, xviii + 727 pp., $23\frac{1}{2}$ cm. Price $79.95.

We have here a fine addition to the textbook literature in computer-aided geometric design (CAGD). The book evolved in several stages from lecture notes (in German) prepared in 1986. The first published edition appeared in German under the Teubner imprint, and the second German edition appeared in 1992. Larry Schumaker undertook the translation and typesetting that resulted in the edition reviewed here.

The text gives a systematic account of CAGD, and is very much up-to-date. In a short review, we can only outline the contents chapter-by-chapter. Chapter 1 discusses transformations, projections, visibility methods, shading and reflection. Chapter 2 reviews elementary differential geometry, classical interpolation methods, and least squares procedures. Chapter 3 is about splines, including tension splines and exponential splines, all in one variable. Chapter 4 gives the theory of B-splines and Bézier curves. This occupies almost 100 pages. Chapter 5 continues with more technical spline topics: "FC" continuity, curvature continuity, Manning's splines, tau-splines, etc. Chapter 6 begins the study of spline surfaces, including tensor-product surfaces and splines on triangulations. Chapter 7 continues the study of surfaces, including multi-patch methods. Chapter 8 is devoted to Gordon-Coons surfaces (blending methods). Chapter 9 deals with scattered data interpolation by several procedures, including radial basis methods, Shepard's method, and methods from the finite element realm. Chapter 10 is devoted to basis transformations for the representation of curves and surfaces. In Chapter 11, methods for problems in high dimensions are discussed. The last five chapters deal with intersections of curves and surfaces, smoothing techniques, blending methods, offset curves and surfaces, and applications to milling processes.

The book has 727 pages, a bibliography of 83 pages, and a wealth of figures (approximately one per page). The authors (and translator) are to be congratulated for producing a comprehensive book on a timely topic.

E. W. C.

12[68-01, 68U05].—JOSEPH O'ROURKE, *Computational Geometry in C*, Cambridge Univ. Press, Cambridge, MA, 1994, xii + 346 pp., 26 cm. Price $59.95 hardcover, $24.95 paperback.

Computational geometry could be an ideal subject for undergraduate study. Students of mathematics would see how challenging it is to apply mathematical