Apparently intended for a course in mathematical software, this book's orientation—overwhelmingly one of endorsing Mathematica as the answer, regardless of the question—seems inappropriate as sole text for such a course. It may be viable as additional "symbolic methods" reading in combination with a numerical methods text.

Part I (Using Mathematica as a Symbolic Pocket Calculator), 140 pages, and Part II (Mastering Mathematica as a Programming Language), 260 pages, fits somewhere between Blachman's introductory book [2] and Maeder's book on advanced programming [3]. Gray's introduction to selected parts of the system is not entirely authoritative (there are even occasional typos in the computer-generated figures) but may be just right for an audience of upper-division applied mathematics students.

Part III in 110 pages illustrates computing in some areas of group theory and differentiable mappings of particular interest to the author. The last 110 pages are answers to problems.

If you wish to learn about ideas of programming languages partly covered in Part II: functional programming, object-oriented programming, the use of a few ideas from lambda calculus, etc., you may find (for example) the text by Abelson et al. [1] far more complete and authoritative than the coverage here.

Re-interpreting such ideas in a *Mathematica* framework has a number of failings, one of which is that it sometimes "reduces" simple ideas to complicated ones; another is that the "implementation" is extremely inefficient in execution time. However, a reader who would like to understand how something might be computed by relating it to an implementation in *Mathematica* may find the systematic development of such ideas as object-oriented programs of some interest.

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1. H. Abelson, G. J. Sussman, and J. Sussman, *Structure and interpretation of computer programs*, MIT Press, Cambridge, MA, 1985.

2. N. Blachman, *Mathematica, a practical approach*, Prentice-Hall, Englewood Cliffs, NJ, 1992.

3. R. E. Maeder, *Programming in mathematica*, 2nd ed., Addison-Wesley, Reading, MA, 1991.

19[14–06, 13–06, 13P10, 14Qxx].—DAVID EISENBUD & LORENZO ROBBIANO (Editors), *Computational Algebraic Geometry and Commutative Algebra*, Istituto Nazionale di Alta Matematica Francesco Severi, Symposia Mathematica, Vol. 34, Cambridge Univ. Press, Cambridge, MA, 1993, x + 298 pp., $23\frac{1}{2}$ cm. Price \$49.95.

This small, attractively bound volume consists of a collection of papers from a conference on the topics in its title held in 1991 in Cortona, Italy. Most of the papers deal with the theory of Gröbner bases, although there are some interesting exceptions. The papers are, as a group, of very high quality, and several of them are first-class contributions to the expository literature on this important and interesting subject. Before discussing some of the papers in more detail, we will briefly consider the role of the Gröbner basis algorithm.

The Gröbner basis algorithm is a generalization of the Euclidean algorithm to systems of polynomial equations in many variables. Using it, one may carry out explicitly many of the fundamental operations in commutative algebra, such as deciding if one polynomial belongs to the ideal generated by a finite list of others, or eliminating variables from a system of polynomial equations. More generally, the algorithm makes effective the process of determining syzygies—algebraic relations—among systems of polynomials. Since many questions in commutative algebra and algebraic geometry eventually boil down to problems involving syzygies, the Gröbner basis algorithm is extremely useful in attacking computational problems in these fields.

One of the more attractive features of the theory of the Gröbner basis algorithm is its inherently interdisciplinary nature. The algorithm is of interest to algebraists and algebraic geometers who wish to carry out explicit computations to answer questions in their "pure" research and to computer scientists interested in complexity bounds. These two groups are driven together because it has become clear that the behavior of the Gröbner basis algorithm applied to a system of polynomials is determined by geometry of the algebraic variety (or scheme) defined by that system. Computer scientists interested in the complexity of the algorithm must understand the geometric significance of that complexity, while algebraic geometers who want to be able to carry out a particular calculation in a reasonable amount of time must understand the significance of the complexity results. Both the complexity theory and practical geometric applications of the Gröbner basis algorithm are addressed in the volume under review, although the geometric applications receive more emphasis.

We turn now to the articles in this volume. The first two articles, entitled "What can be computed in algebraic geometry?" by Dave Bayer and David Mumford, and "Open problems in computational algebraic geometry" by David Eisenbud, are beautiful presentations of the central issues in the field. They should become standard references in this area. The Bayer and Mumford article presents a very clear yet sophisticated introduction to the theory of Gröbner bases, and discusses in detail the relationship between the *regularity* of an ideal, a cohomological measure of the complexity of the ideal, and the performance of the Gröbner basis algorithm. They also present examples of various kinds of worst-case performance, keeping in mind the relationship between geometry and complexity. Finally, they discuss, in general terms, some applications of the algorithm.

Eisenbud's article presents a series of open problems. Some of these problems fall into the general picture susceptible to attack by the Gröbner basis algorithm, but others clearly do not. Some of the problems he discusses include resolving surface singularities and making the classification of surfaces effective; others of a different flavor include finding rational points on varieties, which is clearly of a very different, non-Gröbner character. For the geometer, this article helps to make clear what is meant by "computational" algebraic geometry, since it sharply points out the differences between what can be done "in theory" and what can be done explicitly.

The remainder of the articles in the volume are more specialized. Papers by D. Lazard ("Systems of algebraic equations: algorithms and complexity") and by

T. Mora and L. Robbiano ("Points in affine and projective spaces") consider the theory of zero-dimensional varieties from two points of view. Lazard describes various approaches to finding the solutions to a system of polynomial equations whose common zeros are a finite set of points; in this setting there are a number of algorithms more or less closely related to the Gröbner basis method. Mora and Robbiano consider the opposite problem of finding the ideal of polynomials which vanish on a specified set of points or zero-dimensional subscheme of projective space.

A long article by W. Vasconcelos ("Constructions in Commutative Algebra") surveys methods for solving certain explicit problems in algebra, such as computing integral closure and primary decomposition. The article applies homological techniques (i.e., syzygies) as much as possible to these problems. To take advantage in practice of the methods discussed in this article, the reader should be familiar with algorithms for more elementary constructions, such as computing $(I : J)$ for ideals I and J.

In his article "Sparse elimination theory," Bernd Sturmfels discusses some of the connections of the Gröbner basis algorithm with the combinatorial theory of polytopes and (implicitly) toric varieties. This article seems somewhat out of context in this volume, but in fact is an important signpost to a fascinating related area of beautiful mathematics.

We will briefly mention the other articles in the volume. D. Bayer, A. Galligo, and M. Stillman, present an analysis of the behavior of Gröbner bases under base extension ("Gröbner bases and extensions of scalars"), which among other things provides a very concrete interpretation of the concepts of "flatness" and "faithful flatness." A paper by M. Giusti and J. Heintz ("La détermination des points isolés et de la dimension d'une variété algébrique peut se faire en temps polynomial") analyzes the problem in its title in the spirit of complexity theory. Sheldon Katz shows how Macaulay, a system which actually carries out the Gröbner basis algorithm and computes sheaf cohomology, can be used in practice to attack a problem in geometry ("Arithmetically Cohen-Macaulay curves cut out by quadrics"). Finally, Th. Dana-Picard and M. Schaps, in the only paper in the volume which does not at least mention Gröbner bases ("A computer assisted project: classification of algebras"), consider the problem of classifying finite-dimensional algebras by homological methods.

Physically, this volume has a professional quality binding and the papers were prepared in a reasonably consistent dialect of TEX.

In summary, this is a compact conference proceedings volume containing generally high-quality papers and two excellent expository articles on computational algebraic geometry and commutative algebra.

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