emphasis in this part is on integral representation with singular kernels of solutions. In the first of the chapters, the exterior Neumann problem is discretized using collocation with piecewise constants and a polygonal approximation of the domain. In the second, the ideas are applied to problems in acoustics and hydrodynamics, and in the final chapter, the method of coupling of finite elements and integral representations is discussed.

Even though I would personally have preferred a somewhat bigger dose of mathematical analysis in a text on numerical methods for PDEs for the type of students targeted (and we do include that in the course for engineering students in our university), I have quite a bit of sympathy for the presentation and the list of topics covered in the present book.

V.T.

30[49-02, 49M15, 49N99, 90C30].—R. Bulirsch & D. Kraft (Editors), *Computational Optimal Control*, Internat. Ser. Numer. Math., Vol. 115, Birkhäuser, Basel, 1994, x + 382 pp., 24 cm. Price $94.00.

This book is a collection of selected papers of the ninth IFAC Workshop on Control Applications of Optimization held in Munich in September 1992. There are 30 papers ranging from 6 to 19 pages in length. The collection is divided into five sections. The first section contains four invited papers surveying the field of computational optimal control. Two of the papers describe the transcription of optimal control problems into nonlinear programming problems and discuss sequential quadratic programming (SQP) methods for solving them. The other two papers survey optimal control problems that have been studied in robotics and aerospace applications.

There are five papers in the second section of the book which discusses the theoretical aspects of optimal control and nonlinear programming. These papers discuss recent work on methods of solving boundary value problems, synthesizing adaptive optimal controls, reduced SQP methods, and time-optimal control of mechanical systems. Unfortunately, several of the proofs of the theoretical results have been omitted and are referenced in "forthcoming papers" or theses (that generally take some time to obtain).

Section three contains eight papers presenting algorithms used for optimal control computations. Algorithms discussed include SQP methods, backward procedures for calculating the solvability sets in differential games, repetitive optimization, and interior-point methods. While most of these papers outline algorithms for a class of problems and subsequently illustrate their use on example problems, a couple of the papers are more specialized in that they discuss algorithms for solving particular problems (time-optimal control of a type-2 third-order system and space shuttle reentry with uncertain air density).

In Section four, there are four papers detailing available software and recent efforts in producing software for optimal control calculations. Approaches used in these software packages include symbolic differentiation of equations, using a symbolic manipulation language to generate optimization routines for numeric solution, and interfaces for allowing the user to view the status of numerical results as well as to change design parameters. Examples are given in each of these papers to give the reader a flavor for how the software packages operate.

Finally, nine papers comprise Section five, describing applications of optimal control to a variety of fields such as aeronautics, robotics, and biology. These papers give a sampling of ways optimal control or optimization enter into different disciplines. One paper, discussing an object-oriented approach to developing control systems, does not really describe an application of optimal control and actually seems out of place in this computational optimal control book.

The writing in general is good, but as most of the authors are from non-English speaking countries, there are occasionally awkward sentence structures and grammatical mistakes. The referencing and bibliographic styles vary from paper to paper. Further, there are no section numbers used, but section numbers are occasionally referenced; undoubtedly, section numbers were used in the original conference papers and were removed in the compilation for this book.

Overall, this is a good collection of papers to give the reader an overview of recent theoretical and applied studies in optimal control and optimization. This book is thus recommended as a library holding. However, since detailed proofs of some of the theoretical results are missing, and since the book only gives an overview rather than a comprehensive coverage of any particular application area, most researchers will probably find it difficult to justify purchasing the book for their individual libraries.

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31[65-01, 65F10, 65N55].—WOLFGANG HACKBUSCH, *Iterative Solution of Large Sparse Systems of Equations*, Applied Mathematical Sciences, Vol. 95, Springer, New York, xxii + 429 pp., 24 cm. Price $59.00.

When systems of partial differential equations are discretized, by finite difference methods or finite element methods, corresponding systems on discrete finite-dimensional spaces arise. Also, if this methodology is applied to a linear system of differential equations, a discrete *linear* system is obtained. Hence, such systems can, in principle, be solved by standard elimination methods, well known to any student of linear algebra. Furthermore, it is relatively easy to develop general computer codes for these procedures. However, the necessary amount of work required increases dramatically with the size of the systems. Therefore, in order to solve systems with up to a million unknowns, which often is required in modern scientific computing, one has to search for more efficient algorithms.

Since differential operators are local operators, the corresponding discrete operators will inherit a similar property. Hence, we are led to the study of sparse systems, i.e., systems where only a few of the unknowns are present in each equation. Iterative methods, where a converging sequence of approximations of the solution of the system is generated, are very well suited for large sparse systems. Therefore, such methods have become the dominant class of methods for large systems arising from partial differential equations.

The thesis of D. M. Young [4], from 1950, is often referred to as the beginning of the modern development of iterative methods for the linear systems arising