Finally, nine papers comprise Section five, describing applications of optimal control to a variety of fields such as aeronautics, robotics, and biology. These papers give a sampling of ways optimal control or optimization enter into different disciplines. One paper, discussing an object-oriented approach to developing control systems, does not really describe an application of optimal control and actually seems out of place in this computational optimal control book.

The writing in general is good, but as most of the authors are from non-English speaking countries, there are occasionally awkward sentence structures and grammatical mistakes. The referencing and bibliographic styles vary from paper to paper. Further, there are no section numbers used, but section numbers are occasionally referenced; undoubtedly, section numbers were used in the original conference papers and were removed in the compilation for this book.

Overall, this is a good collection of papers to give the reader an overview of recent theoretical and applied studies in optimal control and optimization. This book is thus recommended as a library holding. However, since detailed proofs of some of the theoretical results are missing, and since the book only gives an overview rather than a comprehensive coverage of any particular application area, most researchers will probably find it difficult to justify purchasing the book for their individual libraries.

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31[65–01, 65F10, 65N55].—WOLFGANG HACKBUSCH, *Iterative Solution of Large Sparse Systems of Equations*, Applied Mathematical Sciences, Vol. 95, Springer, New York, xxii + 429 pp., 24 cm. Price $59.00.

When systems of partial differential equations are discretized, by finite difference methods or finite element methods, corresponding systems on discrete finite-dimensional spaces arise. Also, if this methodology is applied to a linear system of differential equations, a discrete *linear* system is obtained. Hence, such systems can, in principle, be solved by standard elimination methods, well known to any student of linear algebra. Furthermore, it is relatively easy to develop general computer codes for these procedures. However, the necessary amount of work required increases dramatically with the size of the systems. Therefore, in order to solve systems with up to a million unknowns, which often is required in modern scientific computing, one has to search for more efficient algorithms.

Since differential operators are local operators, the corresponding discrete operators will inherit a similar property. Hence, we are led to the study of sparse systems, i.e., systems where only a few of the unknowns are present in each equation. Iterative methods, where a converging sequence of approximations of the solution of the system is generated, are very well suited for large sparse systems. Therefore, such methods have become the dominant class of methods for large systems arising from partial differential equations.

The thesis of D. M. Young [4], from 1950, is often referred to as the beginning of the modern development of iterative methods for the linear systems arising from discretizations of elliptic differential equations. Since then, there has been a steadily increasing activity in this field, leading up to the development of very effective algorithms like multigrid methods and domain decomposition methods in the 1970s and the 1980s. The purpose of the present book is to describe the recent state of the theory for iterative methods.

In the first chapter of the book the linear operator which arises from the simplest discretization of the Poisson equation, i.e., the five-point operator, is studied. In particular, the relation between the difference operator and possible matrix representations of the operator is discussed. This simple example, which is referred to throughout the book, is essential for the presentation since it introduces a family of systems which depend fundamentally on a discretization parameter. Hence, through this example, the author is able to illustrate the main ingredients in the development of elliptic solvers.

In the second chapter a recapitulation of linear algebra is given, while Chapters 3–7 are mostly devoted to the classical iterations (Jacobi, Gauss-Seidel and SOR) and other classical procedures like alternating directions and Chebyshev methods. This part of the book can be described as an updated version of Varga's book [2] from 1962. In Chapter 8 the general concept of "preconditioning" is introduced, and this chapter ends with a discussion of preconditioners derived from incomplete factorizations. The theory of the conjugate gradient method and its variants is discussed in Chapter 9.

The two last chapters cover multigrid methods and domain decomposition methods. These chapters have probably been the most difficult to write since these areas are heavily influenced by research from the last years. For other references on these topics we refer, for example, to the book of Bramble [1] on multigrid methods and the survey article by Xu [3].

The chapter on multigrid methods contains a motivating example in one space dimension and a careful analysis of two-grid methods. Furthermore, results for general multigrid methods are derived from properties of two-grid methods.

The final chapter on domain decomposition is mostly devoted to additive and multiplicative Schwarz iterations, i.e., domain decomposition methods with overlapping subdomains. In particular, attention is focused on the use of these algorithms on parallel computers.

The book is written in a precise mathematical style with theorems, proofs and algorithms. Furthermore, applications and computational issues are frequently discussed. Throughout the book, Pascal procedures of the suggested algorithms are given and numerical results are presented. The book also contains many exercises, which makes it potentially useful as a textbook in a graduate course on iterative methods.

The author deserves a lot of credit for writing a complete book on iterative methods, which starts with elementary matrix theory and ends with a discussion of multigrid methods and domain decomposition methods for elliptic problems. However, a potential textbook in this area also has to include some necessary background on differential equations and discretization techniques. In my opinion, this book would have been improved if the first chapter had been expanded in order to give more of this background. For example, by presenting indefinite problems, discrete Stokes' problems and discrete integral equations in the introductory chapter, it would have been easier for the author to explain the necessary relations between the iterative methods and properties of the systems. In particular, this would have benefited the readers with a weak background in differential equations. Furthermore, it appears to me that in order to fully understand the two final chapters, it is necessary to have a background in partial differential equations far beyond what is given in the book. Still, I am of the opinion that the author has written a book which will be very influential for the development of computational mathematics for many years to come.

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1. J. H. Bramble, *Multigrid methods*, Pitman Research Notes in Mathematics, 1993.
2. R. S. Varga, *Matrix iterative analysis*, Prentice-Hall, Englewood Cliffs, NJ, 1962.
3. J. Xu, *Iterative methods by space decomposition and subspace correction*, SIAM Rev. **34** (1992), 581–613.
4. D. M. Young, *Iterative methods for solving partial differential equations of elliptic type*, Thesis, Harvard University, 1950.

32[31B30, 35J99, 46N40, 65-02, 65D32].—S. L. SOBOLEV, *Cubature Formulas and Modern Analysis: An Introduction* (Translated from the Russian), Gordon and Breach, Philadelphia, PA, 1992, xvi + 379 pp., $23\frac{1}{2}$ cm. Price $120.00.

This book is an abridged translation from the Russian of S. L. Sobolev's 1974 work 'An Introduction to the Theory of Cubature Formulas'. The cover says that the book has been revised and updated. Comparison with the Russian original suggests that the main revision has been the condensation of the first 13 chapters (pp. 1–644) of the original into the first two chapters (pp. 1–199) of the present volume. The claim that the work has been updated is hardly sustainable, given that the most recent reference is dated 1974, and that the material on cubature in Chapters 3 through 8 is a near verbatim translation of Chapters 14 through 19 of the original.

A cubature formula is an approximate expression for an integral over a domain $\Omega \subset R^n$, of the form

$$\int_\Omega \phi(x)\,dx \approx \sum_{k=1}^{N} c_k \phi(x^{(k)}), \tag{1}$$

where $x^{(1)}, \dots, x^{(n)}$ (the 'nodes') are points in Ω, and $c_1, \dots, c_n$ are real numbers, usually required to be positive.

Cubature (or multidimensional quadrature) has many applications in physics, chemistry, statistics, and other fields, but this book is not directed at people with practical problems: it contains essentially no explicit formulas, and does not discuss any practical issues. Rather, its focus is on the theory of 'optimal' cubature formulas of a certain kind (this notion being explained below), and, as the subtitle suggests, on certain related areas of modern analysis. (The analysis literally comes first: not until p. 205 is cubature defined!)

An optimal formula, roughly speaking, is one that, for a given value of N, is 'best' in the sense that its error functional has the smallest norm for all