essary relations between the iterative methods and properties of the systems. In particular, this would have benefited the readers with a weak background in differential equations. Furthermore, it appears to me that in order to fully understand the two final chapters, it is necessary to have a background in partial differential equations far beyond what is given in the book. Still, I am of the opinion that the author has written a book which will be very influential for the development of computational mathematics for many years to come.

RAGNAR WINTHER

Department of Informatics
University of Oslo
Blindern
0316 Oslo, Norway

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32[31B30, 35J99, 46N40, 65-02, 65D32].—S. L. SOBOLEV, *Cubature Formulas and Modern Analysis: An Introduction* (Translated from the Russian), Gordon and Breach, Philadelphia, PA, 1992, xvi + 379 pp., $23\frac{1}{2}$ cm. Price \$120.00.

This book is an abridged translation from the Russian of S. L. Sobolev's 1974 work 'An Introduction to the Theory of Cubature Formulas'. The cover says that the book has been revised and updated. Comparison with the Russian original suggests that the main revision has been the condensation of the first 13 chapters (pp. 1–644) of the original into the first two chapters (pp. 1–199) of the present volume. The claim that the work has been updated is hardly sustainable, given that the most recent reference is dated 1974, and that the material on cubature in Chapters 3 through 8 is a near verbatim translation of Chapters 14 through 19 of the original.

A cubature formula is an approximate expression for an integral over a domain $\Omega \subset R^n$, of the form

$$\int_\Omega \phi(x)\,dx \approx \sum_{k=1}^{N} c_k \phi(x^{(k)}), \tag{1}$$

where $x^{(1)}, \dots, x^{(n)}$ (the 'nodes') are points in Ω, and $c_1, \dots, c_n$ are real numbers, usually required to be positive.

Cubature (or multidimensional quadrature) has many applications in physics, chemistry, statistics, and other fields, but this book is not directed at people with practical problems: it contains essentially no explicit formulas, and does not discuss any practical issues. Rather, its focus is on the theory of 'optimal' cubature formulas of a certain kind (this notion being explained below), and, as the subtitle suggests, on certain related areas of modern analysis. (The analysis literally comes first: not until p. 205 is cubature defined!)

An optimal formula, roughly speaking, is one that, for a given value of N, is 'best' in the sense that its error functional has the smallest norm for all functions ϕ in an appropriate function class. For Sobolev, not unnaturally, the function spaces are the Sobolev spaces $L_2^m(\Omega)$ or $L_2^m(R^n)$ of functions whose (generalized) derivatives of orders up to m are square integrable. He also generally insists that the formula be exact for all polynomials of degree $\leq m-1$, as a result of which two functions are considered to be equivalent if they differ only by a polynomial of degree $\leq m-1$. The norm in this space is then

$$\|\phi\|_m = \left(\sum_{|\alpha|=m} \frac{m!}{\alpha!} k \int (D^\alpha \phi)^2 dx\right)^{1/2}, \tag{2}$$

where the integrals are over Ω or R^n, and where (in the usual notation for partial differential equations) $\alpha = (\alpha_1, \ldots, \alpha_n)$, $\alpha! = \alpha_1! \cdots \alpha_n!$ and $|\alpha| = |\alpha_1| + \cdots + |\alpha_n|$. An optimal formula is then one for which the norm $\|l\|$ of the error functional l for the integral over Ω is as small as possible.

One deficiency of the book, arising from the 20-year delay of its appearance in English, is that it ignores the very large amount of work that has been done, by Russian and other authors, on optimal formulas in other settings, for example in spaces of functions with bounded L_2 norms of mixed derivatives up to the same order in each variable.

The core of the argument in the setting of this book is that, provided we assume $m > n/2$, the error functional $l(\phi)$ is a bounded linear functional on L_2^m, so that the Riesz representation theorem allows $l(\phi)$ to be represented as an inner product in L_2^m,

$$l(\phi) = (\phi, \psi)_m = \int \sum_{|\alpha|=m} \frac{m!}{\alpha!} D^\alpha \phi D^\alpha \psi \, dx, \tag{3}$$

for some $\psi \in L_2^m$. Repeated integration by parts leads to the characterization of ψ as the solution of the polyharmonic equation

$$\Delta^m \psi = (-1)^m \left[\chi_\Omega - \sum_k c_k \delta(\cdot - x_k)\right], \tag{4}$$

with χ_Ω the characteristic function of Ω. Since $\|l\| = \|\psi\|_m$, the question of minimizing $\|l\|$ can now be tackled by methods of potential theory and partial differential equations.

Since equation (4) is obtained by integration by parts, the absence of boundary terms clearly requires some boundary assumptions on ψ or ϕ if Ω is a bounded region, but the precise setting that is intended at each point of the argument is not always clear.

The analysis nevertheless is of impressive weight, as one would expect from an author of Sobolev's standing. The book can therefore be expected to be of some interest to analysts with an interest in partial differential equations.

Should the book be required reading for numerical analysts with an interest in cubature? Perhaps, but we suspect that they will find it hard going, and the effort arguably not worthwhile. It has to be said, first, that the book is marred by many instances of careless presentation, illustrated by three typographical errors in the very first page. (The first is: 'The product of an $m \times n \ldots$ and a $k \times l$ matrix is... ') While some of the faults are trivial, similar errors occur in material that is often far from trivial, and even the preliminary chapters need to be read carefully, because it is here that the notation (much of it nonstandard) is established.

Errors aside, some readers may find it difficult to establish exactly what the assumptions are—even the 1-dimensional example in Chapter 6 stops short of presenting the actual quadrature formulas obtained with this approach. Instead, it refers the reader to work by Shamalov that is unavailable in the West. In the 1-dimensional setting sharper and more explicit results can be found, for example, in the survey [1] (not among the references of the original or the translation).

As noted above, a key question is: what restrictions, if any, are placed on the behavior of ϕ and ψ at the boundary of Ω? This is surprisingly hard to pin down, because the setting at the start of the cubature discussion seems to be $L_2^m(\Omega)$ but then shifts to $L_2^m(R^n)$.

Finally, our friends in numerical analysis who master this book will, we think, be inclined to question the practical value of the optimal rules produced by the methods of this book. Indeed, the author is admirably frank about the difficulties that face optimal cubature: 'Unfortunately, this choice [of function space setting] is partly dictated by the desire to obtain the problem subject to the method of investigation planned by the author, since the natural formulation itself may turn out to be too difficult.' It should also be mentioned that the optimality discussion in this book does not consider in full generality the choice of nodes: instead, guided by results for the periodic case, 'we ... restrict ourselves to the case when the system of nodes ... is a parallelepipedal lattice, and [we] will only change the parameters of this lattice.' It is generally accepted that for nonperiodic smooth functions on bounded regions the cubature points should in some way be concentrated towards the boundary, and this is recognized in Chapter 5 through the ad hoc addition of extra cubature nodes near the boundary. While this is done in such a way as to preserve the optimal order of convergence, many readers will feel that the treatment of boundary effects remains unsatisfactory.

Ian H. Sloan
Christoph Schwab

School of Mathematics
University of New South Wales
Sydney 2052
Australia

Department of Mathematics and Statistics
University of Maryland Baltimore County
Baltimore, MD 21228-5398

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This 88-page booklet is based on lectures given by the author in the DMV seminar on Computational Number Theory held at Schloß Mickeln, Düsseldorf,