MATHEMATICS OF COMPUTATION
Volume 65, Number 214
April 1996, Pages 877–895

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1991 Mathematics Subject Classification can be found in the annual subject index of *Mathematical Reviews* starting with the December 1990 issue.

14[65-01, 65N30, 65N55, 73Cxx]—*The mathematical theory of finite element methods*, by Susanne C. Brenner and L. Ridgway Scott, Texts in Applied Mathematics, Vol. 15, Springer, New York, 1994, xx+294 pp., 24 cm, $39.00

The book is very well done, and it provides a nice basis for an undergraduate course on Finite Element Methods. In particular, Chapters 0 to 5 constitute an easy and effective approach to the mathematical foundations of the method, as they recall in a simple and convincing way the basic ideas and the main results of the whole theory. In Chapter 0 a first presentation of the Finite Element Method is given on very simple one-dimensional problems. The reader can immediately understand the power of the method, its interest and, at the same time, the need for more sophisticated mathematical tools which are to be used for the analysis of more general cases. Such tools are then introduced in the following chapters. In Chapter 1 the basic information on Sobolev spaces is recalled in an easy and elegant way. In Chapter 2 the variational formulation of elliptic boundary value problems is introduced at a rather abstract level, using mainly one-dimensional problems as examples to help the reader. In Chapter 3 the basic features of Finite Element spaces are presented, wisely skipping, for the moment, isoparametric elements. The general theory of polynomial approximation in Sobolev spaces comes then naturally in Chapter 4 with a major emphasis on affine elements and some hints on isoparametric ones. Finally, in Chapter 5 the variational formulation of elliptic boundary value problems is revisited, this time in the n-dimensional case, and the previous results on the approximation theory are used to get the basic error estimates. Up to this point, the book is a very appealing intermediate work between the two other masterpieces in the field, namely the book by P. G. Ciarlet (more detailed) and the book by C. Johnson (more accessible to beginners), although closer to Ciarlet than to Johnson.

In the subsequent chapters various excursions in different streams of development are made. Specifically, in Chapter 6 the basic concepts of multigrid methods are presented, in Chapter 7 the maximum-norm estimates are derived, and in Chapter 8 the case of nonconforming elements is considered. Chapter 9 introduces the reader to plane elasticity problems and to the related *locking* phenomenon, which is dealt with in more detail in Chapter 10 (Mixed methods). Chapter 11 gives a hint on resolution techniques for algebraic problems arising from mixed formulations, and finally Chapter 12 deals with various applications of operator-interpolation theory, essentially showing how to get error estimates in less traditional norms or in classical norms but with intermediate regularity for the solution.

This second part of the book is also very nice, but does not match the expectations. In some sense, I would have expected glimpses into hotter research areas, such as advection-dominated flows, domain decomposition techniques, hierarchical bases. The relationships of finite element methods with other recent techniques, such as spectral elements or p-methods, finite volumes and wavelets, would also have been welcome. Surely, multigrid methods or mixed methods are the object of very active research, but one cannot use this book as a starting point for working in these areas, as the material presented here is too meager and, sometimes, almost misleading. For instance, the use of linear nonconforming methods to avoid *locking* in linear elasticity is recommended in Chapter 9 of the book for Dirichlet boundary conditions, which is a case of no practical interest, but the information that the method does not work for more general cases is hidden in Exercise 10.x.5 at the end of the subsequent chapter. A simple-minded reader might think that he got a sound way for escaping locking, but in truth he got only a mathematical exercise.

More generally speaking, I think that the recent book by A. Quarteroni and A. Valli is a much more powerful instrument as a starting point for somebody who wants to start doing research. But even Johnson's book, in its simplicity, opens much wider perspectives for a beginning applied mathematician. Still the final part of the present book can be useful for a reader with intellectual curiosity, willing to have some ideas on selected topics that go beyond the basic results of the first six chapters, and I strongly recommend it for that.

A final unhappy remark has to deal with citations. As I said, the book is intended (or it should be) as a didactic one more than one oriented toward research. As such, a basic lack of citations can be tolerated. It is also normal for an author to refer explicitly to his own work more often than to the work of others. But the present authors are overdoing it a bit. Finally, I am curious to know what the basic tools contained in the book are that (quoting from the preface) "are commonly used by researchers in the field but never published". I hope they do not refer to all the results obtained by others (and regularly published), presented here without citations.

FRANCO BREZZI
ISTITUTO DI ANALISI NUMERICA–CNR
VIA ABBIATEGRASSO 209
27100 PAVIA, ITALY

15[00A69, 34-01, 35-01, 65-01]—*Industrial mathematics: A course in solving real-world problems*, by Avner Friedman and Walter Littman, SIAM, Philadelphia, PA, 1994, xiv+136 pp., $25\frac{1}{2}$ cm, softcover, \$22.50

This brief, ground-breaking, scholarly text is written for persons who have a two-year basic Calculus background, that includes functions of several variables, along with rudimentary knowledge of ordinary differential equations, linear algebra, infinite series, vector analysis, and who have elementary computer experience (with at least one of these—Fortran, Pascal, C, Maple, Matlab, etc.). In seven concise chapters, mathematical models for the following topics are covered:

(1) Black and white photography, silver crystal growth.
{Ordinary differential equations, theory and methods for numerical solution.}
(2) Air quality.
{Partial differential equations for advection and for advection-diffusion; numerical methods for solution, stability of difference schemes.}