subtle aspects of Maple. The publishers must surely also take some blame for not ensuring adequate proofreading.

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19[49-02, 49K40, 65K05]—*Nonlinear programming*, by Olvi L. Mangasarian, Classics in Applied Mathematics, Vol. 10, SIAM, Philadelphia, PA, 1994, xvi+220 pp., 23 cm, softcover, $28.50

This influential book on the theory of nonlinear optimization was first published in 1969, and went out of print a few years ago. SIAM has now republished it as part of a series "Classics in Applied Mathematics".

This beautiful book contains a lucid exposition of the mathematical foundations of optimization. The presentation is always simple and concise, and contains complete and rigorous proofs of most of the results. This is a formative book, which can be read by a student with a good background of calculus of several variables, in that it exposes the reader to most of the basic mathematical ideas of nonlinear optimization. The book is self-contained: the excellent appendices and the introduction review all the mathematical concepts needed to understand the material of the book.

There is a very good treatment of convexity and its generalizations—quasi-convexity and pseudoconvexity. Optimality and duality are covered in great generality, and the exposition of theorems of the alternative is a pleasure to read. There is also an extensive discussion of constraint qualification. All of these results make the book a valuable reference.

Much work has been done in convexity and duality in the 25 years since this book was published, and our views of the foundations of mathematical programming have changed. Nevertheless, the theory developed in this book remains central to the foundations of nonlinear optimization, and the style remains as effective and delightful today as when the book was first published. Every person interested in nonlinear optimization should own this book.

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20[49-02, 49M35, 49M45, 65K05]—*Interior-point polynomial algorithms in convex programming*, by Yurii Nesterov and Arkadii Nemirovskii, SIAM Studies in Applied Mathematics, Vol. 13, SIAM, Philadelphia, PA, 1994, x+405 pp., 26 cm, $68.50[1]

The appearance of Karmarkar's method ten years ago opened a new chapter in the study of complexity in mathematical programming, which has since resulted

[1] This is an abridged version of a review that appeared in OPTIMA, a newsletter of the Mathematical Programming Society.

in the production of hundreds of papers. Karmarkar's method had a slight theoretical advantage and a very significant computational advantage over the only other polynomial-time algorithm for linear programming at the time—Khachiyan's modification of the ellipsoid method of Yudin and Nemirovsky and Shor, originally devised for nonsmooth convex optimization. The computational developments since Karmarkar's paper, both for interior-point and simplex methods, have been significant. These developments, however, are not the subject of the present book, which provides a truly comprehensive study of the foundations of interior-point methods for convex programming.

Karmarkar used projective transformations and an auxiliary potential function in his algorithm, which was presented for linear programming problems in a rather restrictive form. A large amount of effort went into understanding and extending these ideas and removing the restrictive assumptions over the next few years. Also, connections with classical barrier methods and methods of centers were established, and the first path-following methods, with a superior theoretical complexity bound, were developed by Renegar and soon thereafter Gonzaga. These led to primal-dual algorithms and the explosion of research referred to above.

At the same time as these developments, mainly concerned with complexity issues and practical computation for linear programming, Nesterov and Nemirovsky began their path-breaking research into what the key elements of interior-point methods were, what allowed polynomial complexity bounds to be established, and to what general classes of problems such analyses could be extended. This book is the result of five years of their investigations.

The key idea is that of a *self-concordant barrier* for the constraint set, a closed convex set or cone in a finite-dimensional space. The notion of self-concordance requires that the convex barrier function satisfy certain inequalities between its various derivatives; roughly, its third and first derivatives should be suitably bounded when measured in terms of its second derivative, which defines a seminorm at every point of the interior of the convex set or cone. These conditions ensure, for instance, that Newton's method behaves nicely in a reasonably global sense. From such a barrier one can construct path-following methods, of either barrier or method-of-centers type; if the barrier satisfies an additional property natural for a convex cone, one obtains potential-reduction methods. In all cases, the number of iterations necessary to obtain an ϵ-optimal solution depends polynomially on $\ln(1/\epsilon)$ and θ, a parameter associated with the barrier.

Chapter 1 of the book provides a very useful overview of the ideas underlying the work and the contents of each chapter. Then Chapter 2 contains the basic definitions and properties of self-concordant functions and barriers, including the beautiful result that every convex set in $\Re^n$ admits a self-concordant barrier (the *universal* barrier) with parameter θ of order n. Chapters 3 and 4 are concerned with path-following and potential-reduction algorithms, respectively, and demonstrate that the main requirement for efficiently solving a convex programming problem (without loss of generality, with a linear objective function) is the knowledge of a self-concordant barrier, together with its first two derivatives, for the constraint set, with a reasonably small value for its parameter. (The authors also show concern for the practical efficiency of variants of their methods.)

Chapter 5 provides tools for constructing such barriers and several examples. While the result quoted above assures the existence of a barrier with parameter of the order of the dimension, such a barrier may not be easily computable. For example, the usual barrier for a polyhedral set in $\mathfrak{R}^n$ defined by m inequalities is the standard logarithmic barrier, with parameter m not n. On the other hand, the cone of positive semidefinite matrices of order n, a set of dimension $n(n+1)/2$, admits a barrier of parameter n. (This cone arises frequently in important optimization problems.) Chapter 6 discusses applications of the tools developed previously to a wide range of nonlinear problems, and hence obtains efficient methods for their solution. Chapters 7 and 8 address extensions to variational inequalities and various acceleration techniques, respectively.

This is a book that every mathematical programmer should look at, and every serious student of complexity issues in optimization should own. For a brief idea of the approach, the first chapter, the introductory material in subsequent chapters, and the bibliographical notes at the end of the book can be read. For a more detailed study, a serious commitment is necessary; this is a technically demanding tour-de-force. The authors provide motivation and examples, but many of the beautiful ideas require long technical analyses. The reader is advised to skim forwards and backwards to help understand some of the definitions and results. For example, the standard logarithmic barrier function $-\sum_j \ln x_j$ for the nonnegative orthant is introduced on page 40 (with related barriers on pages 33 and 34), but it is helpful for motivation and illustration where self-concordant functions are first defined on page 12. Likewise, a hint of the barrier-generated family on page 66 would assist in understanding the definition of a self-concordant family on page 58. The authors' overview in Chapter 1 is also very helpful in showing the direction the argument will take.

There seem to be very few misprints. One possibly confusing one appears in (2.2.16): ω^2 should be ω in the numerator. Also, the material on representing problems using second-order cones (§6.2.3) states that the parameter of the barrier F is $2|\mu|$, whereas it is only $2k$; this error propagates throughout the section in the complexity bounds. And the excellent bibliographical notes were prepared for an earlier version of the book; the chapter-by-chapter remarks need the chapter numbers incremented by one. The bibliography itself is somewhat limited and often refers to reports that have since appeared in print.

In summary, this is an outstanding book, a landmark in the study of complexity in mathematical programming. It will be cited frequently for several years, and is likely to become a classic in the field.

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21[65-02, 65F05, 65F10]—*Iterative solution methods*, by Owe Axelsson, Cambridge University Press, Cambridge, 1994, xiv+654 pp., $23\frac{1}{2}$ cm, $59.95

The best place to start reading this book is in Chapter 5, where Gauss is quoted on the subject of iterative methods: "I recommend this modus operandi. You will hardly eliminate directly anymore, at least not when you have more than two un-