

for the Bernstein-Bézier representation of a polynomial curve in a Euclidean space of arbitrary dimension. This leads naturally into the subject of general matrix subdivision schemes. Chapter 2 discusses the de Rham-Chaikin algorithm and the Lane-Reisenfeld algorithm. Here we encounter the *refinement* equation that plays an important role in wavelets, and one section is devoted to applications in that subject. Chapter 3 concerns representation of curves parametrically by spline functions. Many subtopics are dealt with, such as knot insertion, variation-diminishing properties of the B-spline basis, and connection matrices (which relate adjacent parts of the piecewise polynomial). In Chapter 4 the emphasis is on multivariate splines and their use in surface representation. The geometric interpretation of higher-dimensional spline functions as volumes of slices of polyhedra is central. This chapter closes with a historical vignette in the form of letters by I. J. Schoenberg and H. B. Curry. Chapter 5 discusses, among other topics, blossoming, pyramid schemes, and subdivision for multivariate polynomials. This book of 256 pages offers a concentrated mathematical development of the representation of curves and surfaces by one of the most authoritative experts in the field. It certainly establishes the current status of this rapidly unfolding area.

E. W. C.

**24[41-06, 41A30, 41A99, 65-06]**—*Wavelets: theory, algorithms, and applications*, Charles K. Chui, Laura Montefusco, and Luigia Puccio (Editors), *Wavelet Analysis and Its Applications*, Vol. 5, AP Professional, San Diego, CA, 1994, xvi+627 pp., 23½ cm, \$59.95

This is a formidable book to review. It contains twenty-eight papers, broadly sorted into seven categories. It records the transactions of a conference held in October 1993 at Taormina, Italy. Thirteen of the articles were invited specifically for the volume and fifteen were selected from a pool of contributions.

There are four papers in the first group on multiresolution analysis. Authors represented here are Cohen, Heller, Wells, Dahlke, Goodman, and Micchelli. In the second group, on wavelet transforms, there are three papers, by Terrésani, Kautsky, Turcajová, Plonka, and Tasche. The third group, on spline wavelets, contains four articles, by Steidl, Sakakibara, Lyche, Schumaker, Chui, Jetter, and Stöckler. The fourth group, on other mathematical tools for time-frequency analysis, has four papers, by Donoho, Davis, Mallat, Zhang, Suter, Oxley, Courbebaisse, Escudié, and Paul. In the fifth group there are two papers on wavelets and fractals, by Jaffard and Holschneider. The sixth group addresses numerical methods. Here there are five papers, with authors Dahmen, Prössdorf, Schneider, Bertoluzza, Naldi, Ravel, Karayannakis, Montefusco, Fischer, and Defranceschi. The seventh group is concerned with applications and contains six articles, by Wickerhauser, Farge, Goirand, Wesfreid, Cubillo, Mayer, Hudgins, Friehe, Druilhet, Attié, de Abreu Sá, Durand, Bénech, Olmo, Presti, Denjean, and Castanié.

This volume provides an excellent snapshot of the broad activity in wavelet theory. For readers of *Mathematics of Computation*, the papers in the sixth group may be of particular interest. Here one finds a detailed analysis of Galerkin schemes for pseudodifferential equations on smooth manifolds, a discussion of wavelet solutions of two-point boundary value problems on an interval, a paper on bounds for the

Franklin wavelet, an investigation of using orthonormal wavelets in parallel algorithms for numerical linear algebra, and a study of the Hartree-Fock equation by the use of wavelets.

E. W. C.

**25[65-01, 65D07, 65Y25, 68U07]**—*NURB curves and surfaces: from projective geometry to practical use*, by Gerald E. Farin, A K Peters, Wellesley, MA, 1995, xii+229 pp., 24½ cm, \$39.95

First, many readers, such as this reviewer, need to be told that “NURB” means “nonuniform rational B-spline”. NURBS are basic objects that are the building blocks for representing curves and surfaces. Such representations, in turn, are essential in computerized design, drafting, modeling, and so on. NURBS are sufficiently versatile to fit several distinct systems for computerized design. In the 1950s, such systems grew up independently in different companies (mainly in the automobile and aircraft industries) and even in different branches of the same company. NURBS eventually made it possible to avoid the chaos in this field that the industry was apparently facing. The author gives a little of this history in his preface.

The book is intended as a textbook for a course in computer-aided-design at the beginning graduate level. Prerequisites are knowledge of linear algebra, calculus, and basic computer graphics. Since formal geometry is NOT a prerequisite, the author begins with a snappy account of projective geometry. I particularly like his definition of the projective plane, which requires just three simple sentences. By page 17 we have learned all about pencils, Pappus’ theorem, duality, the affine plane, and various models of the projective plane.

Chapter 2 is devoted to projective maps, affine maps, Moebius transformations, perspectivities and collineations. In Chapter 3, conics are introduced in a manner going back to Steiner. The four-tangent theorem and Pascal’s theorem are proved. In Chapter 4, more concrete representations of conics are considered, in parametric form. Here we meet the Bernstein form of a conic and the de Casteljau algorithm for computing points on it. The notion of a control polygon is introduced in this context. Interpolating conics, blossoms, and polars make their entrance. In Chapter 5, emphasis shifts from projective geometry to affine geometry, which is closer to the environment of most applications. Now the parametric form of a conic appears as a rational function containing “control points” and “weights”. In Chapter 6, “conic splines” are introduced. These are curves made up piecewise from conics, with certain smoothness imposed at the junctions. Chapter 7, one of the longer chapters, discusses rational Bézier curves, which are basic to all piecewise rational curve strategies. We have a Bernstein representation, again with control points and weights, either of which can be manipulated to affect the shape of the curve. There is a projective form of the de Casteljau algorithm, due to the author (1983). Degree raising and reduction, reparametrization, blossoming, and hybrid Bézier curves are all treated. Rational cubics are the subject of Chapter 8. Rational cubic splines are treated from the projective viewpoint in Chapter 9, with second-order smoothness imposed by use of the osculants. Chapter 10 is devoted to the general NURBS. Thus, rational B-splines of arbitrary degree are permitted. The basic operation of knot insertion is described.