Franklin wavelet, an investigation of using orthonormal wavelets in parallel algorithms for numerical linear algebra, and a study of the Hartree-Fock equation by the use of wavelets.

E. W. C.

25[65-01, 65D07, 65Y25, 68U07]—*NURB curves and surfaces: from projective geometry to practical use*, by Gerald E. Farin, A K Peters, Wellesley, MA, 1995, xii+229 pp., $24\frac{1}{2}$ cm, $39.95

First, many readers, such as this reviewer, need to be told that "NURB" means "nonuniform rational B-spline". NURBS are basic objects that are the building blocks for representing curves and surfaces. Such representations, in turn, are essential in computerized design, drafting, modeling, and so on. NURBS are sufficiently versatile to fit several distinct systems for computerized design. In the 1950s, such systems grew up independently in different companies (mainly in the automobile and aircraft industries) and even in different branches of the same company. NURBS eventually made it possible to avoid the chaos in this field that the industry was apparently facing. The author gives a little of this history in his preface.

The book is intended as a textbook for a course in computer-aided-design at the beginning graduate level. Prerequisites are knowledge of linear algebra, calculus, and basic computer graphics. Since formal geometry is NOT a prerequisite, the author begins with a snappy account of projective geometry. I particularly like his definition of the projective plane, which requires just three simple sentences. By page 17 we have learned all about pencils, Pappus' theorem, duality, the affine plane, and various models of the projective plane.

Chapter 2 is devoted to projective maps, affine maps, Moebius transformations, perspectivities and collineations. In Chapter 3, conics are introduced in a manner going back to Steiner. The four-tangent theorem and Pascal's theorem are proved. In Chapter 4, more concrete representations of conics are considered, in parametric form. Here we meet the Bernstein form of a conic and the de Casteljau algorithm for computing points on it. The notion of a control polygon is introduced in this context. Interpolating conics, blossoms, and polars make their entrance. In Chapter 5, emphasis shifts from projective geometry to affine geometry, which is closer to the environment of most applications. Now the parametric form of a conic appears as a rational function containing "control points" and "weights". In Chapter 6, "conic splines" are introduced. These are curves made up piecewise from conics, with certain smoothness imposed at the junctions. Chapter 7, one of the longer chapters, discusses rational Bézier curves, which are basic to all piecewise rational curve strategies. We have a Bernstein representation, again with control points and weights, either of which can be manipulated to affect the shape of the curve. There is a projective form of the de Casteljau algorithm, due to the author (1983). Degree raising and reduction, reparametrization, blossoming, and hybrid Bézier curves are all treated. Rational cubics are the subject of Chapter 8. Rational cubic splines are treated from the projective viewpoint in Chapter 9, with second-order smoothness imposed by use of the osculants. Chapter 10 is devoted to the general NURBS. Thus, rational B-splines of arbitrary degree are permitted. The basic operation of knot insertion is described.

In the remaining chapters, attention shifts to surfaces. Rational Bézier patches play a central role. The bilinear and the bicubic cases are singled out. Surfaces of revolution and developable surfaces are considered specially. Triangular patches, quadric surfaces, and Gregory patches are topics considered in the later chapters. The fifteenth and last chapter gives some examples and a discussion of the IGES standards for NURBS. There is a good bibliography and a good index. Copious references to the literature are made throughout the book.

All-in-all, this is a very appealing book that should have a stimulating effect on the teaching of this important subject. It can certainly be recommended for solo study because of the gentle expository style of the writing.

E. W. C.

26[65-06, 65D05, 65D07, 65D17]—*Designing fair curves and surfaces*, Nickolas S. Sapidis (Editor), Geometric Design Publications, SIAM, Philadelphia, PA, 1994, xii+318 pp., $25\frac{1}{2}$ cm, softcover, \$61.50

This volume, the seventh in a series of geometric design publications from SIAM, focuses on the problem of "visually appealing" line/surface construction. Its twelve chapters explore various ways of (i) defining "fairness" or "shape quality" mathematically, (ii) developing new curve and surface schemes that guarantee fairness, (iii) enabling a user to identify and remove local shape aberrations without global disturbance.

A common thread is the use of differential geometry constructs to express fairness. Thus, we encounter *arc length* s, *curvature* κ, *radius of curvature* $\rho = 1/\kappa$, and *torsion* τ in the study of lines, *principal curvatures* k_1, k_2, *mean curvature* $H = (k_1 + k_2)/2$, and *Gaussian curvature* $K = k_1 k_2$ in the study of surfaces. Typically, not these quantities alone, but also their arcwise derivatives (or divided differences), are the determinants of shape quality. In the volume's broadest-gauged chapter, Roulier and Rando list eight different fairness metrics for lines, of which the first, $\mu = \int [\rho^2 \tau^2 + (\rho')^2]^{1/2}\, ds$, is representative. They propose the minimization of μ over a preselected family of design curves as an answer to (i) and (ii) above. Surfaces are to be treated similarly, with at least five double-integral fairness metrics to choose from.

Other chapters present comparable schemes. Moreton and Séquin construct interpolatory quintic spline curves that minimize the functional $\int \|\vec{\kappa}'\|^2\, ds$, and biquintic surface patches wherein (loosely speaking) the total of such functionals over all lines of principal curvature is minimal. Eck and Jaspert work with point sets only. They interpolate data by a polygon, invoke *difference geometry* to obtain discrete curvature and torsion derivatives $\kappa_i, \kappa_i', \kappa_i'', \tau_i, \tau_i'$ at each inner vertex, and perturb these vertices iteratively, so as to minimize $\sum_i [(\kappa_i'')^2 + (\tau_i')^2]$. Feldman obtains discrete curvature in the same way for a planar polygon with vertices (L_i, X_i), and takes the length μ of the derived polygon (L_i, κ_i) as a fairness metric. His aim is to minimize μ by perturbing the ordinates X_i between prescribed tolerance limits.

Several authors prefer inequality constraints on $\kappa, \kappa', \ldots$ to the metric approach. Burchard et al. fit discrete points in the plane by a circular spline with curvature of uniform sign, monotone and log-convex as a function of s, between designated nodes. Ginnis et al. fit the same points by a polynomial spline of nonuniform degree. They allow small perturbation of the data, one point at a time, and local