

**Supplement to
ON THE BETA EXPANSION
FOR SALEM NUMBERS OF DEGREE 6**

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This supplement contains the two tables described in §3. Table 1 contains a list of all quintuples (a, b, c, m, p) where

$$P(x) = x^6 + ax^5 + bx^4 + cx^3 + bx^2 + x + 1$$

is the minimal polynomial of a Salem number β of degree 6 and $\text{trace}(\beta) = -a \leq 5$. The preperiod m and period p of the beta expansion for β are listed in Table 1 if they satisfy $m < 10000$ and $p < 10000$.

Table 2 contains a list of (a, b, c, m, p) for $\text{trace}(\beta) \leq 10$ provided $\min(m, p) \geq 10000$. This includes 18 values for which only a lower bound on $D = m + p$ is known.

Table 1 (page 2)

Table 1 (Preperiods and periods for trace at most 5)

0 -4 -7	1 60	-3 -13 -19	1 219	* 8	-4 -6 -6	1 1	9	-4 0 3	1 1	8	-5 -12 -14	1 45
0 -2 -3	1 7	-3 -12 -18	1 215	* 1	-4 -6 -7	1 20	6085	-4 0 2	1 1	8	-5 -12 -15	1 35
0 -1 -1	1 7	-3 -11 -15	1 113	* 1	-4 -6 -9	1 11	4393	-4 0 1	1 1	8	-5 -12 -16	58 176
0 -1 -2	1 7	-3 -10 -13	1 43	* 1	-4 -6 -10	1 11	6085	-4 0 0	1 1	8	-5 -12 -17	16 138
-1 -1 -7 -11	2438 863	-3 -9 -11	363 14	* 1	-4 -6 -11	5 33	1 13	-4 0 -1	1 1	5	-5 -11 -18	1 35
-1 -1 -5 -7	1 13	-3 -9 -11	1288 859	* 1	-4 -5 -2	1 9	4 0 -4	-4 0 -4	1 1	5	-5 -11 -11	1 11
-1 -1 -4 -2	1 22	-3 -9 -12	256 717	* 1	-4 -5 -3	1 9	4 0 -5	-4 0 -5	6	35	-5 -11 -12	1 11
-1 -1 -4 -2	1 22	-3 -9 -12	3251 5427	* 1	-4 -5 -4	1 9	4 0 -7	-4 0 -7	1 1	12	-5 -11 -13	1 11
-1 -1 -3 -3	1 35	-3 -8 -8	1 57	* 1	-4 -5 -4	1 9	4 1 3	-4 1 3	1 1	12	-5 -11 -14	1 11
-1 -1 -3 -4	1 11	-3 -8 -10	1 57	* 1	-4 -5 -5	1 20	4 1 2	-4 1 2	1 1	8	-5 -11 -15	1 11
-1 -1 -3 -4	1 20	-3 -8 -11	1004 1674	* 1	-4 -5 -6	1 20	4 1 1	-4 1 1	1 1	8	-5 -11 -16	1 11
-1 -1 -2 -1	7 42	-3 -8 -12	1 15	* 1	-4 -5 -7	1 20	4 1 -1	-4 1 -1	1 1	6	-5 -11 -17	1 17
-1 -1 -2 -3	10 22	-3 -7 -7	1 15	* 1	-4 -5 -10	5 33	350 27	-4 1 -1	1 1	6	-5 -10 -9	1 17
-1 -1 -2 -4	1 24	-3 -7 -8	1 15	* 1	-4 -4 -1	1 17	4 1 -2	-4 1 -2	1 1	6	-5 -10 -11	7 42
-1 -1 -1 -1	1 11	-3 -7 -9	1 15	* 1	-4 -4 0	1 17	4 1 -4	-4 1 -4	1 1	6	-5 -10 -12	1 11
-1 -1 -1 0	1 5	-3 -7 -11	1 20	* 1	-4 -4 -1	1 5	4 1 -6	-4 1 -6	1 1	12	-5 -10 -13	1 11
-1 -1 -1 -1	1 5	-3 -7 -11	1 20	* 1	-4 -4 -2	1 5	4 1 -7	-4 1 -7	1 1	12	-5 -10 -14	1 11
-1 -1 -1 -3	12 11	-3 -6 -5	1 125	* 1	-4 -4 -3	1 5	4 2 0	-4 2 0	1 1	8	-5 -10 -15	1 11
-1 -1 0 -1	1 5	-3 -6 -7	90 74	* 1	-4 -4 -3	1 5	4 2 1	-4 2 1	41	56	-5 -10 -16	1 24
-2 -10 -15	1 31	-3 -6 -8	1 125	* 1	-4 -4 -4	1 20	4 2 -2	-4 2 -2	1 1	8	-5 -9 -7	6 28
-2 -5 -14	1 39	-3 -6 -8	1 125	* 1	-4 -4 -5	1 20	4 2 -2	-4 2 -2	1 1	6	-5 -9 -8	1 9
-2 -5 -11	1 13	-3 -6 -10	3635 60	* 1	-4 -4 -5	1 20	4 2 -2	-4 2 -2	1 1	6	-5 -9 -9	1 9
-2 -5 -11	27 69	-3 -6 -10	1 13	* 1	-4 -4 -6	5 80	4 2 5	-4 2 5	1 1	10	-5 -9 -11	44 43
-2 -5 -11	1 23	-3 -5 -4	1 9	* 1	-4 -4 -6	5 80	4 2 6	-4 2 6	1 1	10	-5 -9 -12	1 11
-2 -5 -11	1 23	-3 -5 -4	1 9	* 1	-4 -4 -7	243 73	4 2 7	-4 2 7	1 1	10	-5 -9 -13	1 11
-2 -6 -7	1 21	-3 -5 -7	79 15	* 1	-4 -4 -7	1 13	4 3 1	-4 3 1	1 1	18	-5 -9 -14	1 24
-2 -6 -8	1 31	-3 -5 -8	1 20	* 1	-4 -4 -8	1 13	4 3 2	-4 3 2	1 1	6	-5 -9 -15	1 9
-2 -6 -9	157 88	-3 -5 -9	1 21	* 1	-4 -4 -8	1 13	4 3 2	-4 3 2	1 1	6	-5 -9 -16	1 28
-2 -5 -5	1 35	-3 -4 -1	1 13	* 1	-4 -3 -2	1 5	4 3 4	-4 3 4	1 1	10	-5 -8 -5	1 9
-2 -5 -6	1 11	-3 -4 -2	1 9	* 1	-4 -3 -2	1 5	4 3 6	-4 3 6	1 1	10	-5 -8 -7	1 9
-2 -5 -7	1 20	-3 -4 -3	1 9	* 1	-4 -3 -4	1 14	4 4 -3	-4 4 -3	1 1	10	-5 -8 -8	1 20
-2 -4 -3	1 17	-3 -4 -4	1 9	* 1	-4 -3 -5	1 14	4 4 -4	-4 4 -4	1 1	6	-5 -8 -9	1 7
-2 -4 -4	7 92	-3 -4 -5	1 20	* 1	-4 -3 -6	1 14	4 4 -5	-4 4 -5	1 1	6	-5 -8 -11	7 93
-2 -4 -5	1 24	-3 -4 -7	31 61	* 1	-4 -2 5	1 17	4 4 -7	-4 4 -7	1 1	7	-5 -8 -12	1 11
-2 -4 -6	1 11	-3 -4 -8	1 17	* 1	-4 -2 4	1 7	4 5 -6	-4 5 -6	1 1	7	-5 -8 -13	1 11
-2 -3 -1	1 13	-3 -3 1	1 17	* 1	-4 -2 3	1 7	4 6 7	-4 6 7	1 1	8	-5 -8 -14	15 33
-2 -3 -2	1 9	-3 -3 0	1 5	* 1	-4 -2 2	10 7	5 -23 -33	-5 -23 -33	* 9101	1 8	-5 -7 -3	19 15
-2 -3 -3	1 9	-3 -3 -1	1 5	* 1	-4 -2 1	86 19	5 -21 -31	-5 -21 -31	1108 546	1 9	-5 -7 -4	1 9
-2 -3 -4	97 49	-3 -3 -2	1 5	* 1	-4 -2 -1	1 5	5 -20 -39	-5 -20 -39	1067 6229	1 9	-5 -7 -5	1 9
-2 -3 -5	1 24	-3 -3 -3	1 5	* 1	-4 -2 -2	1 5	5 -19 -37	-5 -19 -37	1 356	1 9	-5 -7 -6	1 9
-2 -3 -6	1 17	-3 -3 -4	1 20	* 1	-4 -2 -3	1 5	5 -18 -36	-5 -18 -36	1 1497	1 9	-5 -7 -7	1 9
-2 -3 -7	1 5	-3 -3 -5	1 20	* 1	-4 -2 -4	1 14	5 -17 -33	-5 -17 -33	1 82	1 13	-5 -7 -8	1 20
-2 -3 -8	1 5	-3 -3 -6	1 20	* 1	-4 -2 -5	1 12	5 -17 -35	-5 -17 -35	22 40	1 40	-5 -7 -9	1 20
-2 -3 -9	1 5	-3 -3 -7	1 20	* 1	-4 -2 -6	1 12	5 -16 -21	-5 -16 -21	1 198	1 198	-5 -7 -11	60 74
-2 -3 -10	1 5	-3 -3 -8	1 20	* 1	-4 -2 -7	1 12	5 -16 -22	-5 -16 -22	1 151	1 151	-5 -7 -12	1 11
-2 -2 -2	5 80	-3 -2 0	1 13	* 1	-4 -1 7	1 16	5 -16 -24	-5 -16 -24	8867 439	1 13	-5 -6 -1	1 13
-2 -2 -3	1 20	-3 -2 -1	1 5	* 1	-4 -1 6	1 12	5 -15 -19	-5 -15 -19	1 40	1 40	-5 -6 -2	1 9
-2 -1 1	1 7	-3 -2 -3	1 14	* 1	-4 -1 5	1 12	5 -15 -20	-5 -15 -20	1 28	1 28	-5 -6 -3	1 9
-2 -1 2	1 5	-3 -2 -4	1 14	* 1	-4 -1 4	1 7	5 -15 -21	-5 -15 -21	9 100	9 100	-5 -6 -4	1 9
-2 -1 -1	1 5	-3 -2 -5	1 14	* 1	-4 -1 3	1 7	5 -14 -17	-5 -14 -17	1 83	1 83	-5 -6 -5	1 9
-2 -1 -2	1 5	-3 -2 -5	1 14	* 1	-4 -1 2	1 7	5 -14 -18	-5 -14 -18	13 44	13 44	-5 -6 -6	1 9
-2 -1 -3	1 14	-3 -1 4	1 9	* 1	-4 -1 1	7 7	5 -14 -19	-5 -14 -19	376 85	376 85	-5 -6 -7	1 20
-2 0 1	1 8	-3 -1 3	1 7	* 1	-4 -1 0	1 5	5 -14 -20	-5 -14 -20	1 15	1 15	-5 -6 -8	1 20
-2 0 0	1 5	-3 -1 2	1 7	* 1	-4 -1 -1	1 5	5 -13 -15	-5 -13 -15	1 17	1 17	-5 -6 -9	1 20
-2 0 -2	1 12	-3 -1 -1	1 5	* 1	-4 -1 -2	1 5	5 -13 -16	-5 -13 -16	573 515	573 515	-5 -6 -11	63 114
-2 0 -3	1 6	-3 -1 -1	1 5	* 1	-4 -1 -3	1 5	5 -13 -17	-5 -13 -17	1 62	1 62	-5 -6 -12	1 33
-2 -2 -3	1 10	-3 -1 -3	1 5	* 1	-4 -1 -5	1 14	5 -13 -18	-5 -13 -18	1 76	1 76	-5 -5 0	1 17
-3 -15 -23	1 35	-3 -1 -3	1 14	* 1	-4 -1 -6	6 23	5 -13 -19	-5 -13 -19	1 15	1 15	-5 -5 1	1 5
-3 -14 -21	1 218	-3 -1 -5	1 12	* 1	-4 0 5	1 12	5 -12 -13	-5 -12 -13	1 1	12	-5 -5 -1	1 5

Table 2
(large preperiods and periods for trace at most 10)

-3	-1	-7	D >	1199978517
-4	-18	-27	19812	21
-4	-11	-15	55251	10256
-5	-22	-33	8604828	5101
-5	-2	-11	D >	1000817207
-6	-26	-33	D >	3574207
-6	-21	-34	D >	9378241
-7	-25	-43	39420662	93236808
-7	-28	-41	D >	1000000
-7	-19	-15	D >	1808319
-7	-19	-15	464594	88425
-7	-19	-15	16784	13119
-8	-33	-49	D >	1777966
-8	-30	-44	D >	1126603
-8	-26	-38	D >	1689756
-8	-24	-33	69544	1623
-8	-23	-34	D >	1643184
-8	-19	-22	10715	855
-8	-12	-17	18203	3405
-8	-9	-17	25796	424
-8	-3	-17	D >	92762937
-8	11	-18	39493	1229
-9	-37	-55	530787	443
-9	-35	-51	D >	1080156
-9	-28	-41	D >	1157554
-9	-23	-28	1979174	11754
-9	-10	-19	102925	421
-9	-6	-20	D >	99964783
-10	-41	-61	D >	1969014
-10	-40	-59	D >	716885
-10	-36	-52	D >	1532616
-10	-31	-45	542819	1291
-10	-25	-32	653506	87061
-10	-23	-26	16230	33
-10	-11	-21	317310	457
-10	-4	-21	D >	1999955

Table 1 (page 3)

-5	-5	-2	1	5	5	-5	-1	1	1	7	5	4	-6	1	6
-5	-5	-3	1	5	5	-5	-1	0	1	5	5	4	-8	1	10
-5	-5	-4	1	5	5	-5	-1	-1	1	5	5	5	-3	1	28
-5	-5	-5	1	20	5	-5	-1	-3	1	5	5	5	-4	1	10
-5	-5	-6	1	20	5	-5	-1	-4	1	5	5	5	-5	1	6
-5	-5	-7	1	20	5	-5	-1	-5	1	14	5	5	-5	1	6
-5	-5	-8	1	20	5	-5	-1	-6	1	14	5	5	-6	1	6
-5	-5	-9	5	60	5	-5	-1	-7	6	23	5	5	-6	1	6
-5	-5	-10	5	124	5	-5	-1	-8	1	12	5	5	-6	1	6
-5	-4	3	397	124	5	-5	0	-9	1	12	5	5	-6	1	6
-5	-4	2	1	13	5	-5	0	4	1	18	5	5	-6	1	150
-5	-4	1	1	5	5	-5	0	3	1	8	5	5	-6	1	10
-5	-4	0	1	5	5	-5	0	2	1	8	5	5	-7	1	8
-5	-4	-1	1	5	5	-5	0	1	1	8	5	5	-7	1	8
-5	-4	-2	1	5	5	-5	0	0	1	8	5	5	-7	1	8
-5	-4	-3	1	5	5	-5	0	0	1	8	5	5	-7	1	8
-5	-4	-4	1	5	5	-5	0	0	1	8	5	5	-7	1	8
-5	-4	-5	1	5	5	-5	0	0	1	8	5	5	-7	1	8
-5	-4	-6	1	14	5	-5	0	-1	1	5	5	5	-8	1	8
-5	-4	-7	1	14	5	-5	0	-2	1	5	5	5	-8	1	8
-5	-4	-8	1	14	5	-5	0	-3	1	5	5	5	-8	1	8
-5	-4	-9	1	14	5	-5	0	-4	1	5	5	5	-8	1	8
-5	-4	-10	1	14	5	-5	0	-5	1	5	5	5	-8	1	8
-5	-4	-11	1	108	5	-5	0	-6	6	35	5	5	-8	1	8
-5	-3	4	8	30	5	-5	0	-7	1	12	12	5	-8	1	12
-5	-3	3	1	13	5	-5	0	-8	1	12	12	5	-8	1	12
-5	-3	2	1	13	5	-5	0	-9	1	12	12	5	-8	1	12
-5	-3	1	1	13	5	-5	1	-1	1	18	18	5	-8	1	12
-5	-3	0	1	32	5	-5	1	-2	1	12	12	5	-8	1	12
-5	-3	-1	17	42	5	-5	1	-3	1	12	12	5	-8	1	12
-5	-3	-2	1	5	5	-5	1	-4	1	8	8	5	-8	1	12
-5	-3	-3	1	5	5	-5	1	-5	1	8	8	5	-8	1	12
-5	-3	-4	1	5	5	-5	1	-6	1	6	6	5	-8	1	12
-5	-3	-5	1	5	5	-5	1	-7	1	6	6	5	-8	1	12
-5	-3	-6	1	14	5	-5	1	-8	1	6	6	5	-8	1	12
-5	-3	-7	1	14	5	-5	1	-9	1	6	6	5	-8	1	12
-5	-3	-8	1	14	5	-5	1	-10	1	6	6	5	-8	1	12
-5	-3	-9	1	12	5	-5	1	-11	1	6	6	5	-8	1	12
-5	-3	-10	1	104	5	-5	2	-1	3	18	18	5	-8	1	12
-5	-2	7	1	19	5	-5	2	0	2	12	12	5	-8	1	12
-5	-2	6	31	9	5	-5	2	1	1	8	8	5	-8	1	12
-5	-2	5	1	7	5	-5	2	0	1	8	8	5	-8	1	12
-5	-2	4	1	7	5	-5	2	-1	1	8	8	5	-8	1	12
-5	-2	3	1	7	5	-5	2	-2	1	6	6	5	-8	1	12
-5	-2	2	1	7	5	-5	2	-3	1	6	6	5	-8	1	12
-5	-2	1	1	7	5	-5	2	-4	1	6	6	5	-8	1	12
-5	-2	0	1	5	5	-5	2	-5	1	6	6	5	-8	1	12
-5	-2	-1	1	5	5	-5	2	-6	1	10	10	5	-8	1	12
-5	-2	-2	1	5	5	-5	2	-7	1	10	10	5	-8	1	12
-5	-2	-3	1	5	5	-5	2	-8	1	10	10	5	-8	1	12
-5	-2	-4	1	5	5	-5	2	-9	1	10	10	5	-8	1	12
-5	-2	-5	1	14	5	-5	2	-10	1	11	11	5	-8	1	12
-5	-2	-6	1	14	5	-5	2	-11	1	8	8	5	-8	1	12
-5	-2	-7	6	23	5	-5	2	-12	1	8	8	5	-8	1	12
-5	-2	-8	*	20	5	-5	2	-13	1	6	6	5	-8	1	12
-5	-2	-9	1	8	5	-5	2	-14	1	6	6	5	-8	1	12
-5	-2	-10	1	8	5	-5	2	-15	1	6	6	5	-8	1	12
-5	-1	8	1	12	5	-5	3	-7	1	10	10	5	-8	1	12
-5	-1	7	1	12	5	-5	3	-8	1	10	10	5	-8	1	12
-5	-1	6	1	9	5	-5	3	-9	1	10	10	5	-8	1	12
-5	-1	5	1	7	5	-5	3	-10	1	20	20	5	-8	1	12
-5	-1	4	1	7	5	-5	3	-11	6	112	112	5	-8	1	12
-5	-1	3	1	7	5	-5	3	-12	1	6	6	5	-8	1	12
-5	-1	2	1	7	5	-5	3	-13	1	6	6	5	-8	1	12
-5	-1	1	1	7	5	-5	3	-14	1	6	6	5	-8	1	12
-5	-1	0	1	7	5	-5	3	-15	1	6	6	5	-8	1	12