9. Definite Integrals,	11 ex,	59 pg.
References (85 listed),		6 pg.
Notation,		2 pg.
Index,		7 pg.

E.I.

39[65N30, 65N50, 65N55, 82D99]—*Multigrid methods for semiconductor device simulation*, by J. Molenaar, CWI Tract, Vol. 100, Centre for Mathematics and Computer Science, Amsterdam, 1993, vi+134 pp., 24 cm, softcover, Dfl. 40.00

In recent years, multigrid methods have found their way into a number of important application areas. Often it seems that the careful analysis which would lead to a useful evaluation of multigrid as a viable method for such problems is ignored or lost along the way. However, from time to time an article or book appears which studies a specific application in depth, and also examines in detail the theoretical and computational properties of the multigrid method for the particular problem. The book is an example of such a study.

The book, based on the author's PhD thesis, is concise and well organized, consisting of seven chapters and two appendices. The material begins in Chapter 1 with an overview of the three main approaches to device simulation, followed by a more detailed description of one of these approaches, namely the numerical solution of the drift-diffusion equations proposed by Van Roosbroeck in the 1950s. The drift-diffusion equations are examined carefully, including the scaling problems, which lead to the use of various formulations as alternatives to the Slotboom variable representation. Basic discretization issues for the drift-diffusion model are also presented, including a discussion of the importance of the well-known Scharfetter-Gummel discretizations, leading into a discussion of the mixed finite element approach (the subject of Chapter 2). The introductory chapter ends with a quick look at methods for the resulting discrete equations (multigrid, Newton-type methods, etc.), and with a detailed outline of the remainder of the book.

Chapter 2 consists of a careful discussion of mixed finite element discretization of the semiconductor drift-diffusion equations, in both one and two dimensions. The motivation for the use of a dual mixed finite element approach is that it can be viewed as a mechanism for extending the Scharfetter-Gummel scheme to more than one dimension, at the same time having available the complete analysis framework that the finite element method provides for an examination of the error. A discretization of a general model elliptic equation is derived in two dimensions on a rectangular mesh. However, as has been shown in other contexts [1], such a discretization is not stable in the sense that an M-matrix is not obtained (a discrete maximum principle is then not available). However, the author obtains an M-matrix through the use of mass-lumping, and his analysis shows that the quadrature rule this corresponds to does not spoil the accuracy of the discretization. (The author does not make it clear that such an approach will not work in three dimensions, since an M-matrix cannot be recovered simply by mass lumping [1].) Applying the discretization to drift-diffusion equations yields the sought-after two-dimensional extension of the Scharfetter-Gummel scheme.

Chapter 3, one of the most interesting chapters of the book, takes a close look first at abstract optimization problems, and then more specifically at special relaxation methods (which find use later in the book as multigrid smoothers). The weak-form equations used in Chapter 2 are reformulated as variational problems in two different ways, yielding constrained optimization problems. Taking this dual view of nonlinear problems is a common technique in developing robust solution algorithms for the weak-form equations. This technique consists of (1) selecting a minimization problem (either the natural one in the Euler-Lagrange sense, or simply the norm of the residual of the weak equations) such that the solution to the minimization problem also solves the weak-form equations, and (2) constructing a solution algorithm for the weak-form equations which is guaranteed to decrease the value of the "cost" function of the associated minimization problem. Convergence of the method thus constructed is then assured. The author considers two relaxation methods (superbox and Vanka-type) for the weak-form equations, and shows that each minimizes an appropriate associated functional, and hence convergence is guaranteed. The chapter finishes with a local mode analysis of each relaxation method in an attempt to evaluate the possible effectiveness of each as a multigrid smoother.

Chapters 4 and 5 consider two- and multigrid methods based on the two relaxation methods of Chapter 3, for a mixed finite element discretization of the Poisson equation. A standard two-grid analysis is performed, yielding bounds on the spectral radii and norm of the error propagator. It is shown that while the "canonical" grid transfer operators may cause a problem in the one-dimensional case for certain orderings in the relaxation, no such problem occurs for the two relaxation methods in two dimensions. While a similar multigrid analysis is not possible, the author describes in detail a nonlinear multigrid algorithm for the full nonlinear drift-diffusion model. He focuses mainly on Gummel's method as an outer nonlinear iteration, and solving the individual equations in each Gummel iteration with multigrid. Many numerical results are presented, illustrating the effectiveness of the two relaxation methods, as well as the overall multigrid method. A claim is made that employing simply (an inexact) Newton's method and possibly a linear multigrid method for the Jacobian systems would not be as effective, owing to the likely ill-conditioning of the Jacobians. However, no comparisons are made numerically, and it is not clear that such a Newton/linear multigrid combination would not be equally effective (see for example [2]).

Chapter 6 considers the use of adaptive mesh refinement with the methods presented in Chapter 5, and a particular refinement criterion is proposed. Some numerical experiments comparing such an adaptive implementation with a uniform mesh implementation show convincingly the utility of adaptive mesh refinement for this problem. The book finishes in Chapter 7 with a careful examination of two different multigrid discretization frameworks, based on two different mixed finite element approaches. It is shown that each is equivalent to a particular box (finite volume) discretization, and leads to either the so-called cell-centered or vertex-centered multigrid methods. It is concluded after some analysis that the vertex-centered approach is both more robust and has faster convergence properties than the cell-centered approach. Two appendices contain some descriptions of various implementation issues which arose in the work.

This book is clearly written and contains few typographical errors. It could be used in a course on numerical semiconductor modeling, or in a course on advanced multigrid techniques for nonlinear elliptic systems.

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40[73-06, 73K05, 73K10, 73K15, 73V25]—*Asymptotic methods for elastic structures*, Philippe G. Ciarlet, Luís Trabucho, and Juan M. Viaño (Editors), de Gruyter, New York, 1995, viii+297 pp., $24\frac{1}{2}$ cm, $128.95

This book is the proceedings of the international conference on "Asymptotic Methods for Elastic Structures" held October 4–8, 1993 in Estoril, Portugal. Twenty-one of the twenty-three speakers at the conference contributed papers to this volume, most of which are between ten and fifteen pages in length. The papers deal with a variety of topics in the theory of beams, plates, rods, shells, and their assemblages. The unifying theme is that all these models are lower-dimensional approximations to higher-dimensional elastic structures which have a small thickness. Some of the topics considered are numerical approximation of the models, existence and uniqueness results, controllability, convergence and error estimation between the original and reduced model, the modelling of problems with junctions, and derivation and justification of models by asymptotic expansions.

R.S.F.

41[65-00, 65-04, 41-00, 41-04, 41A15]—*Handbook on splines for the user*, by Eugene V. Shikin and Alexander I. Plis, CRC Press, Boca Raton, FL, 1995, xii+221 pp., 24 cm, $69.95

According to the authors, this book is intended as a handbook for prospective and active spline users. It is not a textbook, it does not provide any proofs, only states results, and it is limited to the description of cubic splines techniques in one and two dimensions, and their implementations, including a set of programs on diskette.

The book consists of four chapters, each one designed to be read independently of the others. Chapter 1 deals with univariate cubic splines for interpolation (with various end conditions) and smoothing. In Chapter 2, the corresponding tensor-product versions, i.e., interpolating and smoothing bicubic splines, are presented. Spline curves are the topics of Chapter 3. After a short introduction of basic curve theory, there are subsections on cubic Bézier curves, B-spline curves, Beta-splines and one on other approaches such as Hermite, Catmull-Rom and implicitly defined spline curves. Finally, spline surfaces are described in Chapter 4. Again, after some