

which are different from the ones in *The Maple V Handbook*, would quickly become a skilled Maple user.

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47[11-02, 11Gxx]—*Arithmetic of algebraic curves*, by Serguei A. Stepanov, (translated from Russian by Irene Aleksanova), Monographs in Contemporary Mathematics, Consultants Bureau, New York, 1994, xiv+422 pp., 26 cm, \$115.00

Based on a lecture series given at the Tata Institute of Fundamental Research, this book was intended to give a full picture of the state of the art in Diophantine equations. However, owing to space limitations, the author had to leave out analytic aspects connected with the circle method of Hardy and Littlewood. Also the logical aspects in connection with Hilbert's 10th problem had to be limited in size. Even within the remaining techniques from arithmetic algebraic geometry the author had to make choices. What remains, roughly speaking, is an extensive treatment of points on curves over finite fields, the study of integral points on curves over algebraic number fields and a discussion of Hilbert's 10th problem in a (too short) Appendix.

In the study of points on curves over finite fields the author has made an important contribution by giving an elementary proof of the Riemann hypothesis for the zeta-function of such a curve. This proof has later been streamlined by E. Bombieri who used the Riemann-Roch theorem rather than Stepanov's original estimates. This adapted proof, which cannot be found in many other books, is presented in Chapter 5. Before getting to that level, the reader in the preceding chapters was already led through several excursions into topics of number theory. For example in Chapters 1 and 2 we find a treatment of exponential sums and their estimates. In particular, we find a full discussion of Burgess's inequalities which, again, is not often found in other books. One of the interesting consequences of Burgess's estimate is that the least quadratic nonresidue modulo a prime p is bounded by $O_\varepsilon(p^{1/4\sqrt{\varepsilon}+\varepsilon})$ for any $\varepsilon > 0$. The estimates of exponential sums are based on the estimates arising from the Riemann hypothesis for curves and techniques from Vinogradov.

The third chapter deals with rational points on algebraic curves of genus zero and one. The approach is very much in the spirit of Mordell's book on Diophantine

equations and scratches only the surface of the subject. For example, in the case of genus-zero curves, the theorem that an odd-degree curve defined over the rationals has infinitely many rational points is not mentioned. As for genus-one curves, it is clear that one cannot give a concise overview of the theory of their rational points in a single chapter. The fourth chapter contains an introduction into the geometry of curves and the Riemann-Roch theorem. Unlike the previous chapter, this chapter is closely analogous to standard algebraic geometry texts on the subject.

The second main theme of the book is on integral points on plane algebraic curves. Suppose the curve C is given by the equation $f(x, y) = 0$ with $f \in \mathbf{Z}[x, y]$. We look for points $(x, y) \in \mathbf{Z}^2$ on this curve. A classical theorem of Siegel states that the number of such points is finite if $\text{genus}(C)$ is positive or if C has at least three distinct points at infinity. This statement can be generalized to algebraic number fields and their integers and to so-called S -integers, where primes from a fixed finite set are allowed (Siegel-Mahler theorem) in the denominator. The proof of Siegel's theorem contains two ingredients which make the theorem ineffective, that is, it does not give a procedure to compute the finite set of solutions. The first is the rank of the rational points on the Jacobi variety of C , the second is the use of Siegel's method in Diophantine approximation. As suggested by Robinson and Roquette, it is possible to eliminate the ineffective part arising from the rank computation by use of methods from nonstandard arithmetic. It is this proof which is given in Chapter 7 of the present book. Unfortunately, nonstandard analysis/arithmetic is not in the reviewer's area of expertise, so I cannot make any comments here.

As a final remark I should say that the author has relegated much of his material to exercises, which are abundant and form a valuable part of the book. Some of them are quite hard. However, it is certainly worthwhile for a number theorist to browse through them. The author has collected therein many small interesting facts, and they pose interesting challenges for the reader or his or her students.

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