

The paper by Lorenzi deals with finding the characteristics of one-dimensional dispersive media governed by Maxwell's equations.

The book is a valuable guide to the current state of knowledge about inverse problems in diffusion processes.

M. CHENEY

MATHEMATICAL SCIENCES DEPARTMENT
RENSSELAER POLYTECHNIC INSTITUTE
TROY, NY 12180

3[65-02, 45A55, 65D32, 65D30]—*Lattice methods for multiple integration*, by I. H. Sloan and S. Joe, Oxford University Press, New York, 1994, xii+239 pp., 24 cm, \$69.95

When Milton Abramowitz said in his article [1] that a common experience of applied mathematicians was to have a scientist come to the office and say "I have an integral", he was primarily referring to one-dimensional integrals. In the early days of numerical computing, multiple-dimensional integrals (cubatures) were avoided if possible, or manipulated into one-dimensional problems. As computing power increased, however, more attention was directed to the numerical evaluation of practical integrals in several variables. But long before numerical libraries contained reliable cubature methods, statisticians and others were computing high-dimensional integrals using Monte Carlo techniques, and mathematicians were producing elegant numerical approximations, based on rules for integrating polynomials or trigonometric polynomials exactly. (For an interesting history of cubatures, from Maxwell's brick to adaptive techniques, see [2].) Lattice methods provide a link between elegant, if impractical, cubature methods, and the practical, if inaccurate, Monte Carlo methods of the statisticians. They are of interest in themselves, regardless of applications, and are the basis of an algorithm for integration over hypercubes of, in theory, any dimension. The preface of this book states that it is aimed not only at those who might be interested in lattice methods for their own sake, but also those who have practical integrals to compute. The book certainly fulfills the first claim, but it is not clear that it provides practical help to those with high-dimensional integrals to approximate.

Chapter 1 provides a nice introduction to cubature, and to the idea of trigonometric degree. The main topic of the book is introduced in Chapter 2, where lattice rules are defined, and the early history of these is discussed. (The history of lattice rules is scattered throughout the book, with recent results and historical summaries appearing in several chapters.) In Chapter 3 the concept of *rank* of a lattice rule is introduced, together with a canonical form for a general lattice rule, based on *invariants* of the rule. The original lattice rules, called *rank-1* rules, are discussed briefly in Chapter 4, and higher rank rules are dealt with in Chapters 5–7. Since lattice rules are intended for periodic integrands, some modifications are required either to the function or to the formula if the function is not periodic. Methods for periodising the function are discussed in Chapter 2, while modifications of the rules are discussed in Chapter 8. Chapter 9 contains some miscellaneous topics which do not fit anywhere else in the scheme of the book. This takes us to page 164 in a book which, apart from the appendices, contains 215 pages. In the remaining 50 or so pages, practical methods for integration, and comparisons with existing methods,

are dealt with. The only method described is based on rank-1 rules, and a sequence of embedded rules leading to a full rank rule. In addition, the rank-1 rules used are obtained by means of the restricted Korobov search. (It is worth noting that the only other lattice based software available, in the NAG scientific subroutine library, is based on rank-1 rules obtained in a similar manner.) Pseudo-code for the method is provided, and two appendices contain “recommended choices” for the rank-1 rules. It would perhaps have been more useful to have provided an ftp address for the reader to obtain the actual programs and tables electronically.

As an overview of lattice methods and their properties the book reads well. At times, however, the reader gets the feeling that the whole story is not yet known, and that results are obtained by a process of erosion. Chapter 9 is a prime example, where results on the numbers of rules with specific order, or invariants are given. The book provides an accessible entry point for anyone who wishes to understand the essentials of lattice rules, and armed with it, the interested reader will be more ready to deal with the sections on lattice rules in [3] and [4].

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PATRICK KEAST

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE
DALHOUSIE UNIVERSITY
HALIFAX, NOVA SCOTIA B3H 3J5, CANADA

4[44-01, 42A38, 44A10, 44A15, 44A20, 33C10, 33C25]—*Integral transforms and their applications*, by L. Debnath, CRC Press, Boca Raton, FL, 1995, xvii+457 pp., 24 cm, \$69.95

This book gives the standard integral transforms (Fourier, Laplace, Hankel, Mellin, Hilbert, Stieltjes, finite Fourier and Laplace, Z transforms and transforms with orthogonal polynomials (Legendre, Jacobi, Gegenbauer, Laguerre and Hermite)). In all chapters applications are given (Laplace transform applications are given in a separate chapter) for all kinds of boundary value problems, there is an Appendix with main properties of special functions that are used as kernels, and there are thirteen tables of integral transforms. Each chapter has worked examples, applications and exercises, and there is an extensive bibliography and a section with hints and answers to selected exercises.

The book was developed as a result from teaching advanced undergraduates and first-heat graduate students in mathematics, physics and engineering, and the author felt the need to provide lecture notes that were not too advanced for the beginner. This gives the book a quite recognizable place between other texts and