of palindromic continued fractions. There are applications to statements of when the class group is a 2-group. Chapter VII (Infrastructure) discusses a concept of Shanks, namely the "quadratic residue cover" for a function, i.e., a finite set of primes such that every value of the function is divisible by or is a residue of one of the primes. If the function is a sequence of discriminants, there is ready-made information on nonprincipal ideals, which relates to the class number.

Chapter VIII (Algorithms) is a nonidiosyncratic discussion of factorization and primality. The final section of the text, however, on Computation, is a thoughtful commentary, (basically accepting computers as a fact of life). It also almost reads like a personal testament of the author, but for that matter so does the whole book.

This book is followed by an appendix of 85 pages of numerous tables for units and class numbers, etc. There are also commendably dozens of tables in the text illustrating many of the theorems. These show the author's intense desire to inspire experimentation and reader participation.

Yet mathematics books also have browsers, who want to open a book and come up with a result (maybe also with a proof), just the way a brewery has visitors who do not want to buy the brewery, but just want to enjoy a free beer. The author does not make it easy for such casual readers. Most results have some nonstandard symbol, abbreviation, or neologism which requires further cross-references by the reader, perhaps causing the uncommitted reader's curiosity to wane.

The classical style once required that the more important the theorem the fewer the symbols and plainer the prose. The reviewer wishes the author were more so inclined, but the book is still well worth the effort (and the extra effort).

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12[11A41, 11-04, 11N36, 11Y60, 68M15]—*Enumeration to* 10^{14} *of the twin primes and Brun's constant*, by Thomas R. Nicely, Virginia Journal of Science **46** (1995), 195–204

The set $S = \{(3,5),(5,7),(11,13),\dots\}$ of twin prime pairs has been studied by Brun (1919) and more recent authors. It has never been proved that S is infinite, although the appropriate Hardy-Littlewood conjecture and numerical evidence strongly suggest that it is. Brun showed that the sum B of reciprocals of twin primes converges (unlike the sum of reciprocals of all primes). However, the sum defining B converges very slowly and irregularly.

Nicely's paper gives counts $\pi_2(x)$ of the number of twin prime pairs $(q, q+2)$ such that $q \le x$, and the corresponding sum of reciprocals $B(x)$, for various $x \le 10^{14}$. The most extensive previously published computation, by the reviewer (1976), went only to $x = 8 \times 10^{10}$.

Many of Nicely's values of $\pi_2(x)$, including those for $x = 10^{13}(10^{13})10^{14}$, have been confirmed in an independent computation performed by J. Kutrib and J. Richstein (personal communication from J. Richstein, September 21, 1995). Similarly for $B(x)$ (to at least 16 decimal places). We record Nicely's values

$$\pi_2(10^{14}) = 135780321665$$

and

$$B(10^{14}) = 1.82024496813027052889471783861953382834649$$

(believed to be correct to all places given). By extrapolation using the Hardy-Littlewood conjecture, Nicely estimates

$$B = 1.9021605778 \pm 2.1 \times 10^{-9},$$

where the error estimate is not a rigorous bound but corresponds to one standard deviation using a statistical model based on some plausible assumptions. This is consistent with the previous estimate, by the reviewer, of $B = 1.9021604 \pm 5 \times 10^{-7}$ (using a more conservative methodology for the error estimate). A better estimate, based on a subsequent computation to $x = 2.5 \times 10^{14}$, is

$$B = 1.9021605803 \pm 1.3 \times 10^{-9}$$

(personal communication from T. Nicely, April 4, 1996).

Nicely's computations were performed on Intel 80486 and Pentium computers; the sieving speed on a Pentium was about 10^{11} integers per day. In such an extensive computation there is a significant probability of error. The author deserves full credit for his careful error checking which uncovered several potential sources of error, including logical flaws, compiler/library bugs, disk and memory problems, and, last but not least, a design flaw in the floating-point unit on certain Pentium chips. The latter received widespread publicity. On December 20, 1994, Intel offered to replace faulty Pentium processors free of charge.

Nicely's paper is of general interest as a warning of how careful one needs to be in a very extensive computation. Anyone who is interested in large-scale computations should be able to learn something of value from the paper. Computer hardware designers and compiler writers might also learn some useful lessons.

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