the matter; the rigor and detail are out there if you want to look for them.

Given this framework, "Afternotes" is not intended to fulfill all of the needs of a full-fledged textbook. Exercises are few in number and not intended to supplant routine problems or computer projects. Fully two-thirds of the lectures are devoted to matters dealing with linear and nonlinear equations and floating-point arithmetic. Therein lies the strength of the book; Stewart knows just how to illuminate important computational issues, such as effects of conditioning. On the other hand, the remaining third of the lectures deal with standard fare, such as interpolation, numerical integration, and numerical differentiation, and here Stewart manages to hit the high spots with just the right effort for a course at this level.

Notably absent, then, is a section on initial-value problems for ordinary differential equations. One might argue that the course is already packed; nevertheless, inclusion in the notes of a section on ODE's to be used at the discretion of the instructor might be useful.

As befits an expert in the field, Stewart not only covers standard fare skillfully, but it is his introduction to, and treatment of, topics not always encountered at this level, e.g. perturbation theory and backward error analysis, which gives this work much of its value. Other examples of material presented here and not readily found in most texts include a linear-fractional method and a hybrid scheme for solving $f(x) = 0$, and an indication of the role of row vs. column orientation for algorithms for solving linear systems.

As in many texts, a goal here is to point out the nuances and possible pitfalls in numerical computation. This Stewart accomplishes, while at the same time injecting into the presentation an appropriate dose of humor. Notable examples include a spirited conversation between scientist Dr. XYZ and you, the reader, as the numerical analyst, discussing the backward error analysis of summation, and an admonition to "economists and astrologers" on the dangers of unsafe extrapolation (you'll have to read this one for yourselves).

To conclude, we have already seen some reasons (notably, lack of exercises), which prevent this set of notes from being the sole text for a course. We might also ask from the author some guidance concerning his expectation of the reader's knowledge of programming languages, since code segments in differing languages are interspersed throughout the text. But, in the final analysis, what we do have here is an excellent set of notes which might be used to reinforce materials from other sources, or, depending upon the audience, to use as primary lecture material, with a traditional text kept in the bullpen for more detail. Either way, read and enjoy!

STEVEN M. SERBIN
DEPARTMENT OF MATHEMATICS
UNIVERSITY OF TENNESSEE
KNOXVILLE, TN 37996-1300

15[76-02, 65M60]—*Navier-Stokes equations and nonlinear functional analysis*, by Roger Temam, CBMS-NSF Regional Conference Series in Applied Mathematics, Vol. 66, second edition, SIAM, Philadelphia, PA, 1995, xiv + 141 pp., 25 cm, softcover, $26.50

This short monograph is volume 66 of the CBMS-NSF regional conference series in applied mathematics. It is the revised second edition of a text that was published in 1983 as volume 41 of the same series. The author states in the introduction that "the mathematical study of the Navier-Stokes equations is difficult and requires the full force of modern functional analysis. Even now, despite all the important work done on these equations, our understanding of them remains fundamentally incomplete". His aim while writing these notes was to "arrive as rapidly and simply as possible at some central problems in the Navier-Stokes equations", hoping that they would stimulate interest in these equations.

Most of the material presented in the first edition of the book is still relevant and has not been changed, except for the correction of a few (but not all) misprints. The revision consists in the addition of an appendix devoted to inertial manifolds and an update of the rather extensive bibliography (almost 200 items).

The book is divided into three parts, each addressing a group of topics of central importance. Part I addresses questions concerning existence, uniqueness and regularity. Temam begins by recalling the initial-boundary value problems associated with the Navier-Stokes equations (NSE). Then he presents a functional analytic setting for the equations, emphasizing the case of flow in two and three space dimensions with space periodic boundary conditions. This leads to many technical simplifications, while retaining the main mathematical difficulties (except, of course, those related to boundary layers). The modifications needed for physically relevant boundary conditions are given without proofs. This is followed by a presentation of the classical existence and uniqueness results based on the standard Galerkin approximation procedure. The classical *a priori* estimates are derived in detail and the compactness argument leading to convergence of the approximations is outlined. He then proceeds to discuss more advanced topics such as the fractional dimension of the set of singularities of a solution, analyticity in time, compatibility conditions at $t = 0$ (for non-periodic boundary conditions), and the Lagrangian representation of the flow.

Part II deals with the long-time behavior of solutions. Three topics are developed here: Temam first presents a theorem on the number of stationary solutions based on an infinite-dimensional version of Sard's theorem. Then he proves what he calls the "squeezing property": the flow is essentially characterized by a finite number of parameters. Finally he proves that functional invariant sets (e.g. attractors) have finite Hausdorff dimension.

Part III addresses questions related to numerical approximation. Temam presents a completely discrete scheme, that combines an alternating direction method in time with a finite element method in space. Then he gives a detailed convergence proof based on a new (as of 1983) compactness theorem. He also shows that the long-time behavior of eigenfunction Galerkin approximations is completely determined by a fixed finite number of its eigenmodes, a result that adds to the evidence that the long-time dynamics of the NSE is finite dimensional.

Finally, a new appendix is devoted to inertial manifolds. Roughly speaking, an inertial manifold of an evolution system is a smooth finite-dimensional manifold that attracts every solution exponentially fast. Thus, if an inertial manifold exists, then after an initial transient the dynamics of the system is essentially determined by a finite-dimensional system of ODE's, the inertial system. Temam first outlines the basic existence result for inertial manifolds, emphasizing the so-called spectral gap condition: there must be sufficiently large gaps in the spectrum of an associated linear operator. Noting that the gap condition fails for the NSE, Temam then presents a recent attempt to circumvent the difficulty, due to Kwak, that is based on embedding the 2D space periodic NSE into a larger system of reaction-diffusion type, for which the gap condition is less severe. However, after the publication of this book it was found that this argument is incomplete, and it is still unknown whether the NSE possess an inertial manifold or not. The gap in the proof is related to the fact that the linear part of the new system is no longer self-adjoint, as will be explained in the forthcoming second printing of the text.

I think that Temam has achieved his goal: using these notes a reader with a good background in functional analysis will go quickly to some advanced topics in the Navier-Stokes equations. With its ample bibliographical comments the text is also a nice survey of interesting questions, techniques, and results.

STIG LARSSON
CHALMERS UNIVERSITY OF TECHNOLOGY AND GÖTEBORG UNIVERSITY
S-41296 GÖTEBORG, SWEDEN
E-mail address: stig@math.chalmers.se

16[76-02, 76D05, 76Mxx, 65Mxx]—*Numerical solution of the incompressible Navier-Stokes equations*, by L. Quartapelle, International Series of Numerical Mathematics, Vol. 113, Birkhäuser, Basel, 1993, xii + 291 pp., 24 cm, $100.00

The professed aim of this book is to "give a unitary (unified?) view of the methods which reduce the equations for viscous incompressible flows to a system of second-order equations of parabolic and elliptic type". Such methods are popular especially with engineers and have the advantage of avoiding the approximation of the incompressibility condition and the numerical stability issues associated with such approximations. These "nonprimitive variable" methods on the other hand suffer from the difficulty of providing meaningful boundary conditions for the new variables, such as the vorticity, that are introduced. Much of the original work contained in this book consists in deriving appropriate conditions which turn out to be of integral type. Additionally, the equivalence of these various formulations with the original primitive variable equations is demonstrated.

Chapter 1 is introductory in nature and contains a short but interesting discussion of the issue of compatibility of initial data. Chapter 2 is concerned exclusively with two-dimensional flows. The classical vorticity-stream function formulation is derived. The lack of boundary conditions for the vorticity ζ motivates the elimination of ζ and the formulation of a fourth-order (biharmonic) equation for the stream function ψ. Alternatively, integral vorticity conditions are derived which allow the uncoupling of ζ and ψ. The integral nature of these conditions is compatible with the fact that ζ is less regular than the velocity $\mathbf{u}$. Implementations of these integral conditions by means of finite difference, finite element and spectral methods are also given.

In chapter 3, the discussion is extended to three-dimensional flows. The technical issues are vastly different since both the vorticity and stream function are now vector variables. Indeed, there are six unknowns now instead of four in the original formulation.

Chapter 4 is devoted to vorticity-velocity formulations in both 2 and 3 dimensions. Even though there are several advantages associated with them, such "hybrid" formulations have remained relatively unexplored.