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REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

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19[65–02, 65G05]—*Lectures on finite precision computations*, by Françoise Chaitin-Chatelin and Valérie Frayssé, Software, Environments and Tools, SIAM, Philadelphia, PA, 1996, ix + 235 pp., $25\frac{1}{2}$ cm, softcover, $44.50

Serious textbooks on Numerical Analysis have an initial part which considers finite precision computation. Standard topics are the generation and propagation of round-off, condition, forward and backward errors, and numerical stability. The extent to which this material is referenced later varies considerably; but whenever it occurs, a *regular* situation is generally assumed. Only recently, numerical analysis *research* has attempted to gain a more precise understanding of the phenomena governing finite precision computation: On the one hand, normwise perturbation bounds have been replaced by componentwise bounds, and "sufficiently small" perturbations by ones of realistic size; on the other hand, attention has been focused on computations which proceed near a singularity of the data-result mapping, a fact which may strongly alter the expected behavior of the computation.

The volume under review gives the first comprehensive presentation of this new research area to which the authors have heavily contributed themselves. It offers fascinating reading for numerical analysts of any flavor because the results concern fundamental aspects of their work; also the numerous examples throughout the book deal with linear algebra and univariate polynomial zeros which are common ground for everybody and from where one is able to connect to one's own area of interest.

Unavoidably, the treatise begins with the general concepts, formulated in the customary framework. But the second part of the introductory Chapter 2 points clearly beyond the classical syllabus: It introduces the influence of singularities, arithmetically robust convergence, finite precision computability for iterative and approximate algorithms, and a detailed discussion of the chaotic iteration $x_{n+1} := rx_n(1 - x_n)$. Chapter 3 on condition measures for regular problems gives the most comprehensive collection of expressions for normwise and componentwise condition ever published in one place, with many illuminating examples and remarks.

Chapter 4 deals with computation near a singularity. Since $Ax = \lambda x$ is singular at an eigenvalue, the condition of eigen*vectors* is treated here, in the convincing form developed by one of the authors. The reciprocal relation between the condition of a regular linear problem and its distance to the nearest singularity is established. The appropriate extension of this principle to nonlinear problems is discussed; the suggested formulation is later confirmed by computational experimentation.

Chapter 5 on backward errors emphasizes the dependence on the class of admissible perturbations; the definition via the *inf* of the feasible scalings within that class allows a great deal of flexibility. Expressions for many tasks are displayed and interesting situations discussed including the role of iterative refinement. Chapter 6 on the finite precision behavior of iterative and approximate algorithms presents material which has not appeared in textbook form so far; it displays and explains a number of interesting phenomena.

After an account of various other approaches and tools for round-off error analysis in Chapter 7, the toolbox PRECISE for computer experimentation in round-off error analysis is introduced in Chapter 8. This toolbox is meant for a MATLAB environment and consists of two modules: The first one allows experimentation by random perturbation of selected data, with automatic sampling and plots; the other contains tools for sensitivity analysis, plots of pseudospectra, pseudozeros etc. The remainder of the book is largely devoted to a discussion and explanation of experimental results obtained with the help of PRECISE.

Studied are algorithms for the solution of $F(x) = y$ which yield a result $x_\theta = G_\theta(y)$ with $F(x_\theta) =: \eta_\theta$ upon execution with a finite precision θ. A perturbation Δz applied to specified data elements causes changes Δx_θ and $\Delta\eta_\theta := F(x+\Delta x_\theta)-\eta_\theta$, yielding a compound backward error $\Delta y_\theta := F(x + \Delta x_\theta) - y$. The following three *indicators* are fundamental for the assessment of an algorithm at precision θ (sup is over $\|\Delta z\| = \delta$):

the local sensitivity of G_θ	$L_{\theta\delta} := \sup \|\Delta x_\theta\|/\|\Delta z\|$;
the local sensitivity of F^{-1}	$K_{\theta\delta} := \sup \|\Delta x_\theta\|/\|\Delta \eta_\theta\|$;
the "reliability"	$I_{\theta\delta} := \sup \|\Delta y_\theta\|/\|\Delta z\|$.

For fixed precision θ, the indicators are *estimated statistically* for a given δ through (small) samples of random perturbations Δz of norm δ.

$I_{\theta\delta}$ is an important new assessment tool introduced by the authors; it should be approximately *constant* and of order 1 as a function of δ. In the experiments, this holds only within a *reliability interval* $[s, r]$, where s is of the order of the backward error at x_θ. The measured estimate for the condition $K_{\theta\delta}$ should also be constant in δ. Near a singularity, this can only hold up to some δ_0 which is of the order of the distance to the singularity. If $[s, \min(r, \delta_0)]$ is not empty it indicates the perturbation range (within the chosen class) for which the computation will behave regularly and valid results may be expected from the experiments. Chapter 9 presents and discusses PRECISE plots for numerous interesting examples.

Chapter 10 discusses *nonnormality* in matrices and its effect on finite precision computations, a central research area of the authors. Chapter 11 introduces and discusses plots of pseudoeigenvalues and pseudozeros gained with PRECISE. The concept of *qualitative computing* is coined to denote situations where only partial information may be gained through finite precision computation; examples are given. An Annex lists samples of MATLAB codes which have been used to perform the PRECISE experiments in the volume.

The fascination of large parts of this treatise is slightly hampered by unmotivated changes and inconsistencies in the notation. Also, in spite of its essentially narrative style, the text is rather concise; at the first reading, one would welcome more explanations and perhaps repetitions of definitions or facts from previous chapters. The title "Lectures on ..." is definitely not appropriate from this point of view. Naturally, one also finds some loose formulations here and there. But these little flaws do not diminish the high value of this text which surveys a good deal of new territory in a pioneering way. Also it makes one realize where one would like to know more; thus, further reaching investigations will certainly be initiated by it.

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20[65F05, 65G05, 65F10, 65F35]—*Accuracy and stability of numerical algorithms*, by Nicholas J. Higham, SIAM, Philadelphia, PA, 1995, xxv + 688 pp., $23\frac{1}{2}$ cm, $39.00

Nick Higham is well-known for his contributions to error analysis and for his ability to communicate results that are necessarily full of intricate details. Only someone with his renowned writing skills could organize a work of this magnitude and still make it appealing. The book is close to 700 pages in length and is chock full of results, problems, and references.

Higham begins with four chapters that describe what life is like in the presence of roundoff error. A later chapter deals with software issues in floating point arithmetic and completes what I think is one of the best portrayals of finite precision arithmetic in the literature.

Chapters on polynomials, norms, and linear system perturbation theory set the stage for the analysis of $Ax = b$ algorithms, in many ways the real business of the book. Chapters on the LU, block LU, Cholesky, and QR factorizations are complemented by chapters that deal with related issues such as triangular system solving, iterative improvement, condition estimation, and matrix inversion.

Underdetermined systems and full rank least squares problems are also covered. By sticking to the full rank case, the singular value decomposition (SVD) can be avoided. Indeed, the SVD is relegated to a brief appendix and a few problems that are concerned with the pseudo-inverse. As Higham states in the preface, the treatment of singular value and eigenvalue computations requires a book in itself. I appreciate this point but still feel that the SVD should have been introduced as an analytical tool early in the book. It is just too powerful a decomposition to ignore in a major text like this that deals with numerical stability in matrix computations.

Other portions of the book reflect Higham's penchant for answering important stability questions. The reader interested in stationary iterative methods, matrix powers, the Sylvester equation, Vandermonde systems, and fast matrix multiplication will appreciate the author's treatment of these topics. A brief chapter on the FFT includes a nice proof of stability. New results for Newton interpolating polynomial evaluation, iterative refinement, Gauss-Jordan elimination, and the QR factorization are also included.

The overall style is perfect for the specialist who needs detail and rigor. But Higham's legendary expository skills also take care of the casual reader who needs intuition and a passing appreciation of the issues. Especially helpful in this regard are the "Notes and References" that appear at the end of each chapter. Higham is an extraordinary bibliophile and for every topic covered in this book you get the feeling that no stone is left unturned. There are over 1100 entries in the master bibliography and Higham's chapter-ending pointers into the literature are informative and impart a real historical sense.

Practitioners and experimentalists will enjoy several features of this book. There are many references to LAPACK, the software package of choice for solving most of