22[65–00, 65–01]—*Integral equations: Theory and numerical treatment*, by Wolfgang Hackbusch, International Series of Numerical Mathematics, Vol 120, Birkhäuser, Basel, 1995, xiv + 359 pp., 24 cm, $79.50

This is an excellent reference text in the subject of integral equations, their analysis, and their numerical treatment for both the student as well as for the solver of integral equations. The type of integral equations considered include linear and nonlinear, as well as first and second kind Fredholm and Volterra integral equations, and singular integral equations. Integral equation methods can be used for solving differential equations. They are especially important when they enable a considerable saving of computation time resulting from the fact that the solution of the integral equation can be achieved by solving a lower dimensional problem than that for the case of the solution of the differential equation.

The text is organized into the following chapters:

1. *Introduction.* Tools of real and functional analysis are a convenient way to study integral equations, and the author discusses these tools in this introductory chapter. Spaces of continuous (including Hölder continuous) and differentiable functions that house solutions of integral equations are presented via real analysis methods. Also covered in this chapter are some theoretical aspects of general, polynomial, and spline interpolation, quadrature, the convergence of these processes, and the role of the condition number of a system of algebraic equations.
2. *Volterra Integral Equations.* In this chapter we encounter a theoretical treatment of these equations including existence, uniqueness, and regularity properties of solutions of Volterra integral equations of the first and second kind, as well as a discussion of equations of convolution type. Numerical solution methods presented are based on one and multistep methods of solving differential equations, and on the use of simple piecewise linear splines.
3. *Theory of Fredholm Integral Equations of the Second Kind.* Here we find a discussion of compactness of the integral equation operator and its consequences, with application to spaces of continuous functions, to square integrable functions, and to the case of unbounded intervals. Approximability and convergence, as well as properties of the map via the integral equation operator are discussed in general and for several specific types of integral equation operators.
4. *Numerical Treatment of Fredholm Integral Equations of the Second Kind.* The following topics are presented on the subject of replacing a Fredholm integral equation by a system of algebraic equations:
   (a) Stability, consistency, error of approximation, and condition numbers;
   (b) Replacement of the equation by a discrete system;
   (c) Projection methods of approximation;
   (d) Collocation method of approximation;
   (e) Galerkin's method;
   (f) Nyström's method; and
   (g) Supplements to the above, including solving eigenvalue problems, use of extrapolation, defect corrections, and the Fredholm alternative.
5. *Multigrid Methods for Solving Systems Arising from Integral Equations of the Second Kind.* This, too, is a lengthy chapter, which discusses the details of

multigrid implementation, including direct solution, Picard iteration, and the conjugate gradient method. We find here, discussions of the interpolation or projection error, levels of discretizations, two-grid, and more general multigrid iteration, and nested iteration.

6. *Abel's Integral Equation.* Special consideration is given to the solution of this class of Volterra integral equations, which are considered to be difficult to solve numerically. The chapter ends with a discussion of the solution of an Abel equation, for which the singular denominator is the function $(x-y)^{1/2}$; the equation is then solved via piecewise linear approximation of the unknown function.
7. *Singular Integral Equations.* This chapter is concerned mainly with the solution of Cauchy singular integral equations (CSIE). It starts with a careful examination of Hilbert and related transforms, and then leads to methods of approximation of solutions of CSIE, including Fourier series and multigrid methods. Applications are given for solving the interior and exterior Dirichlet problems, and illustrations are made of the use of single and double layer potentials for solving problems over planar regions. At the end of the chapter we encounter a refreshing discussion of hypersingular (Hadamard-type) integrals.
8. *The Integral Equation Method.* This chapter is concerned mainly with the reduction of differential equations to integral equations. Especially important are those cases when the solution of the equivalent integral equation formulation of the differential equation (over the boundary of the region) represents a problem of smaller dimensionality than that for the direct solution of the differential equation. In such cases the Green's function kernels have singularities, and we thus find a relevant study of continuity properties of the mappings via singular integral equation operators. In addition to Green's function kernels for solution of potential problems, which are discussed in detail, one also finds Green's function kernels for the *Helmholtz equation*, for the *biharmonic equation*, for the *Lamé equations*, and for the system of *Stokes equations.*
9. *The Boundary Element Method* (BEM). In this culminating chapter of the text, we encounter various methods of approximating the solution to the integral equation formulations (of differential equations) over the boundary of the region. Topics discussed include collocation, finite element, and multigrid methods. After obtaining a solution (i.e., the boundary "density") of the integral equation, we must integrate the product of this density with the Green's function over the boundary in order to find the corresponding solution of the differential equation in the interior of the region. Special care must be taken to achieve sufficiently accurate approximations of the singular integrals over parts of the boundary, both in the process of setting up the system of algebraic equations that approximate the solution to the boundary integral equation as well as to approximate the solution to the differential equation within the region. Estimates on the error are given for each type of approximation. The chapter ends with a discussion of the *panel clustering algorithm*, a research topic of the author. This algorithm enables an efficient evaluation of the matrix-vector multiplications in the system of algebraic equations whose solution approximates the solution to the integral equation.

There is an unusual inclusion in the text: following the table of contents, we find a list of notations, as well as an explanation of numbers and formulae, the author's method of numbering theorems, and an explanation of the author's use of generic constants. These additions are worthwhile to both reader and student.

At the end of the text we find several pages devoted to a bibliography as well as to an index.

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23[41A05, 41A10, 42A15, 65D05, 65M70, 65T10]—*A practical guide to pseudospectral methods*, by Bengt Fornberg, Cambridge Monographs on Applied and Computational Mathematics, Cambridge Univ. Press, New York, NY, 1996, x + 231 pp., $23\frac{1}{2}$ cm, hardcover, $54.95

This is the first book in the series "Cambridge Monographs on Applied and Computational Mathematics". The stated goal of this series is to publish expositions on all aspects of applicable and numerical mathematics, with an emphasis on new developments in this fast-moving area of research. On the whole, this first book in the series is well written and is clearly in line with the stated goal of the series.

Spectral methods have been under rapid development in the last 20 years. There are many books written in this period, most notably the pioneer book by Gottlieb and Orszag in 1977 [1] and the comprehensive book by Canuto, Hussaini, Quarteroni and Zang in 1988 [2]. The book under review is different from all others in the following aspects. It is not a comprehensive book about spectral methods. The content is restricted to the subject of pseudospectral (PS) methods, which are equivalent mathematically to the interpolation, or collocation, methods. Galerkin methods are thus not covered in the book. Also, the author puts his own research experience into the book, notably the relationship between the finite difference (FD) and the PS methods. The approach of using the limit of FD when stencil is widened to define PS methods is advocated by the author. This book is perhaps the best resource for the readers to fully understand this approach.

The book contains eight chapters and eight appendices. After a brief introduction in Chapter 1, the author introduces spectral methods as expansions in orthogonal functions in Chapter 2. Different ways of determining the expansion coefficients are briefly mentioned, and the goal of the book, namely the discussion of the PS method, is stated. Difficulties of using the spectral method to approximate discontinuous functions, namely the Gibbs phenomenon, is also mentioned early in this chapter. Chapter 3 begins the introduction to PS methods via finite differences. The mechanism of finding the interpolation or differentiation matrices is discussed, and examples given for these matrices for different node distributions. The need to choose special node distributions to avoid divergence (Runge phenomenon) is discussed. Chapter 4 is perhaps the main chapter about the methodology. Several important properties of the PS approximations are discussed. This includes discussion about approximations to both smooth and nonsmooth functions. However, the discussion about approximations to nonsmooth functions seems not comprehensive. In practice there are examples requiring more sophisticated strategies than the ones advocated here. See, e.g. [3]. Chapter 5 discusses PS variations and enhancements in implementations. Several useful tricks in applications are discussed