

to the same, except for in some academic cases. The main emphasis throughout the book is on index 1 and index 2 systems. Often DAE appear in semi-explicit form, i.e. where an algebraic set of equations is appearing separately from the differential part (involving some or all the state variables). Higher index problems may be reduced to lower index ones, although it has a (numerical) price. In chapter 3 multistep methods are treated, with some particular emphasis on BDF methods. The code DASSL of the third author is given a lot of attention in chapter 5, which is anticipated here. Also some of the interesting work of März [2] c.s. is reviewed here. The important class of Runge-Kutta methods is considered in chapter 4, both for index 1 and index 2 systems. Here the material in [3] will be helpful as a more recent update of the state of affairs, as various articles by Petzold, Ascher, Lubich etc. on e.g. projected Runge-Kutta methods will be.

The software in chapter 5 comes in very handy as a reference source for users of the DASSL code. It contains both principles behind the implementation and hints how to use DASSL in various situations.

An application chapter concluded the first version of the book. In the present SIAM edition a new chapter has been added to fill some gaps and make at least the references more up to date. Some of them are really important ones and it is a pity that the authors have not seized the opportunity to rewrite the book for a second edition more thoroughly. As remarked in the beginning there are not many books on numerical ODE and this "SIAM classic" is therefore still a useful introduction and source of references on the subject.

#### REFERENCES

1. C. W. Gear, *The simultaneous solution of differential-algebraic equations*, IEEE Trans. Circuit Theory, CT18 (1971), 89–95.
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3. E. Hairer and G. Wanner, *Solving ordinary differential equations II. Stiff and differential-algebraic problems*, Springer, Berlin, 1991. MR 92a:65016

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**25[65F10, 65K10]**—*Linear and nonlinear conjugate gradient-related methods*, Loyce Adams and J. L. Nazareth (Editors), SIAM, Philadelphia, PA, 1996, xvi + 164 pp., 25½ cm, softcover, \$32.00

This book contains a collection of papers presented at an AMS-IMS-SIAM Summer Research Conference held in July, 1995. Most of the contributions are short notes or surveys containing observations about the conjugate gradient method and its place in optimization and numerical analysis. There are also a few research papers. Most of the authors are from either a sparse linear algebra or an optimization background, and the articles successfully elucidate the many connections between these two areas.

The basic conjugate gradient (CG) algorithm is a method for solving a linear system of equations  $Ax = b$ , where the coefficient matrix  $A$  is symmetric and positive

definite. In exact arithmetic, the algorithm is known to converge to the solution in at most  $n$  iterations, where  $n$  is the dimension of  $A$ . In fact, at each iterate  $x_k$ , the remaining error  $r_k = b - Ax_k$  is projected into a smaller and smaller subspace. The algorithm has maintained its popularity because of its simplicity and economy of implementation and its widespread applicability in sparse matrix computations, numerical analysis (in particular, numerical PDEs) and optimization.

O'Leary's paper (Chapter 1) gives a brief overview of the origins and development of CG and related algorithms, while Nazareth (Chapter 13) surveys the history of nonlinear extensions of the CG algorithm. Nocedal (Chapter 2) also discusses nonlinear variants of CG and their relationship and combination with Newton and quasi-Newton methods for unconstrained optimization. A longer paper by Conn et al. (Chapter 5) describes the "iterated-subspace minimization" technique for unconstrained minimization. In this method, iterates are updated not by searching along a single direction but rather by searching in a low dimensional subspace constructed from the steepest descent direction, the truncated Newton direction, and some iterates generated by the CG algorithm in its search for the Newton direction. Battery testing of their approach is not conclusive, but it shows this to be a promising direction of research.

In Chapter 8, Saunders describes regularization of linear least squares problems, with applications to the linear systems that arise in interior-point methods for linear programming. This contribution has little to do with CG, but it does raise interesting questions about the connections between the regularization strategy and proximal point methods. Mehrotra and Wang (Chapter 11) describe a dual interior-point algorithm for network linear programming, in which the linear system at each iteration is solved with a preconditioned CG method. The preconditioner is based on a minimum spanning tree for the underlying network. Numerical comparisons show the method to be slower than, but within reach of, a state-of-the-art network simplex code.

Davidon (Chapter 6) generalizes the notion of conjugacy to conic functions, while Dixon (Chapter 9) presents some results on two dimensional searches in modified CG algorithms that incorporate a steepest descent direction.

Greenbaum (Chapter 7) discusses the behavior of CG and Krylov methods in the presence of finite precision arithmetic. Barth and Manteuffel (Chapter 10) discuss classes of nonsymmetric matrices for which CG-like algorithms which do not require storage of all preceding iterates can be applied. DeLong and Ortega (Chapter 12) propose the use of SOR as a preconditioner for the nonsymmetric iterative method GMRES on a parallel architecture.

Edelman and Smith (Chapter 3) describe the role of CG-like algorithms in eigenvalue computations. They include a history of this applications class (for which the literature is widely spread) and make the connection between CG and the Lanczos procedure, Rayleigh quotient iterations, and so on. In Chapter 4, a Boeing group describes some of the difficult optimization problems that arise in aircraft design. They show how blind application of standard optimization paradigms yields unsatisfactory results, while smarter, customized strategies (such as merging the optimization and simulation procedures suitably) can be used to advantage.

I found this collection of well written and (for the most part) brief articles by well known researchers to be a refreshing crash course on both the history of CG and related methods and the current state of the art.

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