tation using standard techniques, and the books describe the algorithms in some detail and list the programs in full. Thorough and quite different reviews were given by F. N. Fritsch, Reviews 3 and 4, Math. Comp. **50** (1988), pp. 346–349, and M. M. Gupta, Math. Rev. 87m:65001a,b,c. The original book, which emphasized Fortran, was reworked for C and Pascal in 1988 and 1989. A second edition, published in 1992 in separate volumes for Fortran 77 and C, considerably changed and expanded the algorithmic and software content; see W. Gautschi, Review 3a,b, Math. Comp. **62** (1994), pp. 433–434.

The book under current review is the first to focus attention on Fortran 90. It is a companion to the Fortran 77 volume. The authors see Fortran 90 as a major advance in programming language support for SIMD (single instruction multiple data) parallel computing, and they develop this theme by using the entire Numerical Recipes library as an example. They begin with a very good overview of the language. Next they introduce SIMD and MIMD (multiple instruction multiple data) computing, oriented toward Fortran 90 and anticipated future revisions of Fortran. The remainder of the book gives program listings with useful tips on Fortran 90 and parallel programming along the way.

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4[49N10, 65-01, 65F15, 93C05, 93B40]—*Algorithms for linear-quadratic optimization*, by Vasile Sima, (Pure & Applied Mathematics: A Series of Monographs and Textbooks/200), Marcel Dekker, Inc., Monticello, New York, 1996, vii+366 pp., $23\frac{1}{2}$ cm, hardcover, \$150.00

During the last 30 years, the linear-quadratic optimal control problems in both continuous-time and discrete-time have belonged to the most studied problems in systems and control theory. The theory is now very mature and it is well-known that the optimal control for these problems in case of an infinite-time interval under standard assumptions is obtained via particular solutions of certain algebraic Riccati equations (AREs). In case of continuous-time systems this is an equation of the form

$$0 = \mathcal{R}_c(X) = Q + A^T X + XA - XBR^{-1}B^T X \tag{1}$$

while for discrete-time systems we have

$$0 = \mathcal{R}_d(X) = Q + A^T XA - X - A^T XB(R + B^T XB)^{-1}B^T XA. \tag{2}$$

In both cases, $A, Q, X \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times m}, R \in \mathbb{R}^{m\times m}$, and Q, R as well as the sought-after solution X are assumed to be symmetric. (Throughout the book it is also assumed that Q is positive semidefinite, and most of the time that R is positive definite.) Equation (1) is called the *continuous-time* ARE while (2) is referred to as *discrete-time* ARE. The required solutions X are usually *stabilizing* in the sense that for (1), $A - BR^{-1}B^T X$ has all its eigenvalues in the open left half plane while for (2), $A - (R + B^T XB)^{-1}B^T XA$ has all its eigenvalues in the open unit disk. Under certain assumptions on the underlying linear system, this solution exists, is unique and positive semidefinite.

In recent years, the attention of the control community has been drawn more and more to the field of robust control where the performance index is given by constraints for the closed-loop system rather than by minimizing a quadratic cost functional as in the case of linear-quadratic optimal control problems. It turns out that many methods in this field also lead to the solution of AREs.

Therefore, AREs have been solved and are solved quite frequently in computer-aided control system design (CACSD). This explains the large effort spent on developing algorithms to solve AREs, leading to a variety of computational methods having different benefits and shortcomings. Although still many important problems have not been solved, the field of numerical algorithms for AREs has matured. A textbook treatment giving some advice on which method to use under which circumstances will certainly be most welcome by control engineers. An in-depth treatment of the methods will be appreciated by numerical analysts working in the field of computational methods in control and systems theory. The present book is to the best of my knowledge the first one devoted completely to these goals. The topic has been treated in many survey papers and textbooks, most recently the state of the art is described in [4, 5, 6], but a detailed study of implementation details of the algorithms in question has been missing so far. The author, Vasile Sima, who has implemented himself many of the numerical algorithms considered in the book, addresses this issue using the full power of the fertile field of numerical linear algebra. He provides templates as building blocks for all the considered algorithms in an easy to comprehend pseudo-code that is mostly based on MATLAB notation. The core computations of all algorithms are based on LAPACK or LAPACK-style subroutines. As the discussion of computations goes down to the lowest levels of computations, this enables the reader to implement all discussed algorithms in MATLAB or, using the public domain BLAS and LAPACK packages, in FORTRAN.

The first chapter of this book gives an overview of the theoretical background of linear-quadratic optimization problems as well as of the numerical linear algebra algorithms necessary for the algorithms to be discussed in later chapters. First, the standard linear-quadratic optimal control problem is discussed in the general setting of time-variant systems and arbitrary time intervals. Its solution by Riccati differential equations (RDEs) is derived using the Hamilton-Jacobi equations before specializing the results to time-invariant problems for the interval $[0, \infty]$ such that the RDE reduces to an ARE. The dual problem of estimation is discussed for the discrete-time case showing that this problem can also be solved via AREs. Next, some extensions are considered that are also reducible to the standard problem before turning to topics in robust control where the outlined approach to the $\mathcal{H}_2$ optimal control problem again leads to the solution of AREs. Section 1.4 gives an overview of the available numerical software for linear algebra computations (which are at the heart of any ARE solver!), using some basic algorithms to introduce the description and presentation of algorithmic templates used throughout the book. The remainder of Chapter 1 deals with some special algorithms for difference Riccati equations and sequential state estimation. Somewhat hidden at the end of the first chapter, defect correction for AREs is discussed. As all numerical methods to solve AREs have substantial shortcomings, this technique is of major importance and should always be used to improve an approximate solution computed by any ARE solver. Therefore, a more emphasized presentation of this topic would have been appropriate.

The rest of the book deals with numerical algorithms for AREs, thereby acknowledging the outstanding role played by AREs in linear-quadratic optimization. In Section 2, the Newton iteration for continuous-time and discrete-time AREs is discussed. As AREs can be considered as sets of nonlinear equations, it is not surprising that Newton's method is one of the oldest methods studied for their solution. Used as a solver for AREs, Newton's method requires a stabilizing initial guess which leads the author to describing various techniques for stabilizing constant linear systems (an important topic itself!). At the core of these techniques as well as the Newton iteration itself, Lyapunov equations (i.e., sets of linear matrix equations) must be solved. Well-known algorithms for this problem and also for the solution of the more general Sylvester equations are discussed. The well-known Bartels-Stewart algorithm for the numerical solution of Lyapunov equations requires an initial transformation of the coefficient matrix to Schur form which is usually accomplished using the QR algorithm. This leads the author to describing the QR algorithm in a very detailed way that gives much insight into the tricky implementation issues that lie far beneath the standard textbook treatments as for instance given in [1]. The presentation reveals the enormous amount of work necessary to make a rather simple looking algorithm work on a computer. A criticism of this section evolves around the fact that the author follows the folklore approach followed in the literature concerning Newton's method for AREs. The erroneous or incomplete proofs for its properties are taken over from several sources. Fortunately, in [3], a correct and complete proof using only minimal assumptions on the underlying system is given that shows that Newton's method really has all the properties attributed to it.

Chapters 3 and 4 approach the solution of AREs exploiting the connection of AREs and certain eigenvalue problems. The solution of the ARE (1) can be found by computing the stable invariant subspace of the corresponding Hamiltonian matrix while the ARE (2) can be solved by determining the stable invariant subspace of the corresponding symplectic matrix or, if A is singular, via the stable deflating subspace of the corresponding symplectic matrix pencil. Here stable subspace means that in case of (1) it corresponds to the eigenvalues in the open left half plane while for (2) to the eigenvalues in the open unit disk.

The standard approaches to compute the required invariant/deflating subspaces are the computation of the (generalized) Schur form of the matrix (matrix pencil) in question followed by a re-ordering of eigenvalues. These approaches, called the (generalized) Schur vector methods, are outlined in Chapter 3, making use of the already described QR iteration (Chapter 2) and the QZ algorithm for matrix pencils. The implementation details of the latter algorithm are again given and a special emphasis is laid on the necessary eigenvalue re-ordering routines.

The last chapter of the book deals with methods that are also based on the invariant/deflating subspace approach just described but also exploit the given structure of the matrices/matrix pencils. As a satisfactory structure-exploiting/preserving method for symplectic matrices/pencils is yet not known, the author restricts himself to the more advanced field of structured methods for the continuous-time ARE (1). The methods presented are the *matrix sign function method*, the *Hamiltonian SR algorithm* based on symplectic (but non-orthogonal) similarity transformations, and the *multishift method* which uses only orthogonal and symplectic similarity transformations.

An appendix with a comparison of all solvers for the ARE (1) and another appendix with notation and abbreviations complete the book.

The presentation of methods is very clear. The mathematical background is discussed before presenting the method. Implementation issues are described in a very detailed manner and are put together to provide an algorithmic template for each method. Every algorithm is accompanied by several examples illustrating the use of the discussed algorithm. (Most of the examples are taken from the control literature and provide data from "real" control problems which makes the book a valuable source of examples for testing ARE solvers.) Also, applications and limitations of each algorithm are discussed, where a special emphasis is given to numerical robustness and reliability of the presented methods. For the algorithms discussed in Chapters 3 and 4 it is pointed out that it is usually necessary to iteratively improve the algorithm by the defect correction method presented in Chapter 1, and the results of such improvements are reported. This part of the book benefits from the long experience of the author in implementing and testing the various algorithms available for the numerical solution of AREs.

One criticism of the book is that often referencing is not consistent or up to date. For instance, the fundamental paper describing the matrix sign function method by Roberts dating from 1971 [7] is not mentioned but the method is attributed to later sources. Another example is the optimal scaling problem for discrete-time AREs for which "... it is not yet clear how the problem could be scaled." (page 246, footnote). The solution of this problem has already been published in 1992 [2]. As research around AREs is still very active, many recent publications, particularly new algorithmic developments and results concerning sensitivity and conditioning of AREs, could not be incorporated into the book.

Altogether, Sima's book provides a valuable reference for scientists and engineers working in the field of CACSD and other areas where the numerical solution of linear-quadratic optimization problems and AREs is an issue. It helps choosing the right algorithm and gives detailed guidance how to implement the proposed methods.

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5[65D20, 33-04]—*Computation of special functions*, by Shanjie Zhang and Jianming Jin, John Wiley & Sons, Inc., New York, NY, 1996, xxvi+717 pp., $24\frac{1}{2}$ cm, hardcover, $89.95

This book provides Fortran software on an included diskette for computing numerical values of important special functions. The scope of coverage is a subset of the functions found in Abramowitz and Stegun [1]. Among the omissions are Coulomb wave and Weierstrass elliptic functions, but this hardly alters the fact that the authors undertook a giant task in coding original programs and writing documentation for them in this book. The programs are all encoded in double precision, and the text implies (without making any explicit statement) that results are good to single precision.

What is the audience for this book? Numerical analysts? Unlikely, since there is almost no mathematical development of the mostly standard methods that are used or quantitative derivation of bounds or estimates for rounding and truncation errors. Library developers? Probably not, because a lot of additional work would be required to bring the programs up to current standards of software engineering with respect to accuracy, portability and error handling. Educators? Possibly. As an auxiliary resource in a technical course of study, or for a person interested in self-study, the book gives a realistic flavor of how functions are computed. Engineers and scientists? Maybe. The authors are professors of engineering, and the book does provide an inexpensive source of tolerable software for many functions, some of which are not easy to find elsewhere. However, those who use special functions regularly in computation will be aware of libraries such as IMSL, NAG and SLATEC, engineering packages such as Mathcad, symbolic systems such as Macsyma, Maple and Mathematica, and software repositories such as Netlib, all of which provide considerable support for special functions. Also, the recent books [2, 3, 4], similar in content and style to the book under review here, can be consulted.

Consider a typical chapter, say the one on $J_\nu(z)$ and $Y_\nu(z)$ (other Bessel functions are treated in separate chapters). The chapter has 9 sections. The first gives a collection of basic formulae accompanied by sketchy commentary and a few simple figures. The next four sections are devoted to computation of $J_\nu(z), Y_\nu(z)$ in the cases

(1) z real and $\nu \in \{0, 1\}$,
(2) z real and ν a positive integer,
(3) z complex and ν a positive integer,
(4) z complex and ν real and positive.

This arrangement, which reflects algorithmic conveniences, is consistent with other software packages and libraries. Each of these sections discusses a computational