

## REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1991 Mathematics Subject Classification can be found in the annual subject index of *Mathematical Reviews* starting with the December 1990 issue.

**16[65-02]**—*Acta numerica* 1997, A. Iserles (Managing Editor), Cambridge University Press, New York, NY, 551 pp., 25½ cm, hardcover, \$60.00

This book is Volume 6 (1997) of a series of survey articles on important developments in numerical mathematics and scientific computation written by authors who have made substantial contributions to the topics of their articles. The present volume contains eight articles ranging in length from 38 pages to 174 pages. A review of Volume 4 (1995) of this series by Vidar Thomée appeared in the April 1997 issue of *Mathematics of Computation*.

The first article, *Constructing cubature formulae: The science behind the art*, by Ronald Cools presents “a general, theoretical foundation for the construction of cubature formulae to approximate multivariate integrals.” Quality criteria and several different ways to construct such approximations are discussed, as well as the characterization of minimal cubature formulae.

In *Wavelet and multiscale methods for operator equations*, Wolfgang Dahmen discusses the use of wavelets for the approximation of (mainly elliptic) partial differential equations. Many of the important tools of wavelet analysis can be found in this article along with more general mathematical ideas important to also understanding and analysing other multiscale approaches such as multigrid applied to finite element discretization of partial differential equations.

*A new version of the fast multipole method for the Laplace equation in three dimensions* by Leslie Greengard and Vladimir Rokhlin discusses a new version of the fast multipole method for the evaluation of potential fields in three dimensions. The scheme evaluates all pairwise interactions in large ensembles of particles, i.e., expressions of the form  $\sum_{i=1, i \neq j}^n (q_i) / \|x_j - x_i\|$  for the gravitational or electrostatic potential and  $\sum_{i=1, i \neq j}^n (q_i)(x_j - x_i) / \|x_j - x_i\|^3$  for the field, where  $x_1, x_2, \dots, x_n$  are points in  $R^3$  and  $q_1, q_2, \dots, q_n$  are a set of real coefficients.

The article *Lanczos-type solvers for nonsymmetric linear systems of equations* by Martin J. Gutknecht introduces the basic forms of the Lanczos process and some of the related theory and describes in detail a number of solvers based on it. The author also discusses possible breakdowns of the algorithms and remedies for these breakdowns.

In the paper *Numerical solution of multivariate polynomial systems by homotopy continuation methods*, T. Y. Li discusses the use of homotopy methods to find all isolated solutions of a system of  $n$  polynomial equations in  $n$  unknowns and a variation of this problem in which it is desired to solve such systems for each of several choices of the coefficients in the system.

In *Numerical solution of highly oscillatory ordinary differential equations*, by Linda R. Petzold, Laurent O. Jay, and Jeng Yen, the authors consider several different classes of problems exhibiting oscillatory behavior, including linear oscillatory systems, rigid and flexible mechanical systems, problems in molecular dynamics, and problems from circuit analysis and orbital mechanics. For each class of problems, they discuss “the structure of the equations, the objectives of the numerical simulation, the computational challenges, and some numerical methods that may be appropriate.”

In the article *Computational methods for semiclassical and quantum transport in semiconductor devices*, Christian Ringhofer gives an overview of recently developed numerical methods for several models used in semiconductor device simulation. These include Galerkin methods for the semiclassical and quantum kinetic systems and difference methods for the classical and quantum hydrodynamical systems.

The final article by Steve Smale entitled *Complexity theory and numerical analysis* deals with the complexity analysis of algorithms and involves upper bounds on the time required in a numerical algorithm and estimates of the probability distribution of the condition number of a problem. Among the problems discussed are the solution of polynomial equations, the solution of linear systems of equations, and the calculation of eigenvectors.

This and the other volumes of the series provide an excellent way to learn about current research in a wide variety of areas in numerical analysis and scientific computation.

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**17[26A06, 34A50, 35A40, 65L60, 65M60, 65N30]**—*Computational differential equations*, by K. Eriksson, D. Estep, P. Hansbo, and C. Johnson, Cambridge University Press, New York, NY, 1996, xvi+538 pp., 22½ cm, hardcover, \$100.00

In the philosophy of Leibnitz, analysis and numerical computation are equally important aspects of mathematics and are not considered as separate elements. Unfortunately, this unified view has been neglected for the last hundred years. Mathematical education has treated these elements as distinct and has emphasized the symbolic, analytic aspect of mathematics. Now, however, with the advent of powerful computational tools it is possible to again fuse mathematical analysis and computation. Inspired by the philosophy of Leibnitz, the authors have written an ambitious text for undergraduate/graduate engineers and scientists which combines mathematical modelling, analysis and computation.

In order to plot a course through the vast landscape of material dealing with computation, the authors have chosen a particular method and used it to present a unified treatment of problems ranging from the fundamental theorem of calculus to partial differential equations. The Galerkin method uses linear combinations of special classes of functions to approximate the solutions of differential equations. The special classes of functions may be polynomials, piecewise polynomials, or trigonometric polynomials, to name a few. The Galerkin method can be viewed as a natural extension of the venerable technique of separation of variables, which uses expansions in terms of eigenfunctions.