In *Numerical solution of highly oscillatory ordinary differential equations*, by Linda R. Petzold, Laurent O. Jay, and Jeng Yen, the authors consider several different classes of problems exhibiting oscillatory behavior, including linear oscillatory systems, rigid and flexible mechanical systems, problems in molecular dynamics, and problems from circuit analysis and orbital mechanics. For each class of problems, they discuss "the structure of the equations, the objectives of the numerical simulation, the computational challenges, and some numerical methods that may be appropriate."

In the article *Computational methods for semiclassical and quantum transport in semiconductor devices*, Christian Ringhofer gives an overview of recently developed numerical methods for several models used in semiconductor device simulation. These include Galerkin methods for the semiclassical and quantum kinetic systems and difference methods for the classical and quantum hydrodynamical systems.

The final article by Steve Smale entitled *Complexity theory and numerical analysis* deals with the complexity analysis of algorithms and involves upper bounds on the time required in a numerical algorithm and estimates of the probability distribution of the condition number of a problem. Among the problems discussed are the solution of polynomial equations, the solution of linear systems of equations, and the calculation of eigenvectors.

This and the other volumes of the series provide an excellent way to learn about current research in a wide variety of areas in numerical analysis and scientific computation.

RICHARD S. FALK

17[26A06, 34A50, 35A40, 65L60, 65M60, 65N30]—*Computational differential equations*, by K. Eriksson, D. Estep, P. Hansbo, and C. Johnson, Cambridge University Press, New York, NY, 1996, xvi+538 pp., $22\frac{1}{2}$ cm, hardcover, $100.00

In the philosophy of Leibnitz, analysis and numerical computation are equally important aspects of mathematics and are not considered as separate elements. Unfortunately, this unified view has been neglected for the last hundred years. Mathematical education has treated these elements as distinct and has emphasized the symbolic, analytic aspect of mathematics. Now, however, with the advent of powerful computational tools it is possible to again fuse mathematical analysis and computation. Inspried by the philosophy to Leibnitz, the authors have written an ambitious text for undergraduate/graduate engineers and scientists which combines mathematical modelling, analysis and computation.

In order to plot a course through the vast landscape of material dealing with computation, the authors have chosen a particular method and used it to present a unified treatment of problems ranging from the fundamental theorem of calculus to partial differential equations. The Galerkin method uses linear combinations of special classes of functions to approximate the solutions of differential equations. The special classes of functions may be polynomials, piecewise polynomials, or trigonometric polynomials, to name a few. The Galerkin method can be viewed as a natural extension of the venerable technique of separation of variables, which uses expansions in term of eigenfunctions.

The tone for the book is set in the review chapter on Calculus. The fundamental theorem of the calculus is formulated as an initial value problem for a differential equation. The solution of this problem is then approximated by piecewise constant functions. The accuracy of the approximation is estimated and the estimate is used to suggest a variable mesh algorithm to compute the approximations, to within a given tolerance, as cheaply as possible. The student is immediately made aware of the practical nature of computation and the goal of doing it as efficiently as possible.

Along the way the student is given some basic results on polynomial interpolation and quadrature, and a review of linear algebra.

In Chapter 6 we see the Galerkin method introduced in two basic problems that will be studied in great detail in later chapters. Somewhat surprisingly one of them is the first-order linear equation

$$u'(t) + a(t)u(t) = f(t), \quad u(0) = u_0.$$

Rather than the usual numerical discussion using the Euler method and its refinements, we find polynomial approximations used to reduce the problem to finding solutions of linear systems. Approximation by global polynomials with the basis $1, t, t^2, \dots, t^n$ leads to ill-conditioned systems of linear equations much as it does for least squares polynomial approximation. With this motivation, the authors turn to piecewise polynomial approximate solutions, limiting themselves to (discontinuous) piecewise constant and (continuous) piecewise linear functions.

In a second, more standard approach, the Galerkin finite element method is used to treat the boundary value problem

$$-u''(x) = f(x), \quad u(0) = u(1) = 0.$$

With the stage now set and the basic issues raised, the authors provide a chapter of numerical linear algebra. In Part II, they treat in detail two archetypal problems:

(1) Two point boundary value problems on an interval $[a, b]$ in Chapter 8, and
(2) Initial value problems for systems of first order equations in Chapter 9.

In both chapters the authors carefully discuss the a priori estimates for the Galerkin methods employed and also the a posteriori estimates needed to develop adaptive mesh algorithms. The discussion of error also includes comments about the error arising from the approximation of certain integrals by quadrature formulas. Estimates are made that take into account all the sources of error.

Part III deals with problems in more than one space variable, including the wave equation, the heat equation and the Poisson equation. These chapters provide rather brief derivations of the linearized equations in the usual way. Formulas for the solutions of the heat and wave equations are stated rather than derived. The emphasis in these later chapters is on the geometrical and concrete computational aspects of finite elements in two and three dimensions. The presentation reaches its most advanced level in Chapters 18 and 19, which discuss finite element approximate solutions to convection-diffusion equations in which convection dominates. After 20 chapters of rather concrete, computationally oriented discussion, the authors state and prove the Lax-Milgram lemma and illustrate its application with several examples. This mode of exposition is contrary to that of most mathematics texts and is quite effective.

The authors' writing style is clear and crisp. There is an abundance of quotations, many of them from Leibnitz, but also from sources ranging from Aristotle to Hank Williams. While interesting and amusing, not all of them seem particularly relevant.

Although the authors' stated goal is to fuse mathematical modeling, analysis and computation, the emphasis is clearly on computation. The authors' experience in computation gives them a perspective on the subject that allows them to stress the important issues. Rather than offer simply a collection of methods, they have chosen to unify the text by focusing on a single method—the Galerkin method. However, this comes at a cost because some other very effective techniques, particularly for ordinary differential equations, are not even mentioned.

Interesting problems are inserted throughout the text instead of being gathered together at the end of sections. They are often smaller steps in the proofs of larger propositions. In addition there are many computational projects which may be done with a collection of finite element codes developed at Chalmers University of Technology, Sweden. They are available at no charge at the Chalmers website. The web address that worked for me, different from the one given in the book, is `http://www.md.chalmers.se/Math/Research/Femlab/`.

The text would be suitable for a student having had the usual three semesters of calculus, linear algebra, a course in ordinary differential equations, and some experience with numerical analysis. The first two parts of the book could be used for an advanced undergraduate course, but Part III is graduate level material.

Because of its very definite point of view, this book does not fit the mold of the standard numerical analysis text or of the standard numerical analysis course. However, it is provocative and should be taken seriously by all faculty, not just those in applied mathematics. It takes a fresh look at some of the standard topics in undergraduate analysis, and the ideas of the text could be used in many courses in analysis and computation. While not without its limitations, this book provides a vision of computation and analysis that may become a model for the future.

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18[65-01, 65FXX]—*Applied numerical linear algebra*, by James W. Demmel, SIAM, Philadelphia, PA, 1997, xi+419 pp., $25\frac{1}{2}$ cm, softcover, \$45.00

This book is intended as a textbook in numerical linear algebra for first-year graduate students in a variety of engineering and scientific disciplines. In the preface the author gives a list of goals he was trying to meet. After stating his target audience, he goes on to write, "2. It should be self-contained, assuming only a good undergraduate backgound in linear algebra. 3. The students should learn the mathematical basis of the field, as well as how to build or find good numerical software. 4. Students should acquire a practical knowledge for solving real problems efficiently. In particular they should know what the state-of-the-art techniques are in each area...". Finally he writes, "5. It should all fit in one semester...".

This is a difficult undertaking. In my opinion the author has fallen short in certain ways, but the book is excellent nevertheless. It is clearly written, though somewhat demanding, and it is well organized.