21[68Q40]—*Polynomial algorithms in computer algebra*, by F. Winkler, Springer-Verlag, Wien, New York, 1996, viii+270 pp., 24 cm, softcover, $69.00

Any author in the area of computer algebra is faced with dilemmas forced upon him by the subject matter. Opinions vary on what exactly is computer algebra, but also the angle from which to present the material, and with it the target audience, must be carefully considered.

In *Polynomial algorithms in computer algebra*, Franz Winkler laudably attempts to demarcate clearly what he considers to be "computer algebra", and what part of it he intends to cover. For him computer algebra is synonymous with symbolic algebraic computation, which is concerned with the design, analysis, implementation and application of algebraic algorithms, that is, algorithms dealing with exact (non-numeric) computation in algebraic domains, striving to algebraic solutions. Winkler's interests are in the subarea dealing roughly with "polynomials"—the title refers to the subject, not to the complexity of the algorithms!

Of importance for the potential reader is the point of view chosen by the author to present his subject matter. Computer algebra has been claimed to be both a branch of mathematics and of computer science. Inclusion of Winkler's text in Springer's new series of *Texts and Monographs in Symbolic Computation* does not in itself reveal a view either way, as the publisher seems to aim at practitioners of both disciplines (yet the silver rather than yellow colour of the covers may suggest otherwise). But although definitions for most algebraic notions are briefly recalled, it is clearly expected that the reader is familiar with the elementary theory of (finite/p-adic/number) fields, (finite/polynomial/power series) rings, and, to a lesser extent, groups. Despite its mathematical orientation, this monograph is *not* a systematic account of basic algebra by way of algorithms, but rather an introduction into some of the available (polynomial algebraic) algorithms for accomplishing certain tasks. An analysis of the computational complexity of most algorithms is included, but these and other remarks of an information theoretic nature (such as about computer representation of algebraic objects) are provided at a pleasantly restrained rate (to my taste).

The author also deserves praise for the way he has dealt with the problems of presenting algorithms and of referring to computer algebra *systems*. By presenting most algorithms in a clear and concise combination of plain English and mathematical notation, he is able to avoid the use of lower level programming languages (C, Pascal) and of higher level system languages (like that of MAPLE) and their idiosyncrasies. The text provides no pointers to existing implementations, and remains system independent this way. Yet, users of the major systems will have no difficulty in locating the relevant functions.

Winkler's approach can be outlined as follows: motivated by a few "practical" applications, he develops a toolkit of algebraic techniques and algorithms, and shows in the final chapters how the motivating applications can be rephrased as algebraic problems allowing treatment by the tools he has provided. After an introduction, which includes a discussion of the notion of computer algebra and some other preliminaries, a chapter on arithmetic in basic algebraic domains follows. Just like any developer of a new computer algebra system seems to have to struggle with yet again reimplementing algorithms for basic but crucial operations like long integer arithmetic, and (multivariate) polynomial arithmetic (in sparse and recursive form), Winkler is unable to avoid their fairly lengthy description here. As a consequence we have reached page 50 (out of 250) before getting to the six main chapters on techniques in polynomial algebra, and the three chapters on their application. The inconsistent and confusing notation for residue class rings, finite fields and p-adic rings in the first chapters perhaps only deserves mentioning because of the contrast with otherwise high standards of exposition and consistency, and the reasonably adequate TEX typography and layout.

The six main chapters are concerned with "computing with homomorphic images" (Chinese remaindering, p-adic lifting, discrete Fourier transform), common divisors of polynomials, factorization of polynomials (over finite fields, integers, and number fields), decomposition of polynomials, solving systems of linear equations, and Gröbner bases. Each of these provides a good introduction into the area with many clear examples and exercises. Usually the author cuts a more or less straight path through the jungle of results, and although he does point out certain byways (particularly in the very good bibliographic notes), it is not always clear what their status is. For example, Berlekamp's polynomial factorization algorithm is nicely covered, and the chapter notes mention the existence of alternatives (like Cantor-Zassenhaus), but their relative merits are not discussed at all.

The three motivating applications are quantifier elimination in real closed fields, requiring a decision procedure for systems of polynomial (in)equalities (used in robotics, for example); indefinite summation, in particular of hypergeometric functions; and parametrization of algebraic curves. Gosper's algorithm for solving the second problem and the sketch of Collins's cylindrical algebraic decomposition algorithm for the first both suffer didactically from requiring very little of the background material. The discussion of curve parametrization builds much more nicely on the previous chapters.

In summary, this book provides a good, accessible, and up-to-date account of a particular branch of algorithmic algebra that may perhaps be described as the type practised at RISC, Linz. To me it does not seem suitable as an introductory text on polynomial algebra by way of algorithms, but it will probably prove to be of value to users of computer algebra systems in getting a feeling for the background, capabilities, and difficulties that such systems have in dealing with problems involving polynomial arithmetic.

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22[00A69]—*Computer algebra in industry: Problem solving in practice*, Arjeh M. Cohen (Editor), Wiley, Chichester, 1993, x+252 pp., $23\frac{1}{2}$ cm, \$45.00

Computer algebra systems (e.g., Mathematica, MAPLE, etc.) are now familiar to the academic community. Courses are taught on their use and textbooks are being written using them, which in turn require the reader to use them to solve problems. An example is the differential geometry text by Gray [1]. The publicity for these systems uses phrases such as "environment for technical computing", and questions about how such systems have changed the practice of mathematics are