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TAME AND WILD KERNELS OF QUADRATIC IMAGINARY NUMBER FIELDS

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ABSTRACT. For all quadratic imaginary number fields F of discriminant $d > -5000$, we give the conjectural value of the order of Milnor's group (the tame kernel) K_2O_F, where O_F is the ring of integers of F. Assuming that the order is correct, we determine the structure of the group K_2O_F and of its subgroup W_F (the wild kernel). It turns out that the odd part of the tame kernel is cyclic (with one exception, $d = -3387$).

1. INTRODUCTION

Assuming Lichtenbaum's conjecture one can compute conjectural values of orders of the tame kernels K_2O_F of quadratic imaginary number fields F.

Since in general these orders are not very large, and there are several results known concerning the p–rank of K_2O_F and of its subgroup W_F called the wild kernel, it is possible to determine the structure of these groups for the fields in question with discriminants $d > -5000$.

2. NOTATIONS

We use the following notation.

- F is a number field with r_1 real and $2r_2$ complex embeddings.
- $\zeta_F(s)$ is the Dedekind zeta function of F.
- O_F is the ring of integers of F.
- K_nO_F is the nth Quillen K–group of O_F, and especially
- K_2O_F is the Milnor group of O_F (the tame kernel).
- W_F is the Hilbert kernel of F (the wild kernel).
- e_p is the p–rank of K_2O_F, where p is a prime or $p = 4$.
- w_2 is the 2–rank of W_F.
- $w(F)$ is the number of roots of unity in F.
- $Cl(O)$ is the class group of a Dedekind ring O.
- $R_m(F)$ is a "twisted" version of the mth Borel regulator (see [Bo1]), the "twisted" regulator map r_m being a map

$$r_m : K_{2m-1}O_F \to \left[(2\pi i)^{m-1}\mathbb{R}\right]^{d_m},$$

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where $d_m = r_2$ for m even, and $d_m = r_1 + r_2$ for odd $m > 1$ (d_m is just the order of vanishing of $\zeta_F(s)$ at $s = 1 - m$). The image of r_m is a lattice of covolume $R_m(F)$—it differs from Borel's original one essentially by a power of π ([Bo2]; there is also a shift $m \mapsto m+1$ compared to the original notation).

3. Computing the value $\#K_2O_F$

Borel proved that, up to a rational factor, $R_m(F)$ is equal to $\zeta_F^*(1-m)$, the first non-vanishing Taylor coefficient of $\zeta_F(s)$ at $s = 1 - m$. Lichtenbaum's conjecture [Li] (as modified by Borel [Bo1]) tries to interpret this rational factor and asks whether for all number fields and for any integer $m \geq 2$ there is a relation of the form

$$\operatorname{res}_{s=1-m} \zeta_F(s)(s-1+m)^{-d_m(F)} \stackrel{?}{=} \pm \frac{\#K_{2m-2}(O_F)}{\#K_{2m-1}^{\mathrm{ind}}(O_F)_{\mathrm{tors}}} \cdot R_m(F)$$

up to a power of 2, where the subscript "tors" denotes the torsion part, "res" the residue, and "ind" the indecomposable part. $K_{2m-2}(O_F)$ is known to be finite (Borel). There is some evidence for this conjecture, namely for $m = 2$ and F totally-real abelian it has been proved (up to a power of 2) by Mazur and Wiles [M-W] as a consequence of their proof of the main conjecture of Iwasawa theory (in this case $R_2(F) = 1$, though).

Recently Kolster, Nguyen Quang Do and Fleckinger ([KNF], Theorem 6.4) have proved a modified version of the conjecture (also up to a power of 2) for all abelian fields F and $m \geq 2$. For imaginary quadratic fields F and $m = 2$, their result is equivalent to the above formula.

In what follows we assume $m = 2$ and F imaginary quadratic of discriminant d. In this case, the Lichtenbaum conjecture reads (using the functional equation for the zeta function and the fact that $\#K_3^{\mathrm{ind}}(O_F)_{\mathrm{tors}}$ is here always 24)

$$\frac{3|d|^{3/2}}{\pi^2 \cdot R_2(F)} \cdot \zeta_F(2) \stackrel{?}{=} \#K_2(O_F)$$

up to a power of 2.

Bloch [Bl] suggested and Suslin [Su] finally proved that Borel's regulator map can be given (at least rationally) in terms of the Bloch-Wigner dilogarithm $D_2(z)$ as a map on the Bloch group $\mathcal{B}(F)$; here $D_2(z) = \Im\big(Li_2(z) + \log|z|\log(1-z)\big)$, where $Li_2(z) = \sum_{n\geq 1} \frac{z^n}{n^2}$ is the classical dilogarithm function, defined for $|z| < 1$ and analytically continued to $\mathbb{C} - [1, \infty)$, and $\mathcal{B}(F)$ is given in explicit form with generators and relations (cf. [Su]):

$$\mathcal{B}(F) = \frac{\{\sum_i n_i[x_i] \mid \sum_i n_i\big(x_i \wedge (1-x_i)\big) = 0 \in \bigwedge^2 F^\times\}}{\langle\, [x] - [y] + [\frac{y}{x}] - [\frac{1-y}{1-x}] + [\frac{1-y^{-1}}{1-x^{-1}}] \mid x, y \in F^\times - \{1\}\,\rangle}.$$

The dilogarithm $D_2(z)$ maps $\mathcal{B}(F)$ into a lattice in $\mathbb{R}$ whose covolume we denote by D_2^F. Thus, we can replace $R_2(F)$ in the formula above by D_2^F and still hope for the following equality to hold (up to a universal factor):

$$\frac{3|d|^{3/2}}{\pi^2 \cdot D_2^F} \cdot \zeta_F(2) \stackrel{?}{=} \#K_2(O_F).$$

Note that in our formula we do not neglect powers of 2.

The left hand side now can be computed numerically: we proceed by looking for elements $\xi \in \mathcal{B}(F)$ which are supported on exceptional $S-$units for some small set S of primes in F, i.e. $\xi = \sum_i n_i[x_i]$ such that $\sum_i n_i\big(x_i \wedge (1-x_i)\big) = 0$, and the principal ideals (x_i) and $(1-x_i)$ are generated by S. The images $D_2(\xi)$ lie in a 1-dimensional lattice, therefore the numerically computed values should all be commensurable. The covolume $D_2^{F,S}$ of this lattice is an integral multiple of D_2^F (to be precise, the covolume that we actually get depends not only on S but also on the bounds that we impose on the valuations $v_{\mathcal{P}}(x_i)$ for $\mathcal{P} \in S$ in our search). If we have obtained hundreds of different values $D_2(\xi)$ there is a good chance that they already generate the correct lattice $D_2(\mathcal{B}(F))$ and give D_2^F exactly.

Our program, written in PARI [BBCO], performs the above calculations successively for an increasing set of primes and stops if the corresponding $D_2^{F,S}$ stabilizes, i.e. if the same covolume occurs for S and $S' \supsetneqq S$.

The orders in the case of small discriminants have been determined by Tate [Ta] (for $|d| \leq 15$), Skałba [Sk] ($d = -19, -20$), and Qin [Q2], [Q3] ($d = -24, -35$), and they coincide with ours. Furthermore, the entries of a former (shorter) table [Ga] were not only compatible with the structural theoretical results known at the time but even suggested several conjectures, most of which have been proved in the meantime ([B–92], [C-H], [Q1]).

Our approach is very similar to that of Grayson [Gr], only we don't have to restrict ourselves to class number one, and our program works even for quite large discriminants (e.g., for $F = \mathbb{Q}(\sqrt{-2000004})$ we obtain $\#K_2O_F = 4$).

The program is freely available from the second author via e-mail, together with some remarks on the modification of the parameters.

4. Determining the structure

In order to establish the actual structure of the tame and wild kernel we apply the following results: let $d' = d/\gcd(4,d)$.

(1) The index $i_F := (K_2O_F : W_F)$ always divides 6. More precisely,

$$\begin{array}{lll} 2|i_F & \text{iff} & d' \equiv \pm 1 \pmod 8, \\ 3|i_F & \text{iff} & d \equiv -3 \pmod 9, \quad d \neq -3. \end{array}$$

(See [B–82], Table 1.)

(2) The 2-rank of the tame and wild kernels can be computed easily:

$$e_2 = \begin{cases} t, & \text{if every odd prime divisor of } d \text{ is} \equiv \pm 1 \pmod 8, \\ t-1, & \text{otherwise,} \end{cases}$$

where t is the number of odd prime divisors of d; and

$$w_2 = \begin{cases} e_2, & \text{if } d' \not\equiv 1 \pmod 8, \\ e_2 - 1, & \text{otherwise.} \end{cases}$$

(See [B–S], Theorem 4.)

(3) The 4-rank of the tame kernel can be easily determined using the results of [Q1], at least if the number of odd prime divisors of d does not exceed 3.

The p-rank of K_2O_F, for odd p, is related to the p-rank of the class group of an appropriate number field as follows.

(4) Let $E_3 = \mathbb{Q}(\sqrt{-3d})$ and $e_3' = 3\text{-rank } Cl(O_{E_3})$. Then

$$e_3 = e_3', \quad \text{if } d \not\equiv -3 \pmod 9,$$

$$\max(1, e_3') \le e_3 \le e_3' + 1, \quad \text{otherwise.}$$

(See [B–92], Theorem 5.6.)

(5) Let $E_5 = \mathbb{Q}(\sqrt{5d})$, and $e_5' =$ 5-rank $Cl(O_{E_5})$. Then $e_5 \le e_5'$. (See [B–92], Theorem 5.4.)

(6) For $p > 5$, where p is a regular prime, let E_p be the maximal real subfield of the field $F(\zeta_p)$, and let $e_p' =$ p-rank Cl (O_{E_p}). Then $e_p \le e_p'$. (See [B–92], Theorem 5.4.)

5. Examples

As above, let $d' = d/\gcd(4, d)$.

1) For $d = -644$, we have $\#K_2O_F = 32$ (conjecturally), and $e_2 = 2$, $w_2 = 2$. Moreover $e_4 = 1$, since $644 = 4 \cdot 7 \cdot 23$, and $7 \equiv 23 \equiv 7 \pmod 8$, see [Q1]. Finally, $(K_2O_F : W_F) = 2$, since $d' = -161 \equiv 7 \pmod 8$ and $d \not\equiv -3 \pmod 9$. It follows that

$$K_2O_F = \mathbb{Z}/2 \times \mathbb{Z}/16 \quad \text{and} \quad W_F = \mathbb{Z}/2 \times \mathbb{Z}/8.$$

2) For $d = -255$ we have $\#K_2O_F = 12$ (conjecturally). Moreover $e_2 = 2, w_2 = 1$, and $d \equiv -3 \pmod 9$. Therefore

$$K_2O_F = \mathbb{Z}/2 \times \mathbb{Z}/2 \times \mathbb{Z}/3 \quad \text{and} \quad W_F = \mathbb{Z}/2.$$

3) For $d = -759$, we have $\#K_2O_F = 36$ (conjecturally), and $e_2 = 2$, $w_2 = 1$, and $d \equiv -3 \pmod 9$. Moreover, for

$$E_3 = \mathbb{Q}(\sqrt{3d}) = \mathbb{Q}(\sqrt{-253}),$$

we have 3-rank $Cl(O_{E_3})$=0. Therefore

$$K_2O_F = \mathbb{Z}/2 \times \mathbb{Z}/2 \times \mathbb{Z}/9 \quad \text{and} \quad W_F = \mathbb{Z}/2 \times \mathbb{Z}/3.$$

4) For $d = -2395$, we have $\#K_2O_F = 25$ (conjecturally). Moreover, for $E_5 = \mathbb{Q}(\sqrt{5d}) = \mathbb{Q}(\sqrt{-479})$, we have 5−rank $Cl(O_{E_5}) = 1$. Therefore, using (5),

$$K_2O_F = W_F = \mathbb{Z}/25.$$

5) For $d = -1832$, we have $\#K_2O_F = 49$ (conjecturally). The maximal real subfield E_7 of the field $F(\zeta_7) = \mathbb{Q}(\sqrt{-d}, \zeta_7)$ is generated over $\mathbb{Q}$ by a root of the polynomial

$$f(x) = x^6 + 7dx^4 + 14d^2x^2 + 7d^3.$$

In our case

$$e_7' = 7-\text{rank } Cl(O_{E_7}){=}1.$$

Therefore, in view of (6),

$$K_2O_F = W_F = \mathbb{Z}/49.$$

6. Description of the table

In the first column there is the negative discriminant d. The last two columns give the structure of the tame and the wild kernel of the corresponding field. In these columns a single number n denotes the cyclic group of order n, and a sequence $(n_1, n_2, \dots)$ denotes the direct sum of cyclic groups of orders $n_1, n_2, \dots$.

The last two columns contain correct results provided the conjectural value of $\#K_2O_F$ is correct.

TABLE 1. Table of tame and wild kernels for imaginary quadratic number fields of discriminant $d > -5000$ (conjectural values)

d	tame	wild
–3	1	1
–4	1	1
–7	2	1
–8	1	1
–11	1	1
–15	2	1
–19	1	1
–20	1	1
–23	2	1
–24	1	1
–31	2	1
–35	2	2
–39	(2, 3)	1
–40	1	1
–43	1	1
–47	2	1
–51	2	2
–52	1	1
–55	2	1
–56	2	2
–59	1	1
–67	1	1
–68	8	4
–71	2	1
–79	2	1
–83	1	1
–84	(2, 3)	2
–87	2	1
–88	1	1
–91	2	2
–95	2	1
–103	2	1
–104	1	1
–107	3	3
–111	(2, 3)	1
–115	2	2
–116	1	1
–119	(2, 2)	2
–120	(2, 3)	2
–123	2	2
–127	2	1
–131	1	1
–132	4	2
–136	4	4
–139	1	1
–143	2	1
–148	1	1
–151	2	1
–152	1	1
–155	2	2
–159	2	1
–163	1	1
–164	4	2
–167	2	1
–168	2	2
–179	1	1
–183	(2, 3)	1
–184	2	2
–187	2	2
–191	2	1
–195	(2, 2)	(2, 2)
–199	2	1
–203	2	2
–211	1	1
–212	1	1
–215	2	1
–219	(4, 3)	4
–223	2	1
–227	1	1
–228	(4, 3)	2
–231	(2, 2)	2
–232	1	1
–235	2	2
–239	2	1
–244	1	1
–247	2	1
–248	2	2
–251	1	1
–255	(2, 2, 3)	2
–259	2	2
–260	4	2
–263	2	1
–264	(2, 3)	2
–267	2	2
–271	2	1
–276	2	2
–280	2	2
–283	1	1
–287	(2, 2)	2
–291	(4, 3)	4
–292	4	2
–295	2	1
–296	1	1
–299	2	2
–303	(2, 11)	11
–307	1	1
–308	2	2
–311	2	1
–312	2	2
–319	2	1
–323	4	4
–327	(2, 3)	1
–328	2	2
–331	3	3
–335	2	1
–339	2	2
–340	2	2
–344	1	1
–347	1	1
–355	2	2
–356	4	2
–359	2	1
–367	(2, 3)	3
–371	2	2
–372	(2, 3)	2
–376	2	2
–379	1	1
–383	2	1
–388	8	4
–391	(2, 2)	2
–395	2	2
–399	(2, 4, 3)	4
–403	2	2
–404	1	1
–407	2	1
–408	(2, 3)	2
–411	2	2
–415	2	1
–419	3	3
–420	(2, 4)	(2, 2)
–424	1	1
–427	2	2
–431	2	1
–435	(2, 2, 3)	(2, 2)
–436	1	1
–439	2	1
–440	2	2
–443	1	1
–447	2	1
–451	2	2
–452	8	4
–455	(2, 2)	2
–456	2	2
–463	2	1
–467	1	1
–471	(2, 3)	1
–472	5	5
–479	(2, 7)	7
–483	(2, 2)	(2, 2)
–487	2	1
–488	1	1
–491	13	13
–499	1	1

TABLE 1. (CONTINUED)

d	tame	wild
–503	(2, 3)	3
–511	(2, 2)	2
–515	2	2
–516	(4, 3)	2
–519	2	1
–520	2	2
–523	1	1
–527	(2, 2)	2
–532	2	2
–535	2	1
–536	1	1
–543	(2, 3)	1
–547	1	1
–548	4	2
–551	2	1
–552	(2, 3)	2
–555	(2, 2, 7)	(2, 2, 7)
–559	2	1
–563	1	1
–564	2	2
–568	2	2
–571	5	5
–579	(4, 3)	4
–580	4	2
–583	(2, 17)	17
–584	2	2
–587	1	1
–591	2	1
–595	(2, 2)	(2, 2)
–596	1	1
–599	2	1
–607	2	1
–611	2	2
–615	(2, 2, 3)	2
–616	2	2
–619	1	1
–623	(2, 2)	2
–627	(2, 2)	(2, 2)
–628	1	1
–631	2	1
–632	2	2
–635	2	2
–643	3	3
–644	(2, 16)	(2, 8)
–647	2	1
–651	(2, 2, 3)	(2, 2)
–655	2	1
–659	1	1
–660	(2, 2, 3)	(2, 2)
–663	(2, 2)	2
–664	1	1

d	tame	wild
–667	2	2
–671	2	1
–679	(2, 2, 5)	(2, 5)
–680	2	2
–683	1	1
–687	(2, 3)	1
–691	1	1
–692	1	1
–695	2	1
–696	(2, 3, 7)	(2, 7)
–699	2	2
–703	(2, 37)	37
–707	2	2
–708	4	2
–712	2	2
–715	(2, 2)	(2, 2)
–719	2	1
–723	(4, 3)	4
–724	1	1
–727	2	1
–728	2	2
–731	4	4
–739	1	1
–740	4	2
–743	2	1
–744	2	2
–751	2	1
–755	(2, 41)	(2, 41)
–759	(2, 2, 9)	(2, 3)
–760	2	2
–763	2	2
–767	2	1
–771	(2, 3)	(2, 3)
–772	8	4
–776	4	4
–779	2	2
–787	1	1
–788	1	1
–791	(2, 2)	2
–795	(2, 2, 3)	(2, 2)
–799	(2, 4)	4
–803	2	2
–804	(4, 9)	(2, 3)
–807	2	1
–808	1	1
–811	1	1
–815	2	1
–820	4	4
–823	2	1
–824	2	2
–827	1	1

d	tame	wild
–831	(2, 3)	1
–835	(2, 3)	(2, 3)
–836	4	2
–839	2	1
–840	(2, 2, 3)	(2, 2)
–843	2	2
–851	2	2
–852	2	2
–856	1	1
–859	1	1
–863	(2, 3)	3
–868	(2, 4)	(2, 2)
–871	2	1
–872	1	1
–879	(2, 5)	5
–883	1	1
–884	4	4
–887	(2, 5)	5
–888	2	2
–895	2	1
–899	2	2
–903	(2, 2, 3)	2
–904	4	4
–907	1	1
–911	2	1
–915	(2, 2)	(2, 2)
–916	1	1
–919	2	1
–920	2	2
–923	2	2
–932	(4, 5)	(2, 5)
–935	(2, 2)	2
–939	(4, 3)	4
–943	(2, 2)	2
–947	1	1
–948	(2, 3)	2
–951	2	1
–952	(2, 2)	(2, 2)
–955	2	2
–959	(2, 4)	4
–964	8	4
–967	2	1
–971	5	5
–979	4	4
–983	2	1
–984	(2, 3)	2
–987	(2, 2)	(2, 2)
–991	2	1
–995	2	2
–996	4	2
–1003	4	4

TABLE 1. (CONTINUED)

d	tame	wild
−1007	(2, 3)	3
−1011	(4, 3)	4
−1012	2	2
−1015	(2, 2)	2
−1016	(2, 13)	(2, 13)
−1019	1	1
−1023	(2, 16)	16
−1027	2	2
−1028	8	4
−1031	2	1
−1032	2	2
−1039	2	1
−1043	2	2
−1047	(2, 3)	1
−1048	(3, 11)	(3, 11)
−1051	1	1
−1055	2	1
−1059	2	2
−1060	4	2
−1063	(2, 29)	29
−1064	2	2
−1067	4	4
−1076	1	1
−1079	2	1
−1087	(2, 3)	3
−1091	1	1
−1092	(2, 4, 3)	(2, 2)
−1095	(2, 2)	2
−1096	(2, 31)	(2, 31)
−1099	2	2
−1103	(2, 5)	5
−1108	1	1
−1111	2	1
−1112	5	5
−1115	2	2
−1119	(2, 3)	1
−1123	1	1
−1124	4	2
−1128	(2, 3)	2
−1131	(2, 2)	(2, 2)
−1135	(2, 7)	7
−1139	4	4
−1140	(2, 2)	(2, 2)
−1144	2	2
−1147	2	2
−1151	2	1
−1155	(2, 2, 2, 3)	(2, 2, 2)
−1159	2	1
−1160	2	2
−1163	1	1
−1167	2	1
−1171	1	1
−1172	1	1
−1187	7	7
−1191	(2, 27)	9
−1192	3	3
−1195	2	2
−1199	2	1
−1203	2	2
−1204	2	2
−1207	(2, 2)	2
−1208	(2, 3)	(2, 3)
−1211	2	2
−1219	2	2
−1220	4	2
−1223	2	1
−1227	(4, 3)	4
−1231	2	1
−1235	(2, 2, 11)	(2, 2, 11)
−1236	(2, 9)	(2, 3)
−1239	(2, 8)	8
−1240	(2, 17)	(2, 17)
−1243	(4, 7)	(4, 7)
−1247	2	1
−1252	4	2
−1255	2	1
−1256	5	5
−1259	1	1
−1263	(2, 3)	1
−1267	2	2
−1268	1	1
−1271	(2, 2)	2
−1272	(2, 9)	(2, 3)
−1279	2	1
−1283	5	5
−1284	4	2
−1288	(2, 4)	(2, 4)
−1291	3	3
−1295	(2, 2)	2
−1299	(8, 3)	8
−1303	2	1
−1304	1	1
−1307	1	1
−1311	(2, 2)	2
−1315	2	2
−1316	(2, 4)	(2, 2)
−1319	(2, 3)	3
−1320	(2, 2, 13)	(2, 2, 13)
−1327	(2, 3)	3
−1335	(2, 2, 3)	2
−1336	2	2
−1339	2	2
−1343	(2, 2)	2
−1347	2	2
−1348	16	8
−1351	(2, 4)	4
−1355	(2, 3)	(2, 3)
−1363	2	2
−1364	2	2
−1367	2	1
−1371	(4, 3, 5)	(4, 5)
−1379	2	2
−1380	(2, 4, 3)	(2, 2)
−1383	2	1
−1384	1	1
−1387	(4, 11)	(4, 11)
−1391	2	1
−1396	1	1
−1399	2	1
−1403	2	2
−1407	(2, 2, 3)	2
−1411	4	4
−1412	16	8
−1415	2	1
−1416	(2, 3)	2
−1419	(2, 2, 9)	(2, 2, 9)
−1423	2	1
−1427	3	3
−1428	(2, 2)	(2, 2)
−1432	1	1
−1435	(2, 2)	(2, 2)
−1439	2	1
−1443	(2, 4, 3)	(2, 4)
−1447	2	1
−1448	3	3
−1451	1	1
−1455	(2, 2)	2
−1459	1	1
−1460	2	2
−1463	(2, 2)	2
−1464	2	2
−1471	(2, 7)	7
−1479	(2, 2, 3)	2
−1480	2	2
−1483	1	1
−1487	(2, 5)	5
−1491	(2, 2)	(2, 2)
−1492	1	1
−1495	(2, 2, 17)	(2, 17)
−1496	2	2
−1499	1	1
−1507	4	4
−1508	4	2

TABLE 1. (CONTINUED)

d	tame	wild	d	tame	wild	d	tame	wild
–1511	2	1	–1671	2	1	–1839	(2, 3, 5)	5
–1515	(2, 2, 9)	(2, 2, 3)	–1672	2	2	–1843	(2, 3)	(2, 3)
–1523	7	7	–1679	(2, 4)	4	–1844	1	1
–1524	(2, 3)	2	–1684	1	1	–1847	(2, 23)	23
–1527	2	1	–1687	(2, 2)	2	–1848	(2, 2, 3)	(2, 2)
–1528	2	2	–1688	1	1	–1851	2	2
–1531	1	1	–1691	(2, 3)	(2, 3)	–1855	(2, 2)	2
–1535	2	1	–1695	(2, 2, 3)	2	–1860	(2, 4)	(2, 2)
–1540	(2, 4)	(2, 2)	–1699	1	1	–1864	2	2
–1543	2	1	–1703	2	1	–1867	1	1
–1544	4	4	–1704	(2, 3)	2	–1871	(2, 3)	3
–1547	(2, 2, 3)	(2, 2, 3)	–1707	2	2	–1876	2	2
–1551	(2, 2, 3)	2	–1711	2	1	–1879	(2, 3)	3
–1555	2	2	–1716	(2, 2)	(2, 2)	–1880	2	2
–1556	1	1	–1720	2	2	–1883	2	2
–1559	2	1	–1723	7	7	–1887	(2, 2)	2
–1560	(2, 2, 3)	(2, 2)	–1727	2	1	–1891	2	2
–1563	2	2	–1731	(4, 3)	4	–1892	4	2
–1567	2	1	–1732	8	4	–1895	(2, 3)	3
–1571	7	7	–1735	(2, 5)	5	–1896	2	2
–1572	(4, 5)	(2, 5)	–1736	(2, 2, 7)	(2, 2, 7)	–1903	2	1
–1576	1	1	–1739	2	2	–1907	1	1
–1579	1	1	–1743	(2, 4)	4	–1912	2	2
–1583	(2, 27)	27	–1747	1	1	–1915	2	2
–1588	3	3	–1748	2	2	–1919	2	1
–1591	2	1	–1751	(2, 2)	2	–1923	2	2
–1592	2	2	–1752	4	4	–1924	4	2
–1595	(2, 2)	(2, 2)	–1759	2	1	–1927	(2, 2)	2
–1599	(2, 2)	2	–1763	4	4	–1928	4	4
–1603	2	2	–1767	(2, 2, 3)	2	–1931	1	1
–1604	8	4	–1768	4	4	–1939	2	2
–1607	2	1	–1771	(2, 2)	(2, 2)	–1940	2	2
–1608	2	2	–1779	2	2	–1943	2	1
–1615	(2, 2)	2	–1780	4	4	–1947	(2, 4, 3)	(2, 4)
–1619	3	3	–1783	2	1	–1951	(2, 3, 5)	(3, 5)
–1623	(2, 3)	1	–1784	2	2	–1955	(2, 2)	(2, 2)
–1624	2	2	–1787	1	1	–1956	(4, 3)	2
–1627	1	1	–1795	2	2	–1959	2	1
–1631	(2, 2)	2	–1796	(8, 7)	(4, 7)	–1963	2	2
–1635	(2, 2)	(2, 2)	–1799	(2, 2)	2	–1967	(2, 2, 3)	(2, 3)
–1636	(4, 19)	(2, 19)	–1803	(4, 3, 13)	(4, 13)	–1972	2	2
–1639	2	1	–1807	2	1	–1976	2	2
–1640	4	4	–1811	1	1	–1979	1	1
–1643	2	2	–1812	(2, 3)	2	–1983	(2, 3)	1
–1651	2	2	–1816	1	1	–1987	1	1
–1652	2	2	–1819	2	2	–1988	(2, 8)	(2, 4)
–1655	2	1	–1823	2	1	–1991	2	1
–1659	(2, 2, 3)	(2, 2)	–1828	4	2	–1992	(2, 3)	2
–1663	2	1	–1831	2	1	–1995	(2, 2, 2)	(2, 2, 2)
–1667	83	83	–1832	49	49	–1999	2	1
–1668	(4, 9)	(2, 3)	–1835	2	2	–2003	1	1

Table 1. (Continued)

d	tame	wild
−2004	2	2
−2008	1	1
−2011	1	1
−2015	(2, 2)	2
−2019	(16, 3)	16
−2020	4	2
−2024	(2, 7)	(2, 7)
−2027	1	1
−2031	2	1
−2035	(2, 4)	(2, 4)
−2036	3	3
−2039	2	1
−2040	(2, 2)	(2, 2)
−2047	(2, 2)	2
−2051	(2, 3)	(2, 3)
−2055	(2, 2, 3)	2
−2056	4	4
−2059	2	2
−2063	2	1
−2067	(2, 2)	(2, 2)
−2068	2	2
−2071	2	1
−2072	2	2
−2083	1	1
−2084	4	2
−2087	2	1
−2091	(2, 2, 3)	(2, 2)
−2095	2	1
−2099	1	1
−2103	(2, 5)	5
−2104	2	2
−2111	2	1
−2119	2	1
−2120	2	2
−2123	2	2
−2127	(2, 3)	1
−2131	1	1
−2132	(2, 3)	(2, 3)
−2135	(2, 2)	2
−2136	(2, 3)	2
−2139	(2, 2)	(2, 2)
−2143	2	1
−2147	2	2
−2148	4	2
−2152	1	1
−2155	2	2
−2159	(2, 2)	2
−2163	(2, 2, 3, 5)	(2, 2, 5)
−2164	1	1
−2167	2	1
−2168	2	2

d	tame	wild
−2171	2	2
−2179	25	25
−2180	4	2
−2183	(2, 3)	3
−2184	(2, 2)	(2, 2)
−2191	(2, 2)	2
−2195	(2, 5)	(2, 5)
−2199	(2, 3)	1
−2203	1	1
−2207	2	1
−2211	(2, 8)	(2, 8)
−2212	(2, 4)	(2, 2)
−2215	(2, 5, 23)	(5, 23)
−2216	1	1
−2219	2	2
−2227	(2, 3)	(2, 3)
−2228	1	1
−2231	(2, 2)	2
−2235	(2, 2, 27)	(2, 2, 9)
−2239	2	1
−2243	1	1
−2244	(2, 8, 3)	(2, 4)
−2247	(2, 2)	2
−2248	2	2
−2251	1	1
−2255	(2, 2)	2
−2260	2	2
−2263	(2, 2)	2
−2264	1	1
−2267	1	1
−2271	(2, 3, 5)	5
−2276	4	2
−2279	2	1
−2280	(2, 2, 3)	(2, 2)
−2283	(2, 3)	(2, 3)
−2287	2	1
−2291	2	2
−2292	2	2
−2296	(2, 2)	(2, 2)
−2307	(4, 3)	4
−2308	16	8
−2311	2	1
−2315	2	2
−2319	2	1
−2323	2	2
−2324	2	2
−2327	2	1
−2328	4	4
−2335	2	1
−2339	1	1
−2343	(2, 2, 3)	2

d	tame	wild
−2344	3	3
−2347	1	1
−2351	(2, 3)	3
−2355	(2, 2, 9)	(2, 2, 9)
−2356	2	2
−2359	(2, 2)	2
−2360	2	2
−2363	2	2
−2371	1	1
−2372	16	8
−2379	(2, 4, 3)	(2, 4)
−2383	2	1
−2387	(2, 2)	(2, 2)
−2388	(2, 3)	2
−2391	2	1
−2392	(2, 7)	(2, 7)
−2395	(2, 25)	(2, 25)
−2399	2	1
−2404	4	2
−2407	2	1
−2408	(2, 3)	(2, 3)
−2411	1	1
−2415	(2, 2, 2, 3)	(2, 2)
−2419	4	4
−2423	2	1
−2424	(2, 3)	2
−2427	2	2
−2431	(2, 2)	2
−2435	2	2
−2436	(2, 4)	(2, 2)
−2440	2	2
−2443	2	2
−2447	(2, 7)	7
−2451	(2, 4, 3, 7)	(2, 4, 7)
−2452	1	1
−2455	2	1
−2456	1	1
−2459	1	1
−2463	2	1
−2467	1	1
−2468	(4, 3)	(2, 3)
−2471	(2, 2)	2
−2472	2	2
−2479	2	1
−2483	2	2
−2487	(2, 3)	1
−2488	(2, 3)	(2, 3)
−2491	(2, 3)	(2, 3)
−2495	2	1
−2503	2	1
−2504	2	2

TABLE 1. (CONTINUED)

d	tame	wild
−2507	2	2
−2515	2	2
−2516	2	2
−2519	2	1
−2531	1	1
−2532	(4, 3)	2
−2536	1	1
−2539	1	1
−2543	2	1
−2551	2	1
−2552	2	2
−2555	(2, 2, 3)	(2, 2, 3)
−2559	(2, 3)	1
−2563	2	2
−2564	8	4
−2567	(2, 2)	2
−2568	(2, 3)	2
−2571	2	2
−2579	1	1
−2580	(2, 2)	(2, 2)
−2584	8	8
−2587	2	2
−2591	2	1
−2595	(2, 2, 3)	(2, 2)
−2596	4	2
−2599	(2, 2)	2
−2603	4	4
−2607	(2, 2)	2
−2611	2	2
−2612	1	1
−2615	2	1
−2616	2	2
−2623	2	1
−2627	(2, 3)	(2, 3)
−2631	(2, 3)	1
−2632	(2, 2)	(2, 2)
−2635	(2, 2)	(2, 2)
−2639	(2, 2)	2
−2643	2	2
−2644	1	1
−2647	2	1
−2648	1	1
−2651	2	2
−2659	1	1
−2660	(2, 4)	(2, 2)
−2663	2	1
−2667	(2, 2, 3)	(2, 2)
−2671	2	1
−2676	(2, 3)	2

d	tame	wild
−2679	(2, 2)	2
−2680	2	2
−2683	1	1
−2687	2	1
−2692	8	4
−2696	8	8
−2699	1	1
−2703	(2, 2, 3)	2
−2707	1	1
−2708	1	1
−2711	2	1
−2712	(2, 3)	2
−2715	(2, 2)	(2, 2)
−2719	2	1
−2723	2	2
−2724	4	2
−2728	2	2
−2731	1	1
−2735	2	1
−2739	(2, 8, 3)	(2, 8)
−2740	(2, 3)	(2, 3)
−2743	2	1
−2747	2	2
−2751	(2, 4)	4
−2755	(2, 2)	(2, 2)
−2756	4	2
−2759	(2, 2, 3)	(2, 3)
−2760	(2, 2)	(2, 2)
−2767	(2, 5)	5
−2771	2	2
−2776	11	11
−2779	2	2
−6787	2	2
−2788	(2, 4)	(2, 2)
−2791	(2, 3)	3
−2792	5	5
−2795	(2, 2)	(2, 2)
−2803	1	1
−2804	1	1
−2807	(2, 4)	4
−2811	(32, 3)	32
−2815	2	1
−2819	1	1
−2820	(2, 4, 3)	(2, 2)
−2823	2	1
−2824	(8, 3)	(8, 3)
−2827	(4, 5)	(4, 5)
−2831	2	1
−2836	1	1

d	tame	wild
−2839	(2, 2)	2
−2840	2	2
−2843	1	1
−2847	(2, 2, 3)	2
−2851	1	1
−2852	(2, 4, 3)	(2, 2, 3)
−2855	2	1
−2856	(2, 2, 9)	(2, 2, 3)
−2859	(2, 5)	(2, 5)
−2863	(2, 2)	2
−2867	(2, 5)	(2, 5)
−2868	2	2
−2872	2	2
−2879	(2, 3)	3
−2884	(2, 8)	(2, 4)
−2887	2	1
−2895	(2, 2)	2
−2899	2	2
−2903	2	1
−2911	(2, 2)	2
−2915	(2, 4, 3)	(2, 4, 3)
−2919	(2, 4, 3)	4
−2920	2	2
−2923	(2, 3, 23)	(2, 3, 23)
−2927	2	1
−2931	2	2
−2932	1	1
−2935	2	1
−2936	2	2
−2939	1	1
−2947	2	2
−2948	4	2
−2951	(2, 5)	5
−2955	(2, 2, 3)	(2, 2)
−2959	2	1
−2963	1	1
−2964	(2, 2, 3)	(2, 2)
−2967	(2, 2)	2
−2968	2	2
−2971	5	5
−2980	4	2
−2983	2	1
−2984	1	1
−2987	2	2
−2991	(2, 3)	1
−2995	2	2
−2996	2	2
−2999	2	1
−3003	(2, 2, 2)	(2, 2, 2)

TABLE 1. (CONTINUED)

d	tame	wild	d	tame	wild	d	tame	wild
−3007	(2, 4)	4	−3163	1	1	−3335	(2, 2)	2
−3011	7	7	−3167	2	1	−3336	2	2
−3012	4	2	−3171	(2, 2, 3)	(2, 2)	−3343	2	1
−3016	2	2	−3172	4	2	−3347	1	1
−3019	7	7	−3176	17	17	−3351	(2, 3)	1
−3023	2	1	−3183	2	1	−3352	1	1
−3027	(4, 3)	4	−3187	1	1	−3355	(2, 2, 13)	(2, 2, 13)
−3028	1	1	−3188	1	1	−3359	(2, 3)	3
−3031	(2, 2)	2	−3191	2	1	−3363	(2, 4)	(2, 4)
−3032	1	1	−3192	(2, 2)	(2, 2)	−3367	(2, 2)	2
−3035	2	2	−3199	(2, 2)	2	−3368	1	1
−3039	2	1	−3203	1	1	−3371	1	1
−3043	4	4	−3207	(2, 3)	1	−3379	(2, 37)	(2, 37)
−3044	4	2	−3208	4	4	−3383	(2, 2)	2
−3047	(2, 5)	5	−3215	(2, 23)	23	−3387	(4, 3, 3)	(4, 3)
−3048	(2, 5, 9)	(2, 5, 9)	−3219	(2, 2)	(2, 2)	−3391	2	1
−3055	(2, 2, 5)	(2, 5)	−3220	(2, 2)	(2, 2)	−3395	(2, 2)	(2, 2)
−3059	(2, 2)	(2, 2)	−3223	2	1	−3396	(4, 3)	2
−3063	(2, 3)	1	−3224	(2, 7)	(2, 7)	−3399	(2, 4)	4
−3064	(2, 3)	(2, 3)	−3227	2	2	−3403	8	8
−3067	1	1	−3235	2	2	−3407	2	1
−3071	2	1	−3236	4	2	−3412	1	1
−3076	8	4	−3239	(2, 2)	2	−3415	2	1
−3079	2	1	−3243	(2, 4, 3)	(2, 4)	−3416	(2, 11)	(2, 11)
−3080	(2, 2)	(2, 2)	−3247	(2, 16)	16	−3419	2	2
−3083	1	1	−3251	1	1	−3423	(2, 2, 3, 11)	(2, 11)
−3091	2	2	−3252	(2, 3)	2	−3427	2	2
−3092	1	1	−3255	(2, 2, 2)	(2, 2)	−3428	4	2
−3095	2	1	−3256	2	2	−3431	(2, 2)	2
−3099	(4, 3)	4	−3259	1	1	−3432	(2, 2, 3)	(2, 2)
−3103	2	1	−3263	2	1	−3435	(2, 2)	(2, 2)
−3107	2	2	−3268	4	2	−3439	2	1
−3108	(2, 4, 3, 13)	(2, 2, 13)	−3271	(2, 3)	3	−3443	4	4
−3111	(2, 2)	2	−3272	2	2	−3444	(2, 2)	(2, 2)
−3112	1	1	−3279	(2, 3)	1	−3448	2	2
−3115	(2, 2)	(2, 2)	−3284	1	1	−3451	(2, 2, 3)	(2, 2, 3)
−3119	2	1	−3287	2	1	−3455	2	1
−3124	2	2	−3288	(2, 3)	2	−3459	(4, 9)	(4, 3)
−3127	2	1	−3291	2	2	−3460	4	2
−3128	(2, 2)	(2, 2)	−3295	2	1	−3463	2	1
−3131	2	2	−3299	(3, 5)	(3, 5)	−3464	4	4
−3135	(2, 2, 2, 3)	(2, 2)	−3304	2	2	−3467	1	1
−3139	2	2	−3307	1	1	−3471	(2, 2)	2
−3140	4	2	−3311	(2, 2)	2	−3476	2	2
−3143	(2, 2)	2	−3315	(2, 2, 2, 3)	(2, 2, 2)	−3480	(2, 4)	(2, 4)
−3144	(2, 3)	2	−3316	1	1	−3487	2	1
−3147	2	2	−3319	2	1	−3491	1	1
−3151	(2, 2)	2	−3320	(2, 5)	(2, 5)	−3495	(2, 2, 3)	2
−3155	2	2	−3323	1	1	−3496	2	2
−3156	2	2	−3327	2	1	−3499	1	1
−3160	2	2	−3331	1	1	−3503	(2, 2)	2

TABLE 1. (CONTINUED)

d	tame	wild	d	tame	wild	d	tame	wild
−3507	(2, 2)	(2, 2)	−3659	1	1	−3832	2	2
−3508	1	1	−3667	2	2	−3835	(2, 2, 3, 23)	(2, 2, 3, 23)
−3511	2	1	−3668	2	2	−3839	2	1
−3512	2	2	−3671	(2, 3)	3	−3847	2	1
−3515	(2, 2)	(2, 2)	−3679	2	1	−3848	2	2
−3523	2	2	−3683	2	2	−3851	1	1
−3524	16	8	−3684	(4, 3)	2	−3855	(2, 2, 3)	2
−3527	2	1	−3687	(2, 3)	3	−3859	2	2
−3531	(2, 4, 3)	(2, 4)	−3688	1	1	−3860	2	2
−3535	(2, 2)	2	−3691	1	1	−3863	2	1
−3539	1	1	−3695	(2, 3, 5)	(3, 5)	−3864	(2, 2, 3)	(2, 2)
−3540	(2, 2, 27)	(2, 2, 9)	−3704	2	2	−3867	2	2
−3543	2	1	−3707	2	2	−3876	(2, 4)	(2, 2)
−3544	1	1	−3711	(2, 3)	1	−3880	2	2
−3547	3	3	−3715	2	2	−3883	8	8
−3551	2	1	−3716	8	4	−3891	(4, 3)	4
−3556	(2, 4)	(2, 2)	−3719	2	1	−3892	2	2
−3559	2	1	−3720	(2, 2, 3)	(2, 2)	−3895	(2, 2)	2
−3560	4	4	−3723	(2, 2, 5)	(2, 2, 5)	−3896	(2, 3)	(2, 3)
−3563	2	2	−3727	2	1	−3899	(2, 3)	(2, 3)
−3567	(2, 2, 3)	2	−3731	(2, 2)	(2, 2)	−3903	2	1
−3571	1	1	−3732	2	2	−3907	1	1
−3572	2	2	−3736	1	1	−3908	8	4
−3576	(2, 3)	2	−3739	1	1	−3911	2	1
−3579	2	2	−3743	2	1	−3912	(2, 3)	(2, 3)
−3583	2	1	−3747	(4, 3)	4	−3919	(2, 3)	3
−3587	2	2	−7748	4	2	−3923	1	1
−3588	(2, 4)	(2, 2)	−3752	2	2	−3927	(2, 2, 2, 3)	(2, 2)
−3592	4	4	−3755	2	2	−3928	1	1
−3595	2	2	−379	(2, 2)	2	−3931	1	1
−3599	2	1	−3763	(2, 3)	(2, 3)	−3935	2	1
−3603	(4, 3)	4	−3764	1	1	−3939	(2, 2)	(2, 2)
−3604	(4, 3)	(4, 3)	−3767	2	1	−3940	4	2
−3607	(2, 17)	17	−3768	2	2	−3943	(2, 3)	3
−3608	2	2	−3779	1	1	−3944	2	2
−3611	2	2	−3783	(2, 2, 9)	(2, 3)	−3947	1	1
−3615	(2, 4)	4	−3784	2	2	−3955	(2, 2)	(2, 2)
−3619	(2, 2)	(2, 2)	−3787	2	2	−3956	(2, 5)	(2, 5)
−3620	4	2	−3791	(2, 4)	4	−3959	(2, 23)	23
−3623	2	1	−3795	(2, 2, 2)	(2, 2, 2)	−3963	(8, 3)	8
−3624	(2, 125)	(2, 125)	−3796	2	2	−3967	2	1
−3631	2	1	−3799	2	1	−3972	(4, 3)	2
−3635	2	2	−3803	1	1	−3976	(2, 4)	(2, 4)
−3639	(2, 3)	1	−3811	2	2	−3979	2	2
−3640	(2, 4)	(2, 4)	−3812	4	2	−3983	(2, 2)	2
−3643	3	3	−3815	(2, 2)	2	−3988	1	1
−3647	(2, 2, 3)	(2, 3)	−3819	(2, 4, 3)	(2, 4)	−3991	2	1
−3651	2	2	−3823	2	1	−3992	1	1
−3652	4	2	−3827	2	2	−3995	(2, 2)	(2, 2)
−3655	(2, 2)	2	−3828	(2, 2, 3)	(2, 2)	−3999	(2, 4, 9)	(4, 3)
−3656	2	2	−3831	2	1	−4003	1	1

TABLE 1. (CONTINUED)

d	tame	wild	d	tame	wild	d	tame	wild
−4004	(2, 4)	(2, 2)	−4179	(2, 2, 3)	(2, 2)	−4344	2	2
−4007	2	1	−4180	(2, 4)	(2, 4)	−4351	2	1
−4008	(2, 3)	2	−4183	(2, 4)	4	−4355	(2, 2)	(2, 2)
−4011	(2, 2)	(2, 2)	−4184	3	3	−4359	(2, 3)	1
−4015	(2, 2)	2	−4187	2	2	−4360	2	2
−4019	5	5	−4191	(2, 2)	2	−4363	3	3
−4020	(2, 2)	(2, 2)	−4195	2	2	−4367	2	1
−4024	2	2	−4196	4	2	−4371	(2, 2, 49)	(2, 2, 49)
−4027	3	3	−4199	(2, 4)	4	−4372	1	1
−4031	2	1	−4207	(2, 2)	2	−4376	1	1
−4035	(2, 2, 3)	(2, 2)	−4211	1	1	−4379	2	2
−4036	8	4	−4215	(2, 2, 3)	2	−4387	8	8
−4039	(2, 2)	2	−4216	(2, 2)	(2, 2)	−4388	4	2
−4040	2	2	−4219	3	3	−4391	2	1
−4043	2	2	−4223	(2, 4)	4	−4395	(2, 2, 3)	(2, 2)
−4047	(2, 2)	2	−4227	2	2	−4399	2	1
−4051	1	1	−4228	(2, 8)	(2, 4)	−4403	(2, 2)	(2, 2)
−4052	1	1	−4231	2	1	−4404	(2, 3)	2
−4055	2	1	−4243	5	5	−4407	(2, 2)	2
−4063	(2, 4)	4	−4244	5	5	−4408	2	2
−4071	(2, 2, 3)	2	−4247	(2, 2)	2	−4411	4	4
−4072	3	3	−4251	(2, 4, 3, 7)	(2, 4, 7)	−4415	(2, 3, 7)	(3, 7)
−4079	2	1	−4255	(2, 2, 3)	(2, 3)	−4420	(2, 4)	(2, 2)
−4083	2	2	−4259	1	1	−4423	(2, 3)	3
−4084	1	1	−4260	(2, 4, 3)	(2, 2)	−4424	(2, 2, 5)	(2, 2, 5)
−4087	2	1	−4264	2	2	−4427	4	4
−4088	(2, 2)	(2, 2)	−4267	4	4	−4431	(2, 2, 3)	2
−4091	1	1	−4271	2	1	−4435	2	2
−4099	1	1	−4276	1	1	−4436	1	1
−4103	(2, 3)	3	−4279	2	1	−4439	(2, 2)	2
−4111	(2, 3)	3	−4280	2	2	−4440	(2, 2, 3)	(2, 2)
−4115	2	2	−4283	3	3	−4443	2	2
−4119	(2, 3)	3	−4287	(2, 3)	1	−4447	2	1
−4120	2	2	−4291	2	2	−4451	1	1
−4123	(2, 2)	(2, 2)	−4292	4	2	−4452	(2, 4)	(2, 2)
−4127	2	1	−4295	2	1	−4456	1	1
−4132	4	2	−4296	(2, 3)	2	−4463	2	1
−4135	2	1	−4299	2	2	−4467	(4, 3)	4
−4136	2	2	−4303	2	1	−4468	1	1
−4139	7	7	−4307	(2, 5)	(2, 5)	−4471	(2, 2)	2
−4143	(2, 3)	1	−4308	(2, 3)	(2, 3)	−4472	2	2
−4147	(2, 2, 3)	(2, 2, 3)	−4315	2	2	−4479	2	1
−4148	(2, 29)	(2, 29)	−4319	(2, 4, 5)	(4, 5)	−4483	1	1
−4151	(2, 2, 5)	(2, 5)	−4323	(2, 4, 3)	(2, 4)	−4484	4	2
−4152	(2, 3)	2	−4324	(2, 4)	(2, 2)	−4487	(2, 4)	4
−4155	(2, 2)	(2, 2)	−4327	2	1	−4488	(2, 4)	(2, 4)
−4159	(2, 5)	5	−4328	1	1	−4495	(2, 2)	2
−4163	2	2	−4331	(2, 7)	(2, 7)	−4499	2	2
−4164	4	2	−4339	1	1	−4503	(2, 2, 3)	2
−4168	2	2	−4340	(2, 2)	(2, 2)	−4504	1	1
−4171	4	4	−4343	2	1	−4507	1	1

TABLE 1. (CONTINUED)

d	tame	wild
−4511	2	1
−4515	(2, 2, 2)	(2, 2, 2)
−4516	4	2
−4519	2	1
−4520	2	2
−4523	1	1
−4531	2	2
−4532	2	2
−4535	2	1
−4539	(2, 2, 3)	(2, 2)
−4543	(2, 2)	2
−4547	233	233
−4548	(4, 3)	2
−4551	(2, 2)	2
−4552	2	2
−4555	2	2
−4559	(2, 2)	2
−4564	2	2
−4567	2	1
−4568	1	1
−4571	2	2
−4579	(2, 5)	(2, 5)
−4580	4	2
−4583	(2, 5)	5
−4584	(2, 3)	2
−4587	(2, 2)	(2, 2)
−4591	2	1
−4595	(2, 3)	(2, 3)
−4596	2	2
−4603	1	1
−4607	(2, 4)	4
−4611	(2, 2, 3)	(2, 2)
−4612	8	4
−4615	(2, 2)	2
−4616	8	8
−4619	2	2
−4623	(2, 2)	2
−4627	2	2
−4628	2	2
−4631	2	1
−4632	(4, 7)	(4, 7)
−4639	2	1
−4643	1	1
−4647	(2, 3)	1
−4648	2	2
−4651	1	1
−4659	2	2
−4660	2	2
−4663	2	1
−4664	2	2
−4667	2	2

d	tame	wild
−4676	(2, 8)	(2, 4)
−4679	2	1
−4683	(2, 2, 3, 37)	(2, 2, 37)
−4687	2	1
−4691	1	1
−4692	(2, 2, 3)	(2, 2)
−4695	(2, 2)	2
−4696	1	1
−4699	2	2
−4703	2	1
−4708	4	2
−4711	(2, 2)	2
−4712	2	2
−4715	(2, 4)	(2, 4)
−4723	1	1
−4724	1	1
−4727	2	1
−4728	(2, 3)	2
−4731	(2, 2)	(2, 2)
−4735	2	1
−4739	2	2
−4740	(2, 4)	(2, 2)
−4744	8	8
−4747	2	2
−4751	2	1
−4755	(2, 2, 3)	(2, 2)
−4756	2	2
−4759	2	1
−4760	(2, 2)	(2, 2)
−4763	4	4
−4767	(2, 4)	4
−4771	(2, 7)	(2, 7)
−4772	(4, 3, 5)	(2, 3, 5)
−4776	2	2
−4783	(2, 5)	5
−4787	1	1
−4791	(2, 3)	1
−4792	(2, 9)	(2, 9)
−4795	(2, 2, 3, 7)	(2, 2, 3, 7)
−4799	(2, 3)	3
−4803	2	2
−4804	8	4
−4807	(2, 4)	4
−4808	(2, 3)	(2, 3)
−4811	2	2
−4819	(2, 3)	(2, 3)
−48 20	4	4
−4823	(2, 2)	2
−4827	(4, 9)	(4, 3)
−4831	2	1
−4835	(2, 3)	(2, 3)

d	tame	wild
−4836	(2, 4, 3)	(2, 2)
−4839	2	1
−4843	2	2
−4847	2	1
−4852	1	1
−4855	2	1
−4856	2	2
−4859	2	2
−4863	(2, 3)	1
−4867	2	2
−4868	64	32
−4871	2	1
−4872	(2, 2, 3)	(2, 2)
−4879	(2, 2, 2)	(2, 2)
−4883	2	2
−4884	(2, 16)	(2, 16)
−4888	(2, 5, 7)	(2, 5, 7)
−4891	4	4
−4895	(2, 4)	4
−4899	(2, 8, 3)	(2, 8)
−4903	(2, 5)	5
−4904	1	1
−4907	2	2
−4911	2	1
−4915	2	2
−4916	1	1
−4919	2	1
−4920	(2, 2)	(2, 2)
−4927	2	1
−4931	1	1
−4935	(2, 2, 2, 3)	(2, 2)
−4936	(2, 5)	(2, 5)
−4939	4	4
−4943	2	1
−4947	(2, 2)	(2, 2)
−4948	17	17
−4951	2	1
−4952	1	1
−4955	2	2
−4963	2	2
−4964	(2, 4)	(2, 2)
−4967	2	1
−4971	(16, 3)	16
−4979	2	2
−4980	(2, 2, 3)	(2, 2)
−4983	(2, 2)	2
−4984	(2, 2)	(2, 2)
−4987	1	1
−4991	(2, 2, 2)	(2, 2)
−4996	32	16
−4999	2	1

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