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REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1991 Mathematics Subject Classification can be found in the annual subject index of *Mathematical Reviews* starting with the December 1990 issue.

1[65-01, 65Fxx]—*Numerical linear algebra*, by Lloyd N. Trefethen and David Bau, III, SIAM, Philadelphia, PA, 1997, xii+361 pp., 25 cm, softcover, $34.50

For most people numerical linear algebra is part of the infrastructure of numerical computations—roadways and bridges that connect one part of a calculation to another. People assume that the road will be smooth, that the bridge will not fall down, and they give the matter little additional thought. For the most part their confidence is justified. The matrix computation community has taken pains to market its algorithms in attractive, reliable formats, whether in packages such as LAPACK or in interactive systems such as MATLAB.

But numerical linear algebra is also a fascinating subject, well worth studying in its own right. Moreover, packages cannot solve every problem, and at some point most people performing numerical computations will have to involve themselves with the details of a matrix algorithm. Once again these people have been well served by the community. The area of matrix computations is supplied with well-written books ranging from introductory textbooks to advanced monographs.

The book under review is a textbook and a most unusual one. Introductions to the field tend to come in two flavors. The first is the classroom textbook, of which Datta's *Numerical Linear Algebra and Applications* is a fine example. The second is the grand survey, of which Golub and Van Loan's *Matrix Computations* is definitive. The present book is unusual in its stress on the *ideas* of matrix computations. This is not to say that ideas are absent from other works. But the senior author, Nick Trefethen, is known for taking a broad view of things, and that view is what informs the entire book.

The coverage is what you would expect in an introductory text—fundamentals, the QR decomposition and least squares, condition and stability, Gaussian elimination, eigensystems, and iterative methods. It is not to be expected that the treatment of this wide range of topics in a book of 350 pages would be exhaustive. However, it is good enough to bring an intelligent student to the point where he or she can proceed alone. An unusual feature of the book is that it treats the QR decomposition before Gaussian elimination. The justification is that the QR decomposition is easier to understand than Gaussian elimination, which teeters on the brink of instability. Another reason, I suspect, is that the QR approach to least squares gives the authors more scope for geometrizing. In any event, the decision is defensible and the result is a refreshing change.

As hinted above, geometry plays an important role in this book. The singular value decomposition is related to the mapping by a matrix of the unit ball into a hyperellipsoid. I particularly liked the treatment of projectors, where algebra and geometry are played off against each other. Geometry of another sort plays a role in the discussion of Arnoldi's eigenvalue method where the authors use "Arnoldi leminscates" to illustrate the convergence of the method.

The authors also stress the interrelation between algorithms. For example they use a four-way division of Krylov sequence methods (linear systems vs. eigenproblems and Hermitian vs. non-Hermitian) to guide their discussion. Again, the authors make an amusing distinction between "orthogonal structuring" and "structured orthogonalization" to illustrate the difference between algorithms based on Householder transformations and those based on orthogonalizing a sequence of vectors.

The book concludes with an essay by Trefethen on the definition of numerical analysis. One does not have to agree with the definition itself to appreciate the important issues Trefethen raises so entertainingly.

I have two reservations about the book—neither damning. First, there could be more stress on implementation issues (e.g., convergence criteria for the QR algorithm). It is natural that a book of this sort would not spend a great deal of time on the minutiae, but given the many ways you can shoot yourself in the foot while computing with matrices, a few more examples of the pitfalls would be helpful. Second, the material and presentation was developed for graduate students at two high-powered institutions (MIT and Cornell). I would certainly not say that the book is unsuitable for other schools, but the instructor who uses it should be prepared to field some difficult questions.

These reservations aside, I can strongly recommend this book. The authors are to be congratulated on producing a fresh and lively introduction to a fundamental area of numerical analysis.

G. W. Stewart

2[65-01, 65Lxx, 65Mxx]—*A first course in the numerical analysis of differential equations*, by Arieh Iserles, Cambridge Texts in Applied Mathematics, Cambridge University Press, New York, New York, 1996, xvi+378 pp., softcover, $27.95, hardcover, $74.95

This is a lively textbook that is suited for mathematics graduate students or for well-prepared (mathematically) engineering students. This text is, on the one hand, rigorous and concise in its presentation of mathematical ideas and, on the other hand, verbose in its discussion of the big picture, i.e., "the ways and means whereby computational algorithms are implemented" and developed. To quote Professor Iserles: "In this volume we strive to steer a middle course between the complementary vices of mathematical nitpicking and of hand-waving".

This monograph is devoted to the numerical analysis of both ordinary and partial differential equations but, as needed, many other traditional topics are introduced and studied. These include interpolatory quadrature, Newton's method (and its variants) in $\mathbf{R}^d$, Gaussian elimination, iterative methods for sparse linear systems, and the FFT. There are several appendices, called "*Bluffer's guide to useful mathematics*", wherein important definitions and theorems from linear algebra, approximation theory, and ordinary differential equations are presented. Each chapter concludes with a short but challenging list of exercises and a Comments and Bibliography section. The text is organized into three parts. Part I consists of