

a vehicle for the description of the FEM that is presented as a Galerkin method for the differential equation as well as a Ritz method for the minimization of the appropriate functional. More general self-adjoint elliptic problems and the corresponding FEM are described later in the chapter with careful statements of important tools (for existence of the FEM solution and error analysis) such as Cea's lemma and the Lax-Milgram theorem. 9. *Gaussian elimination for sparse linear equations*. This brief chapter examines the issue of "fill in" in Cholesky factorization and the use of graphs to investigate the sparsity structure and factorization of matrices. 10. *Iterative methods for sparse linear equations*. The classical Jacobi, Gauss-Seidel, and SOR methods are analyzed with emphasis on SOR. Unfortunately, the powerful conjugate gradient method is relegated to the Comments and Bibliography sections at the end of the chapter. 11. *Multigrid techniques*. The author motivates the multigrid technique by demonstrating the smoothing property of Gauss-Seidel thereby revealing how one may accelerate via a hierarchy of grids. Then the basic ideas of the V -cycle and full multigrid iteration are discussed. No error analysis is presented. 12. *Fast Poisson solvers*. This chapter is concerned with the use of FFT techniques to efficiently solve block Toeplitz, symmetric tridiagonal systems that arise in certain finite difference (element) approximations.

The final two chapters constitute Part III, namely partial differential equations of evolution.

13. *The diffusion equation*. The analysis, stability and convergence, of semidiscrete and fully discrete schemes for parabolic initial-boundary value problems is presented. The discussion is, by and large, limited to Euler and Crank-Nicolson time discretizations. 14. *Hyperbolic equations*. Professor Iserles motivates the development of numerical schemes for hyperbolic problems by considering the advection equation $\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0$ and its numerical solution by Euler and Crank-Nicolson with particular attention to stability. The remainder of the chapter deals with the wave equation and Burgers equation. (On p. 308, Burgers is incorrectly stated; the expression $\frac{\partial u^2}{\partial x^2}$ should read $\frac{\partial u^2}{\partial x}$.)

This is a well-written, challenging introductory text that addresses the essential issues in the development of effective numerical schemes for the solution of differential equations: stability, convergence, and efficiency. The softcover edition is a terrific buy—I highly recommend it.

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3[65M06, 65M12, 65M20]—*Numerical methods for the three dimensional shallow water equations on supercomputers*, by E. D. de Goede, CWI Tract, Vol. 88, Stichting Mathematisch Centrum, Amsterdam, 1993, x+124 pp., 24 cm, softcover, Dfl. 40.00

This book is a collection of articles on the development of a numerical method for the three dimensional shallow water equations. They are obtained by simplifying the Navier-Stokes model: the unknowns are the horizontal velocity and the water elevation as for the two dimensional model, but the velocity may depend on the vertical coordinate. The pressure gradient is directly linked to the water elevation,

and the vertical velocity can be deduced from the incompressibility condition. The equation for the water elevation is nonlocal since it involves the integrals in the vertical direction of the velocity components.

After presenting very clearly the physical model and the spatial discretization, the author focuses on the development of good time-stepping schemes for the three dimensional shallow water equations. He mainly proposes two schemes, which are first studied for a simplified model:

- A semi-implicit scheme (the vertical diffusion is treated implicitly and the scheme is fully explicit for the water elevation) leading to a block triangular linear system which can be solved in parallel by direct methods. For stabilization, smoothing techniques are applied and may deteriorate the accuracy.
- A time splitting scheme where the diffusion terms are decoupled from the terms responsible for the water waves. The linear systems are solved iteratively and preconditioners are used. The scheme is unconditionally stable and seems more robust and accurate than the previous one.

Finally, the parallel implementation is thoroughly discussed.

The theoretical part of the book is written in a very interesting way, and numerous and systematic tests are given. In addition to being of evident interest for the field specialists, this book can be seen as an example of how to tackle a complex numerical problem, and proves if necessary that numerical analysis can be very helpful for concrete engineering problems. I have enjoyed reading this book, although, since it is a collection of articles, it contains repetitions and redundancies.

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4[65-00, 65-01]—*Boundary element method, fundamentals and applications*, by Frederico Paris and Jose Canas, Oxford University Press, New York, NY, 1997, xv+392 pp., 24 cm, hardcover, \$65.00

It is frequently possible to express the solution of a partial differential equation (PDE) in a region \mathcal{B} in terms of an integral over the boundary of the region of some *Green's* function times a *potential density*. In such cases, the problem of solving the PDE at a discrete set of interior points in \mathcal{B} can often be reduced to solving an integral equation for the *potential density* on the boundary of \mathcal{B} , and then using the integral expression for the solution to compute the solution in the interior of \mathcal{B} . This is the *boundary element method* (BEM). In the case when the BEM can be applied, it usually enables considerable saving of effort compared with direct solution of the PDE via classical methods. Other difficulties ensue, due to the presence of corners on the boundary of \mathcal{B} which give rise to singularities of the *potential density*, and due to the singularities in the *Green's function*.

This text is about the BEM. It stems from a Ph.D. course taught by the authors in Blacksburg, USA, in 1991–92. It covers, in essence, the use of the BEM for solving first *Laplace-type* and then *elastic-type* problems. Initially, *Green's* theorem is used to transform the PDE to an integral equation, and the integral equation is then solved for the two dimensional case via *finite element method* (FEM). Piecewise