and the vertical velocity can be deduced from the incompressibility condition. The equation for the water elevation is nonlocal since it involves the integrals in the vertical direction of the velocity components.

After presenting very clearly the physical model and the spatial discretization, the author focuses on the development of good time-stepping schemes for the three dimensional shallow water equations. He mainly proposes two schemes, which are first studied for a simplified model:

- A semi-implicit scheme (the vertical diffusion is treated implicitly and the scheme is fully explicit for the water elevation) leading to a block triangular linear system which can be solved in parallel by direct methods. For stabilization, smoothing techniques are applied and may deteriorate the accuracy.
- A time splitting scheme where the diffusion terms are decoupled from the terms responsible for the water waves. The linear systems are solved iteratively and preconditioners are used. The scheme is unconditionally stable and seems more robust and accurate than the previous one.

Finally, the parallel implementation is thoroughly discussed.

The theoretical part of the book is written in a very interesting way, and numerous and systematic tests are given. In addition to being of evident interest for the field specialists, this book can be seen as an example of how to tackle a complex numerical problem, and proves if necessary that numerical analysis can be very helpful for concrete engineering problems. I have enjoyed reading this book, although, since it is a collection of articles, it contains repetitions and redundancies.

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4[65-00, 65-01]—*Boundary element method, fundamentals and applications*, by Frederico Paris and Jose Canas, Oxford University Press, New York, NY, 1997, xv+392 pp., 24 cm, hardcover, $65.00

It is frequently possible to express the solution of a partial differential equation (PDE) in a region $\mathcal{B}$ in terms of an integral over the boundary of the region of some *Green's* function times a *potential density.* In such cases, the problem of solving the PDE at a discrete set of interior points in $\mathcal{B}$ can often be reduced to solving an integral equation for the *potential density* on the boundary of $\mathcal{B}$, and then using the integral expression for the solution to compute the solution in the interior of $\mathcal{B}$. This is the *boundary element method* (BEM). In the case when the BEM can be applied, it usually enables considerable saving of effort compared with direct solution of the PDE via classical methods. Other difficulties ensue, due to the presence of corners on the boundary of $\mathcal{B}$ which give rise to singularities of the *potential density*, and due to the singularities in the *Green's function.*

This text is about the BEM. It stems from a Ph.D. course taught by the authors in Blacksburg, USA, in 1991–92. It covers, in essence, the use of the BEM for solving first *Laplace-type* and then *elastic-type* problems. Initially, *Green's* theorem is used to transform the PDE to an integral equation, and the integral equation is then solved for the two dimensional case via *finite element method* (FEM). Piecewise linear and piecewise quadratic elements are used for regions where the boundary is smooth, and the authors derive singular elements in the cases when the nature of the singularity is known. Such "singular elements" are not yet known for every type of corner and edge in three dimensions. Many simple examples are presented, enabling a student to get a good understanding of the BEM. In addition, at the end of each chapter one finds a set of problems which further facilitate the understanding of the BEM, as well as a bibliography on the subject matter of the chapter.

The text is organized into the following chapters.

1. *Preliminary concepts.* This chapter presents some basic mathematical concepts that are required for the conversion of a PDE to a boundary integral equation, and for replacing the boundary integral equation by a system of algebraic equations. Included are Gauss' divergence theorem, the concept of a delta function, the concept of a singular integral, including the Cauchy principal value. A brief description of approximate methods of solving boundary integral equations consists of the Galerkin scheme based on the use of basis functions, such as polynomials, trigonometric polynomials, finite elements, and delta functions. Also included is a discussion of the merits of the BEM versus the FEM for solving PDE's. The chapter then concludes with a listing of some problems that can be effectively dealt with by the BEM.
2. *Integral formulation of the Laplace equation.* In this chapter one encounters a derivation of the fundamental solution in two and three dimensions, a detailed derivation of the integral expression for the solution in terms of the fundamental solution by means of Green's theorem, and a derivation of the boundary integral equation from this integral representation.
3. *BEM applied to the Laplace equation.* Finite element approximations are extensively discussed for the two dimensional case using piecewise constant, piecewise linear, and piecewise quadratic elements. The use of singular elements is also discussed. Also discussed are the approximations to the solution in the interior of the region, once the solution to the BEM has been computed.
4. *A computer program to solve the 2D Laplace equation based on BEM.* The authors present a FORTRAN program for approximation of a solution to a Poisson problem based on the BEM under the assumption that the boundary is given in terms of piecewise linear elements. The resulting system of linear equations is then solved via Gaussian elimination. Some examples are given illustrating the use of the program. These include a heat conduction problem, a torsion problem, a Motz problem, and a ground water flow problem.
5. *Integral formulation of the elastic problem.* After deriving in detail the PDE of the elastic problem, the authors express in detail the solution to the PDE in the interior to the region as integrals over the boundary of the region in terms of Green's functions, and they then use these integral expressions to derive in detail the boundary integral equations. This is done in both two and three dimensions.
6. *BEM applied to elastic theory.* Just as for the case of Laplace's equation, the authors illustrate in detail for the two dimensional case the setting of the system of algebraic equations based on piecewise constant, piecewise linear, and piecewise linear discontinuous boundary elements. In addition, explicit evaluation of boundary integrals are also presented for the case of isoparametric elements, circular elements, and singular elements. Also discussed are

the approximation to the solution in the interior of the region via use of the integral expressions for the solution.

7. *A computer program to solve 2D elastic problems based on BEM.* This chapter discusses the numerical solution to the integral equations developed in the previous chapter using a FORTRAN computer program, which they present, the use of which is described in detail. They illustrate the use of their program for obtaining the stresses for the cases of: (i) a square plate in traction; (ii) the semi-infinite half space uniformly loaded along half the length; (iii) a plate with central hole in traction; and (iv) a plate with central crack in traction.

There are also the following appendices.

1. *Bringing the integral equation to the boundary without deforming it.* This appendix presents an alternative procedure for taking the integral expressions of the solution to the boundary.
2. *Numerical integration by the Gauss method.* Appendix 2 presents methods of numerical integration via: (i) the trapezoidal formula; (ii) Simpson's rule; and (iii) Gauss-Legendre quadrature. In addition, a FORTRAN program is given to evaluate the zeros and weights for Gauss-Legendre quadrature.
3. *Definition of a stress vector in terms of the principal directions obtained from the displacement field.* This appendix expresses the stress vector in the neighborhood of a corner, based on a piecewise linear finite element approximation.
4. *Analytic expressions of integration constants for parabolic elements in elasticity when the collocation point belongs to the element.*

Finally, the authors include a description of a diskette which is attached to the book inside the back cover. This is for *Mac* and *PC* use, and contains the files of the FORTRAN programs given in Chapters 4 and 7 of the text, including programs for the examples which they have presented.

The final pages of the text consists of a four page index, starting on page 289.

FRANK STENGER