

incomplete LU factorization is always possible for invertible diagonally dominant matrices, and the algorithm is presented.

For those who teach courses in numerical linear algebra or those who are simply interested in the subject, this is a well-written modern book that deserves a place on the office bookshelf. The book is packed with information and insights from one of the leaders in the field.

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**7[65-02, 65F10]**—*Iterative methods for solving linear systems*, by Anne Greenbaum, *Frontiers in Applied Mathematics* 17, SIAM, Philadelphia, PA, 1997, xiii + 220 pp., 25½ cm, softcover, \$41.00

This is a stimulating book. It describes recent developments in the theory of iterative techniques for solving large sparse systems of linear equations. The book is not a complete survey; Axelsson's treatise [1] is more definitive in this respect. Instead, as the author points out in the preface, "With this book, I hope to discuss a few of the most useful algorithms and the mathematical principles behind their derivation and analysis... I have tried to include the most *useful* algorithms... and the most *interesting* analysis from both a practical and a mathematical point of view." This selective treatment of the theory is what sets Greenbaum's book apart.

The body of the book is preceded by an introductory chapter containing a concise overview of the state of the art. The remainder of the book is split into two parts. Chapters 2 through 7 form the first part of the book and describe "basic" Krylov subspace methods. The second part of the book (Chapters 8 through 12) is devoted to preconditioning aspects.

The distinction between Hermitian/symmetric and non-Hermitian/nonsymmetric linear systems is made obvious to the reader in Chapter 1, and is reinforced in Chapters 2 and 3 ("Some iteration methods" and "Error bounds for CG, MINRES and GMRES"). When solving (real-) symmetric systems, Krylov methods like MINRES (or CG in the positive definite case) generate a best approximation from a subspace of increasing dimension with a fixed computational effort at every iteration. Furthermore, the concept of preconditioning has a sound theoretical basis, since the convergence is completely determined by the eigenvalue distribution of the coefficient matrix. The explanation of why the convergence of CG/MINRES is described by the eigenvalue spectrum even in the presence of rounding errors is outlined in Chapter 4, "Effects of finite precision arithmetic". These fundamental results are an outcome of the author's research, and their inclusion in a textbook for the first time is very valuable. The inherent difficulty in the nonsymmetric/non-Hermitian case is unravelled in Chapter 6, which provides the answer to the question, "Is there a short recurrence for a near-optimal approximation?" The upshot is a plethora of "open problems" in the nonsymmetric case; in particular, the optimal choice of Krylov subspace method is problem specific, and preconditioning has a very limited theoretical basis. (A consequence of the latter is that the book has little to offer practitioners interested in solving problems other than the model

diffusion and neutron transport problems discussed in Chapter 9.) A number of methods that have been proposed as being effective in the nonsymmetric case are described in Chapter 5, “BICG and related methods”. While the basic BICGSTAB algorithm is discussed, there is no mention of the more sophisticated BICGSTAB( $\ell$ ) methods for  $\ell > 1$ .

Much of the material in the second part of the book is application specific. Chapter 10, “Comparison of preconditioners”, includes an excellent review of the classical SOR theory, and then discusses aspects of Perron-Frobenius theory in the context of the two model problems introduced in Chapter 9. Chapter 11, “Incomplete decomposition”, is largely forgettable. In contrast, the concluding chapter, “Multigrid and domain decomposition methods”, has an appealing simplicity. I especially liked the train of argument that is developed in order to provide motivation for multigrid methods. Unfortunately, the citation to the literature is rather narrow throughout the second part of the book, effectively limiting its scope. Hopefully, this will be addressed in a future edition.

My overall opinion of the book is very positive. It is very well written and it is easy to be carried along by the prose, which is both lively and illuminating. Notwithstanding the informality, the mathematical arguments are precise and invariably well constructed. The outcome is a mathematical content that is deeper than that of Saad [2] or Kelley [3], and an exposition that is more insightful than that of Hackbusch [4]. With the exception of the rather limited Chapter 5, the first part of the book is uniformly excellent. I strongly recommend the book as mandatory reading for mathematically able research students who are interested in working in this rapidly developing area of numerical analysis.

#### REFERENCES

- [1] O. Axelsson, *Iterative Solution Methods*, Cambridge University Press, 1994. MR **95f**:65005
- [2] Y. Saad, *Iterative Methods for Sparse Linear Systems*, PWS Publishing, Boston, 1996.
- [3] C. T. Kelley, *Iterative Methods for Linear and Nonlinear Equations*, SIAM, Philadelphia, 1995. MR **96d**:65002
- [4] W. Hackbusch, *Iterative Solution of Large Sparse Systems of Equations*, Springer, Berlin, 1994. MR **94k**:65002

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