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REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1991 Mathematics Subject Classification can be found in the annual subject index of *Mathematical Reviews* starting with the December 1990 issue.

2[65-00, 65-02]—*Handbook of numerical analysis, Vol. VI, Numerical methods for solids* (*Part* 3), *Numerical methods for fluids* (*Part* 1), P. G. Ciarlet and J. L. Lions (Editors), North-Holland, Amsterdam, 1998, x+689 pp., $24\frac{1}{2}$ cm, hardcover, \$164.00

According to its general preface, each volume of the *Handbook of Numerical Analysis* presents in an expository fashion either "basic methods of numerical analysis" or "the numerical solution of actual problems of contemporary interest in applied mathematics." The goal of these volumes is to "thoroughly cover all the major aspects of numerical analysis, by presenting in-depth surveys, which include the most recent trends." Volume VI is dedicated to surveys of numerical methods for solids and numerical methods for fluids. It contains three technical sections plus an obituary of Professor Juan Carlos Simo, the author of the section on numerical analysis and simulation of plasticity.

The first technical section is "Iterative finite element solutions in nonlinear mechanics" by R. M. Ferencz and T. J. R. Hughes. The section begins with a short and clear development of nonlinear solid mechanics and conjugate gradient methods. The overall solution strategy presented is based on finite element discretization, Newton iteration for the nonlinearity with preconditioned conjugate gradient methods for the linearized systems. The primary focus of the article is the presentation of element-by-element preconditioners, developed extensively by Hughes and his coworkers. Element-by-element preconditioning has become a standard engineering tool for solving linearized systems arising in complex applications. Remarkably, it has been studied comparatively little by the numerical analysis community. One way to describe element-by-element preconditioning is as follows. Given a finite element mesh, the elements are "colored" with (as many as necessary) J colors so that all same-color elements do not share nodes. With this coloring, the stiffness matrix A can then be additively decomposed as

$$A = A_1 + A_2 + \cdots + A_J, \quad J \text{ "colors"}.$$

Because of the way the mesh is colored, each color's associated stiffness matrix A_j is similar to a block diagonal matrix with the color-j element stiffness matrices as diagonal blocks. Thus, any calculation with A_j can be performed in a fully parallel fashion using only elemental stiffness matrices. The element-by-element preconditioner is given by the product

$$\prod_{j=1}^{J} (I + A_j)^{-1}.$$

Elemental data is the natural logical unit for parallel computation of nonlinear problems as well as for adaptive algorithms because finite element assembly is "embarassingly parallel" at the elemental level. Thus, element-by-element preconditioning is a brilliant idea for massively parallel solution of practical engineering problems. However, this most straightforward version of it has not, however, proven competitive serially on the model Poisson problem with more global preconditioners such as multigrid methods. It seems likely that it has become a preferred method for complex engineering problems not only because of its parallel efficiency but also because of the excellent robustness that such methods seem to have. This article will focus more attention on element-by-element methods and issues of robustness in preconditioners.

The second technical section is "Numerical Analysis and Simulation of Plasticity" by J. C. Simo. This section is a pleasure to read. It begins with a mathematically precise and physically lucid presentation of classical continuum models of plasticity. This is followed by an overview of time integrators satisfying the nonlinear stability conditions necessary for use in plasticity problems. Next there is a mathematically precise development of the nonlinear continuum mechanics of finite strain plasticity followed by one section containing interesting examples of calculations on thermomechanical problems.

The third and last technical section, "Navier-Stokes Equations: Theory and Approximation" by M. Marion and R. Temam, gives a thorough survey of the developments arising from the authors' work on analysis and computation of turbulent flows. This section is the first of several planned articles on numerical methods for fluids. (For work related to this approach, see the recent book of Dubois, Jauberteau and Temam [1]. Complementary treatments of finite elements and fluids are given in the books of Girault and Raviart [2], Gresho and Sani [3] and Gunzburger [4].) Chapter I contains a synopsis of the mathematical theory of the Navier-Stokes equations. Chapters II and III present error analysis for discretizations of the Navier-Stokes equations. With this background, Chapter IV describes nonlinear Galerkin methods, a special method developed by the authors for simulating turbulent flows efficiently. This chapter concludes with several most interesting experiments using a dynamic adaptivity version of the nonlinear Galerkin method.

References

[1] T. Dubois, F. Jauberteau and R. Temam, *Dynamic Multilevel Methods and the Numerical Simulation of Turbulence*, Cambridge University Press, Cambridge, 1999.

[2] V. Girault and P. A. Raviart, *Finite Element Methods for the Navier-Stokes Equations*, Springer-Verlag, Berlin, 1986.

[3] P. M. Gresho and R. L. Sani, *Incompressible Flow and the Finite Element Method*, John Wiley and Sons, Chichester, 1998.

[4] M. D. Gunzburger, *Finite Element Methods for Viscous Incompressible Flow*, Academic Press, Boston, 1989.

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