(block) methods for linear systems with multiple right-hand sides, since they are useful in many applications and also make natural use of BLAS2 and BLAS3 computations. The chapter terminates with a discussion on testing iterative methods.

Preconditioning is an important step in iterative approaches; this and parallel implementations are thoroughly presented in Chapter 9. Besides the classical incomplete schemes, a few pages are devoted to the presentation of recent approaches, such as Sparse Approximate Inverse, and Element-by-Element preconditioning, which typically exhibit their best performance in a parallel context. It would have been nice if the authors had opted for a more detailed presentation of domain decomposition methods.

Chapters 10 and 11 describe methods for the standard and generalized eigenvalue problems. After a survey of the most widely used approaches, the very successful package ARPACK is described together with its parallel implementation P_ARPACK (written using MPI). Several important issues are discussed at length, providing the reader with some implementation hints and with a good feeling of the expected performance. Finally, the Appendices gives the necessary information to practically deal with some of the described codes.

In conclusion, in spite of what the authors say in the Preface (" ... Any book that attempts to cover these topics must necessarily be somewhat out of date before it appears"), the book contains a lot of up-to-date material, and I recommend it to computational scientists who deal with linear algebra methods on any of parallel, vector and sequential (!) computer environments. Readers who already own the previous edition will find that this book has been significantly expanded to include recent important advances in numerical linear algebra tools and HPC environments that will make their "HP computational life" much easier.

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5[65F15, 65F10]—*ARPACK Users' Guide, Solution of Large-Scale Eigenvalue Problems with Implicitly Restarted Arnoldi Methods*, by R. B. Lehoucq, D. C. Sorensen, and C. Yang, SIAM, Philadelphia, PA, 1998, xv+142 pp., $25\frac{1}{2}$ cm, softcover, \$39.00

The chief impediment to solving large eigenvalue problems is lack of sufficient memory—a difficulty that has two aspects. In the first place, if the order of the matrix in question is large, the matrix must be represented in some compact form. This limits what we can do with the matrix to simple operations like multiplying it by a vector or, if we are lucky, factoring it so that we have a representation of its inverse. The second aspect is that we cannot hope to store the entire matrix of eigenvectors and must content ourselves with computing a few selected eigenpairs. We are also limited in the number of extra working vectors that we can use to compute these eigenpairs.

Krylov sequence methods are popular in part because they can be made to respect these limitations. The methods proceed by orthogonalizing a Krylov sequence $u, Au, A^2u, \ldots$. When the resulting vectors are arranged in a matrix

$U_k = (u_1 \cdots u_k)$, they satisfy the relation

$$AU_k = U_k H_k + F_k, \tag{1}$$

where H_k is Hessenberg and F_k is nonzero in only its last column, which is orthogonal to the columns of U_k. Theory (and practice) show that as k increases, the spaces spanned by the U_k contain increasingly good approximations to eigenvectors whose eigenvalues lie on the periphery of the spectrum of A. However, Krylov sequence methods are not necessarily confined to finding such eigenpairs. If we can factor the matrix, we have the option of working with $(A - \sigma I)^{-1}$, which moves the part of the spectrum of A near σ to the periphery—a process known as shift-and-invert enhancement.

When A is Hermitian, the method is known as the Lanczos method. In this case the matrix H_k is tridiagonal, and the vectors u_k satisfy a three-term recurrence. Thus, in principle, it is not necessary to save all the u_i to expand the sequence. In practice, however, the u_i can loose orthogonality, and it is necessary to reorthogonalize them, a costly procedure. It was eventually realized that it is not necessary to reorthogonalize against all preceding vectors, and the algorithm became the method of choice for large Hermitian eigenvalue problems [2, Ch. 13].

When A is non-Hermitian, the method is called the Arnoldi method. Here there is no three-term recurrence, and each new vector must be orthognozied against all the previous vectors. This not only increases the computational work, but raises the possibility that the method will consume the available storage before the required eigenpairs have converged. A cure is to restart the Krylov sequence with a vector containing information on the required eigenvectors. Unfortuantely, good starting vectors are hard to find [1].

Sorensen's implicitly restarted Arnoldi [3] is based on the observation that the matrix H_k contains approximations, called Ritz values, to the eigenvalues of A. If the QR algorithm is used to triangularize H_k in such a way that the eigenvalues that are desired are at the top and the transformations are accumulated in the Arnoldi factorization (1), the factorization can then be truncated to one containing only approximations to the desired eigenpairs. Thus the implicitly restarted Arnoldi algorithm breathes in and out, first expanding the Arnoldi factorization to get better approximations to the desired eigenpairs and then contracting it to get rid of the undesired eigenpairs. Of course there is nothing to keep A from being Hermitian, in which case the algorithm becomes implicitly restarted Lanczos.

The book under review documents the software that Sorensen and his colleagues have built around the idea of implicit restarting. In judging a software package there are three things to take into account: the organization of the software, the accessibility of the user documentation, and the quality of the technical documentation.

ARPACK is soundly designed. An important problem in a package like this is how to get the user to perform the matrix-vector multiplications needed to generate the Arnoldi sequence. The authors have wisely chosen to use reverse communication, a contrived but effective device in which a called routine returns to the calling routine and asks it for further input. The package handles single and double precision, real and complex, and Hermitian and non-Hermitian matrices. It will solve both ordinary and generalized eigenvalue problems with or without shift-and-invert enhancement. The package provides options for tracing and check-pointing the calculations. Calling sequences are necessarily complicated, but there are drivers which cover the majority of cases occuring in practice.

The user documentation is excellent. After leading the reader through a simple example, the authors give a general overview of the capabilities of the package. An appendix contains detailed descriptions of the various drivers. Nothing can make learning to use a package of this magnitude actually easy, but the authors have taken care to see that it is not unnecessarily difficult. I asked students in a class of mine to get ARPACK up and running on problems of their choice. They had little trouble with the project.

Technical documentation can be divided into program details and mathematical underpinnings. Of the former there is none, and the reader must go to the programs to find out what is going on. Fortunately, they are well formatted and commented. I was disappointed in the mathematical description of the algorithm in Chapter 4. There is a lot of information there, but it is not very well organized, and I found parts very tough reading. Important topics (e.g., locking in eigenpairs after they have converged) are slighted while peripheral topics (e.g., block methods) are given undue attention. Since the guide itself is not long, the authors could have easily found extra space for a more leisurely, didactic treatment—a treatment not to be found in the literature.

But this lost opportunity will not be missed by most of the users of ARPACK. The authors, starting from an elegant idea, have produced a sound, well-documented package, which has deservedly become widely popular. We may hope that others with new ideas for solving large eigenvalue problems will hew to the authors' high standards.

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6[65-02, 65D32, 65Y05, 65Y10, 65Y15, 65Y20]—*Computational Integration*, by Arnold R. Krommer and Christoph W. Ueberhuber, SIAM, Philadelphia, PA, 1998, xx + 445 pp., $25\frac{1}{2}$ cm, softcover, $64.00

The book under review has three major parts entitled, respectively, *Introduction* (86 pages), *Symbolic Integration* (20 pages), *Numerical Integration* (288 pages), and a 23-page bibliography of some 450 items dating mostly from the last 15 years.

Part I contains three chapters. The first deals with various concepts of integrals and their properties: proper and improper Riemann integrals, and Cauchy principal value and Hadamard finite part integrals in one and several variables. Chapter 2 briefly describes selected areas in scientific computing that rely on numerical integration, while Chapter 3 spells out more concretely the types of integration problems occurring in practice. Also discussed are matters of conditioning, available