coverage of spectral method but on a practical and rapid introduction to the readers of how the spectral method works through the Matlab codes. The advantage of this approach is that many more students and researchers are expected to learn the basics of the spectral methods through this book and the Matlab codes in it. Perhaps some of them will get so interested in the method that they will ask deeper questions which this book cannot answer, but then they will already be adequately prepared to move on to read a more comprehensive spectral method book.

There are fourteen chapters in the book. The first six chapters cover the basic topics in spectral methods, such as the differentiation matrices and fast Fourier transforms. Chapters 7 through 14 give more applications. A reader who cares less about the underlying ideas of the spectral method but more about the applications could probably skip the first six chapters. However, it would be much more effective if one went through all the chapters and played with the Matlab codes as soon as they appear in the book.

This book is a very nice addition to the collection of books on spectral methods, from a totally different angle. It should attract more students and researchers to the powerful spectral methods.

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4[65-02, 65N06, 65N30]—*Generalized difference methods for differential equations. Numerical analysis of finite volume methods*, by Ronghua Li, Zhongying Chen, and Wei Wu, Marcel Dekker, New York, NY, 2000, xv+442 pp., 23 1/2 cm, hardcover, $175.00

This book provides a framework for construction and analysis of finite volume approximations of partial differential equations. The approach falls into the general class of Petrov–Galerkin methods that presents the boundary value problem in a weak form with the corresponding bilinear form defined over two different spaces: the solution space and the test space. In the book, the approximate solution is taken in the finite element space of piecewise polynomial functions over a partition of the domain into simplices or quadrilaterals (in most of the cases these are conforming spaces), while the test space consists of piecewise constant functions over a different (a dual) partition of the domain. Integrating a convection-diffusion-reaction equation over a particular finite volume produces a balance equation, which is the sum of surface (line) integrals of the diffusive and convective flux through the volume boundary and volume integrals of the reaction and the source terms. Replacing the derivatives in the balance equation by finite differences has been successfully used in the last 50 years. Alternatively, one can replace the exact solution by its finite element interpolant. This approach, consistently used in the book, is often called finite volume element method, a term that describes quite accurately its essence. The finite volume method and its applications to problems in science and engineering has been a major direction in computational mathematics in the last fifteen years (see, e.g., the Proceedings of the First and Second International Conferences on Finite Volumes for Complex Applications [5, 6]).

In the book the finite volume element approach is applied to second and fourth order elliptic equations, to parabolic and hyperbolic equations as well as convection-dominated diffusion problems, elasticity and Maxwell's equations. A merit of the book is that it gives a general framework for presenting this approach in a unified and consistent way. In this respect the book is quite timely and useful. The presentation is clear and compact (except some lengthy computations that could have been left as exercises).

Chapter 1 contains the usual material of Sobolev spaces and the corresponding abstract Hilbert theory of elliptic problems. Chapter 2 is the most elaborate part of the book and describes the main steps in the construction and analysis of the schemes on one-dimensional problems. In my opinion, this chapter is the least interesting since most of the schemes and theory are the same as in the standard finite element method, including the results on convergence and super-convergence error estimates. At the same time this part also shows the limitations of the method. For example, this method is missing important schemes with harmonic averaging of the coefficients of the differential equation and other high-order 3-point finite difference schemes of the sixties (see, e.g., [3]).

Chapter 3 is the most important and useful part of the book. The Petrov–Galerkin approach is developed fully and demonstrated on self-adjoint elliptic equations of second order in a polygonal domain. The construction of the test spaces based on finite volumes are given for linear, quadratic, and cubic elements, and many constructions are carried out to an explicit form. The coercivity of the bilinear form is studied in detail which is not so trivial since the bilinear form is defined on the product of the solution and test spaces and, therefore, one has to verify the corresponding inf-sup condition. Further, error estimates in a discrete energy, L^2- and maximum-norms are derived. After reading this chapter, one realizes that the construction of the schemes and their analysis depend heavily on the finite element method and its theoretical tools.

Subsequently, the same approach is applied to fourth order elliptic equations (Chapter 4), and to parabolic (Chapter 5) and hyperbolic equations (Chapter 6). Chapter 7 contains the construction and analysis of finite volume methods for convection-dominated diffusion problems. In my opinion, this is an interesting part of the book. It demonstrates the flexibility of the method and its capability to easily generate various schemes that combine characteristic and upwind approximation. It is shown that for diffusion and convection-diffusion problems, the presented Petrov–Galerkin method produces discretizations that have the property of local volume-by-volume mass conservation, an important and desired property for many applications. The analysis also proves stability in maximum norm by the maximum principle argument, a technique that has been widely used in the theory of finite differences (see, e.g., [3]).

I found numerous misprints and minor inaccuracies. For example, the term interpolation projection that is used in the book sounds very strange. In fact, almost everywhere it has the meaning of the finite element interpolant (see, e.g., [1]). On page 118 alone I found two inaccuracies: as stated, the estimate (3.2.8) does not make sense for $m = 2$, while in (3.2.12) the interpolation projection is not well defined for $u \in H^1$. There are also claims (in the introduction) that the estimates in the finite difference schemes theory are "usually not optimal". In the Russian literature there are a number of papers on this subject, and the optimality of the error estimates has been resolved by using the fundamental Bramble–Hilbert lemma. This research has been summarized in [4] for rectangular and in [2] for simplicial and quadrilateral meshes. The book under review does an excellent job by giving a fairly complete bibliography on the finite volume research published in Chinese. The references to the English literature are far from complete, but they represent the mainstream research in the area, while the references to the Russian literature are scarce and misleading.

As noted above, the book has various merits and is a useful and timely text in the area of finite volume methods for partial differential equations. It could be used as a textbook for an advanced course on finite volume methods or as a supplement to a course on discretization methods for differential equations.

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5[65-01, 65L05, 65L06, 65L10, 65L12, 65L99]—*Computer methods for ordinary differential equations and differential-algebraic equations*, by Uri M. Ascher and Linda R. Petzold, SIAM, Philadelphia, PA, 1998, xvii+314 pp., 25 cm, softcover, $36.50

This book is intended to be a textbook introducing students of mathematics as well as of other fields like computer science, mechanical, electrical, and chemical engineering, physics, into the field of the numerical solution of ordinary differential equations (ODEs).

In addition to initial value problems (IVPs) and boundary value problems (BVPs) in regular ODEs, also specific, singular ODEs, namely, differential-algebraic equations (DAEs) are considered. While regular IVPs are discussed on about 140 pages, about 70 pages each are devoted to regular BVPs and DAEs.

Each attempt to write a concise textbook on such a broad and complex subject constitutes a hazardous business. It always means a tightrope walk trying to present facts as simple as possible and as complex as necessary. The selection of the topics and the necessary restriction of the material to be presented will be subjective in any case.

Many nice, partly quite extensive monographs are available on the numerical solution of regular IVPs, even on single methods, like the good, old Adams book [1] by Lawrence F. Shampine and Marilyn Gordon. Flicking through this book is of great benefit still today. More recent books already contain sections on DAEs. Corresponding extensive monographs are available on regular BVPs, too. Concerning the integration of those three aspects, IVPs, BVPs, and DAEs, in the necessary brevity but in adequate detail for an independent course, this new textbook by Uri M. Ascher and Linda R. Petzold is a welcome and useful novelty.