bijective mappings (as for BVPs for example) and of the qualitative, asymptotic properties of flows (like stability in the sense of Lyapunov for IVPs).

In my opinion the representation of the practical applicability of (multiple) shooting methods for regular ordinary BVPs is somewhat misleading. The much older representation in [2] seems to me to be more mature.

Writing a good textbook always requires, in addition to the authors' expert knowledge, that the development in the field concerned has been finished to a certain extent. A certain distance is necessary to be able to restrict oneself to the most essential things. In the case of the DAEs in Part IV, the stage of development essential for a really good textbook has not yet been reached, in my opinion, and the two authors are themselves too strongly involved in this development to keep the necessary distance. Hence, the nice character of a textbook gets lost in Part IV. This is rather a part of a monograph with a great amount of subproblems and approaches strung together. For example, in spite of the mentioned sound restriction to globally Lipschitz-continuous vector fields in the beginning, the authors do not introduce a global notion of index for DAEs then. Just for these already complicated equations, they start with a local notion of index, which is confusing not only for beginners.

As intended by the authors, this new textbook is a strongly advisable aid for introductory courses to the numerics of regular ODEs. In particular, I consider the IVP part to be so exceptionally successful that I will advise students to use it as first literature for studying on one's own. For the BVP part it requires a few additional comments to achieve a more balanced education.

The abundant source of instructive examples and exercises in all parts of this book will be extremely valuable for all teachers.

Altogether, a gain!

References

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[3] Gene H. Golub and James M. Ortega, *Scientific computing and differential equations. An introduction to numerical methods*, Academic Press, Inc., 1981, 1992.

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6[65L05, 34A50, 58F99]—*Geometric integration: numerical solution of differential equations*, C. J. Budd and A. Iserles (Editors), Philosophical Transactions of the Royal Society, Mathematical, Physical and Engineering Sciences, The Royal Society, UK, April 1999, vol. 157, no. 1754, pp. 943–1133

The present issue of volume 357 of the *Philosophical Transactions of the Royal Society of London, Series A*, is entirely devoted to *geometric integration*. Under this heading, several recent developments in the numerical treatment of differential equations are collected. They have as a common theme the idea to preserve as far as possible structures (symmetries) of the exact flow in numerical discretization.

This book starts with a paper by C. J. Budd and A. Iserles with the title of the book, "Geometric integration: numerical solution of differential equations on manifolds". This article provides an interesting introduction to the topic of geometric integration: it briefly mentions different numerical approaches that have been developed in this context, and it addresses their importance for a large number of real applications. The remaining contributions of this issue treat different aspects of geometric integration; they are research articles, and they are independent of each other.

Much recent research is devoted to the solution of differential equations on manifolds having a Lie group action. Their discretization by a numerical algorithm usually involves computations in the corresponding Lie algebra. For reasons of efficiency, the number of appearing commutators has to be kept as small as possible. The article by H. Munthe-Kaas and B. Owren "Computations in a free Lie algebra" extends Witt's formula on the number of commutators of a fixed length (of the Hall basis) to graded Lie algebras. This allows one to get upper bounds on the number of necessary commutators in a numerical method. A substantial improvement of Runge–Kutta methods for Lie-type equations has been obtained by suitably regrouping the arguments of the commutators.

Differential equations of the form $y' = a(t)y$, where the solution $y(t)$ evolves in a Lie group and where $a(t)$ is a smooth function in its Lie algebra, are the subject of the paper "On the solution of linear differential equations in Lie groups" by A. Iserles and S. P. Nørsett. It is based on an explicit formula of the solution given by the Magnus series. This contribution presents a new one-to-one correspondence between the individual terms of the Magnus series and binary trees, which allows one to derive explicit recurrence relations as well as a convergence proof of the series. By suitably truncating the series and by ingeniously evaluating the appearing multiple integrals and commutators, new efficient methods are proposed for this class of methods.

In the contribution "Geometric integration using discrete gradients" by R. I. McLachlan, G. R. W. Quispel, and N. Robidoux, it is pointed out that ordinary differential equations with first integrals and/or Lyapunov functions can be written as "linear-gradient systems" $\dot{x} = L(x)\nabla V(x)$, where $L(x)$ is a matrix-valued function. Using discrete gradients, numerical methods are derived that preserve exactly first integrals and Lyapunov functions. This method is successfully applied to Hamiltonian, Poisson, and gradient systems, and also to many dissipative systems.

Geometric aspects of partial differential equations are the subject of the article "Self-similar numerical solutions of the porous-medium equation using moving mesh methods" by C. J. Budd, G. J. Collins, W. Z. Huang, and R. D. Russel. The role of conservation laws and similarity solutions is discussed, and adaptive numerical discretizations are studied which admit discrete forms of conservation laws and which have discrete self-similar solutions. It is shown that such methods capture correctly the long-time dynamics of the underlying partial differential equation.

The composition of simple methods with favorable geometric properties (symplecticity, volume preservation, ...) automatically inherits these properties, and it also allows us to increase the order of accuracy. One possibility to get the corresponding order conditions is by using the Baker-Campbell-Hausdorff formula. The article "Order conditions for numerical integrators obtained by composing simpler integrators" by A. Murua and J. M. Sanz-Serna presents a different approach based on a new type of rooted tree. As a result, the authors derive a simple presentation of the order conditions, which is obtained by a simple transcription of the structure of the corresponding graph.

The stable computation of trajectories of N-body problems is important for astronomical applications as well as for studies of atomic systems. The article "Reversible adaptive regularization: perturbed Kepler motion and classical atomic trajectories" by B. Leimkuhler discusses the impact of symplectic and time-reversible integrators on the energy error, it studies time transformations and the use of variable step sizes, and it presents a series of interesting numerical experiments with a new code that is especially written for the simulation of perturbed Kepler motions and classical atomic trajectories.

To sum up, this issue shows several important aspects of the wide field of geometric integration, all of which are written by experts in this topic.

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7[37Mxx, 65Pxx]—*Dynamical systems and numerical analysis*, by A. M. Stuart and A. R. Humphries, Cambridge University Press, New York, NY, 1998, xxi+685 pp., 23 cm, hardcover, \$64.95, softcover \$39.95

To come to the point first: This is an extraordinarily well-made monograph on dynamical systems which are generated by application of linear multistep methods and Runge–Kutta methods to explicit ordinary differential equations (ODEs). In particular, emphasis is put on the qualitatively correct reflection of properties, resp. the structure of the dynamical system generated by the ODE itself.

The principle aim of this book is—according to the two authors—to address two questions:

I. Assume that the differential equation has a particular invariant set. Does the numerical method have a corresponding invariant set which converges to the true invariant set as $\Delta t \to 0$? If so, what is the rate of convergence?
II. Assume that the vector field defining the differential equation has a particular structural property which confers certain properties on the dynamical behaviour of the equation. Find special numerical methods which inherit these structural properties under mild or no restrictions on the time-step or, for general numerical methods, find conditions on Δt under which these structural properties are inherited.

During the last 15 years, considerable progress has been made in answering these questions. Numerous articles have been written, by numerical mathematicians as well as analysts, among them, not least the two authors of the present monograph. In this 700-page monograph they collect these results to form a comprehensive and cogent account which "is intended to be accessible to anyone familiar with either dynamical systems or numerical analysis theory and, with a little work, to someone familiar with neither."

No doubt, this book is a considerable gain for all those who want to get familiar with this field of mathematics as well as for someone requiring a reference text for