

# Classification of the $N=1$ Seiberg-Witten theories

Csaba Csáki

*Theoretical Physics Group, Ernest Orlando Lawrence Berkeley National Laboratory, University of California, Berkeley, California 94720  
and Department of Physics, University of California, Berkeley, California 94720*

Witold Skiba

*Department of Physics, University of California at San Diego, La Jolla, California 92093*

(Received 4 February 1998; published 10 July 1998)

We present a systematic study of  $N=1$  supersymmetric gauge theories which are in the Coulomb phase. We show how to find all such theories based on a simple gauge group and no tree-level superpotential. We find the low-energy solution for the new theories in terms of a hyperelliptic Seiberg-Witten curve. This work completes the study of all  $N=1$  supersymmetric gauge theories where the Dynkin index of the matter fields equals the index of the adjoint ( $\mu=G$ ) and, consequently, all theories for which  $\mu<G$ . [S0556-2821(98)06414-5]

PACS number(s): 11.30.Pb, 11.15.Tk, 12.60.Jv

## I. INTRODUCTION

The past four years have witnessed tremendous progress in our understanding of strongly coupled supersymmetric gauge theories. The number of theories for which exact results have been established is ever growing since the initial breakthrough by Seiberg and Witten [1], who gave a complete solution of the  $N=2$   $SU(2)$  theory, and by Seiberg, who described the low-energy dynamics of  $N=1$  supersymmetric QCD with varying number of flavors [2].

From these solutions the following basic picture emerges. There are six known phases of supersymmetric gauge theories: Higgs, confining, Abelian Coulomb, non-Abelian Coulomb, free magnetic, and infrared free phases. The actual phase of a given theory usually depends on the size of its matter content. This size can be characterized by the relative value of the Dynkin index  $\mu$  of the chiral superfields compared to the value of the Dynkin index  $G$  of the vector superfields.

In this paper we will focus on the  $N=1$  theories, which are in the Abelian Coulomb phase everywhere on their moduli spaces. We examine theories based on simple gauge groups and no tree-level superpotential. We will argue that in order for such a theory to be in the Coulomb phase, the theory has to satisfy the index condition  $\mu=G$ . The essence of the argument can be summarized in the following: one expects the low-energy solutions of such a theory to be given in terms of an auxiliary Riemann surface, defined by a curve (which in most cases is hyperelliptic). The classical curve is smoothed out by quantum corrections, which are proportional to the dynamical scale  $\Lambda$  of the theory. We will show that in the absence of a tree-level superpotential the condition for  $\Lambda$  to appear in the curve is  $\mu=G$ . Theories with an adjoint chiral superfield (the pure  $N=2$  theories) satisfy this condition, and we will give a complete list of other theories which do so as well. After restricting ourselves to these theories, it is easy to actually find all of those which are in the Coulomb phase by checking the unbroken gauge group on a generic point of the moduli space. This way we obtain a complete list of  $N=1$  Seiberg-Witten theories based on

simple gauge groups and no tree-level superpotential. For these theories we determine the Seiberg-Witten curves, providing the solution for the low-energy effective gauge kinetic couplings by flowing to theories for which the curve is already known.

This work completes the study of theories satisfying the index condition  $\mu=G$ . It has been known for a while that confining theories with a quantum-deformed moduli space have to satisfy this index condition [3,4]. All such confining theories have been systematically analyzed in Refs. [3–6]. We find that if the matter content is in a faithful representation of the gauge group, then a  $\mu=G$  theory is confining with a quantum-deformed moduli space. However, if the matter content is not in a faithful representation of the gauge group, then the theory is in the Coulomb phase and the low-energy solution can be given in terms of a Seiberg-Witten curve. Therefore, the low-energy dynamics of all theories with  $\mu=G$  has now been determined. Since all theories with  $\mu<G$  can be obtained from the  $\mu=G$  theories by adding mass terms for fields in vectorlike representations, it is a straightforward task to determine the low-energy behavior of all  $\mu<G$  theories as well.

The paper is organized as follows. In Sec. II we review the basic properties of the Seiberg-Witten solutions and their applications to  $N=1$  theories. In Sec. III we give our general arguments which help us classify all  $N=1$  Seiberg-Witten theories based on simple groups and no tree-level superpotential. In Sec. IV we give the actual low-energy solutions of these theories, and we conclude in Sec. V. The derivation of the curves for the new theories is explained in Appendixes A and B.

## II. REVIEW OF THE SEIBERG-WITTEN SOLUTION AND ITS APPLICATION TO $N=1$ THEORIES

Seiberg and Witten showed how to employ electric-magnetic duality to obtain exact solutions to the low-energy dynamics of  $N=2$  theories [1]. The original example of  $SU(2)$  theory was subsequently generalized to other classical groups in Refs. [7–17]. An alternative derivation of these

solutions using the confining phase superpotential method has been given in Ref. [18], while the connection to integrable systems has been described in [13,14]. The dynamics of the low-energy theory following from these solutions has been analyzed in Refs. [19,20]. Below, we review the basic features of these solutions of the  $N=2$  theories and the application of the Seiberg-Witten methods to  $N=1$  theories [21].

Since  $N=2$  theories contain scalar fields in the adjoint representation, their classical moduli space has a submanifold with unbroken  $U(1)$  gauge symmetries. In general, in addition to the Coulomb submanifold, there is usually a subspace where the gauge group is completely broken. The Higgs branch does not receive perturbative or nonperturbative quantum corrections in  $N=2$ , while the Coulomb branch is affected by both perturbative and nonperturbative effects.

The low-energy Lagrangian of  $N=2$  theories can be characterized in terms of a single holomorphic function, the prepotential  $\mathcal{F}$ . In the more interesting case of the Coulomb branch, the prepotential can be computed in terms of the original “electric” fields and their dual “magnetic” degrees of freedom. It turns out that both kinds of variables are necessary for a consistent description of the theory.

The classical pattern of symmetry breaking by a field in the adjoint representation,  $G \rightarrow U(1)^r$ , where  $r$  is the rank of the gauge group, persists in the quantum theory everywhere on the Coulomb branch. Classically, there are points with a larger unbroken subgroup, when the vacuum expectation values (VEVs) of the adjoint field happen to coincide. Because of quantum effects, there are no points of enhanced gauge symmetry; however, some states become massless on certain submanifolds of the moduli space. These additional massless states are indicated by singularities in the effective description. Since these massless particles carry magnetic charges, magnetic variables are more suitable for a description of the theory near singularities.

Another indication that there are singularities on the moduli space is the presence of nontrivial monodromies. At large expectation values, the monodromy can be calculated using perturbation theory. The remaining strong-coupling monodromies are guessed using symmetry arguments and the requirement that the product of strong-coupling monodromies has to equal the monodromy at infinity. Knowledge of all monodromies, that is, the singularity structure and the behavior at infinity, is sufficient to determine the full form of a holomorphic quantity, in this case the prepotential  $\mathcal{F}$ .

The low-energy solution is obtained by introducing an auxiliary Riemann surface of genus usually equal to the rank of the gauge group. All holomorphic quantities can be computed as line integrals over this surface. This surface will also encode the singularity structure of the theory. In most cases, this surface turns out to be a hyperelliptic surface, which can be defined by an equation of the form

$$y^2 = P(x, u_i, \Lambda),$$

where  $P$  is a polynomial in  $x$ ,  $u_i$ , and  $\Lambda$ . The variables  $x$  and  $y$  are auxiliary parameters, while the  $u_i$ ’s are the coordinates on the moduli space and  $\Lambda$  is the dynamical scale of the

theory. Therefore, finding the exact low-energy action is equivalent to finding the polynomial  $P$ . From the curve described by  $P$ , one can extract information about the dynamics of the theory. In particular, the effective gauge coupling, the metric on the moduli space, and the spectrum of the Bogomol’nyi-Prasad-Sommerfield (BPS) states can be calculated.

For example, the simplest theory in the Abelian Coulomb phase is  $N=2$   $SU(2)$  with no hypermultiplets. In that case,

$$y^2 = (x^2 - u)^2 - 4\Lambda^4,$$

where  $u = \frac{1}{2} \text{Tr} \Phi^2$  and  $\Phi$  is the adjoint superfield. This equation describes a torus, which is a surface of genus 1. Any VEV of an adjoint field can be rotated into the Cartan subalgebra, which for  $SU(2)$  means that  $\langle \Phi \rangle = \text{diag}(a, -a)$ . One needs also to introduce a dual scalar field  $a_D$ . The above curve has three singularities, for  $u = \pm \Lambda^2$  and  $u = \infty$ . From the curve one can calculate the monodromies of the  $(a_D, a)$  vector around the singularities. The monodromies are elements of the duality group, which acts on the vector  $(a_D, a)$ . The singularity at infinity is the perturbative singularity, while the two singularities at  $u = \pm \Lambda^2$  occur at strong coupling. The nonperturbative singularities arise because of a monopole or a dyon becoming massless at these points of the moduli space. The charges of the massless fields are the left eigenvectors of the respective monodromy matrices.

From the curve one can also compute  $a$  and  $a_D$  as functions of the parameter of the moduli space  $u$ . These are determined as integrals over the periods  $\gamma_{1,2}$  of the torus:  $a_D = \oint_{\gamma_1} \lambda$ ,  $a = \oint_{\gamma_2} \lambda$ , where  $\lambda$  is the so-called Seiberg-Witten differential. In the case of  $SU(2)$ ,  $\lambda \propto x^2 dx/y$ . The gauge coupling

$$\tau = \frac{da_D/du}{da/du} = \frac{\partial^2 \mathcal{F}}{\partial a^2},$$

from which one can also establish the metric on the moduli space,  $\partial \mathcal{F} / \partial a$ .

Many features of the Seiberg-Witten solution of  $N=2$  theories persist in  $N=1$  theories in the Coulomb phase. Intriligator and Seiberg pointed out that the  $U(1)$  gauge coupling, which is a holomorphic quantity, can be described by the methods used in  $N=2$  [21]. The dependence of the gauge coupling on the parameters of the moduli space can be found once a curve describing the theory is established. The gauge-kinetic term is

$$\mathcal{L} = \frac{1}{4\pi} \text{Im} \int d^2 \theta \tau_{ij} W^i_\alpha W^{\alpha,j},$$

where  $\tau_{ij}$  is the effective gauge coupling matrix, whose eigenvalues are related to the effective gauge coupling and theta parameter of the  $k$ th  $U(1)$  factor by  $\tau_k = i4\pi/g_k^2 + \theta_k/2\pi$ . The effective gauge coupling function  $\tau_{ij}$  is not related by supersymmetry to the Kähler potential. Hence no information about the Kähler potential is provided by the  $N=1$  solution. Likewise, there is no central extension of the  $N=1$  supersymmetry algebra which would incorporate BPS

particles. However, one can learn about the charges of massless states associated with singularities. The monodromy around a singularity still encodes the information about the charge. These methods have been used in Refs. [21–23] to obtain solutions to several  $N=1$  theories in the Coulomb phase or to Coulomb branches of  $N=1$  theories with tree-level superpotential terms.

### III. NECESSARY CRITERIA FOR SEIBERG-WITTEN THEORIES

In this section we will show how to systematically find all  $N=1$  supersymmetric theories based on a simple gauge group and no tree-level superpotential, which are in the Coulomb phase. We will call such theories the  $N=1$  Seiberg-Witten theories. First, we show that such theories have to satisfy the index condition  $\mu=G$  mentioned in the Introduction and, then, decide which  $\mu=G$  theories are actually in the Coulomb phase.

#### A. Index condition

We have seen in the previous section that the effective gauge kinetic function  $\tau_{ij}$  can be identified with the period matrix of an auxiliary Riemann surface, usually a hyperelliptic curve  $y^2=P(x, u_i, \Lambda)$ . The curve obtained in the limit  $\Lambda \rightarrow 0$  is singular everywhere on the moduli space, reflecting the fact that turning off the gauge coupling will result in additional massless gauge bosons independently of the VEVs of the scalars, since there is no Higgs mechanism in the  $\Lambda \rightarrow 0$  limit. This singularity must be smoothed out by effects proportional to  $\Lambda$ , except at a submanifold where the singularity persists, indicating the existence of additional massless states. The lesson which should be learned from this is that the dynamical scale  $\Lambda$  has to appear as a parameter of the full Seiberg-Witten curve to smooth out the classical singularities. In the absence of a tree-level superpotential coupling, this requirement will severely restrict the matter content of the theory.

We now discuss the constraint which arises from requiring that the dynamical scale  $\Lambda$  appear in the Seiberg-Witten curve. This curve must respect all symmetries of the original theory. In particular, one can consider a  $U(1)_R$  symmetry under which all fields  $\Phi_i$  carry zero charge. This symmetry is anomalous under the gauge group  $G$ : that is, the  $G^2 U(1)_R$  anomaly is nonvanishing. One can, however, restore this symmetry by promoting the dynamical scale  $\Lambda$  to a background chiral superfield [24]. The reason for this is that an anomalous  $U(1)$  rotation by an angle  $\alpha$  acts like a shift

$$\theta \rightarrow \theta + \sum_i \mu_i q_i \alpha$$

on the  $\theta$  parameter of the theory, where  $q_i$  is the charge of the  $i$ th fermion under the  $U(1)$  transformation and  $\mu_i$  is the

Dynkin index of the  $i$ th fermion.<sup>1</sup> This means that the dynamical scale

$$\Lambda^b = \mu^b e^{-8\pi^2/g^2(\mu) + i\theta}$$

of the theory has charge  $\sum_i \mu_i q_i$  under the anomalous  $U(1)$  symmetry, where  $b$  is the coefficient of the one-loop beta function. In the case of the  $U(1)_R$  symmetry defined above, the fermions from the chiral superfield have charge  $-1$ , while the gauginos have charge  $+1$ ; thus, the charge of  $\Lambda^b$  is  $G - \mu$ , where  $G$  is the Dynkin index of the adjoint representation and  $\mu = \sum_i \mu_i$  is the sum of the Dynkin indices of the matter fields. After assigning this charge  $G - \mu$  to the dynamical scale, one can require that the Seiberg-Witten curve be invariant (or at least covariant) under this anomalous  $U(1)_R$  symmetry as well.

Let us now consider the  $\Lambda \rightarrow 0$  limit again. In this limit we obtain the classical curve  $P_{cl}(y, x, u_i) = 0$ . Since we expect that the full curve defined by  $x$  and  $y$  describes a Riemann surface, we expect that  $P_{cl}$  is not a homogeneous polynomial in  $x$  and  $y$  (which is true for every known solution). However, this implies that  $x$  and  $y$  have to have zero  $R$  charge as well. But now considering the full curve  $P(y, x, u_i, \Lambda) = 0$ , one can see that  $\Lambda$  can appear in a nontrivial way in the curve only if its  $R$  charge is zero as well, implying that  $\mu = G$ . Note that all pure  $N=2$  theories without matter fields satisfy this condition; however, the  $N=2$  theories with hypermultiplets do not. The way these theories evade this constraint is that  $N=2$  supersymmetry automatically requires a tree-level superpotential coupling between the hypermultiplets and the adjoint from the  $N=2$  vector superfield, which explicitly breaks the  $U(1)_R$  symmetry used above.

The  $\mu = G$  condition is exactly the same as one gets for theories with a quantum-deformed moduli space. The coincidence of these two conditions is not very surprising, since in both cases we are requiring that  $\Lambda$  appear in an equation involving the moduli in a nontrivial way. In the case of the quantum-deformed moduli space, we require that a term proportional to  $\Lambda$  appear on the right-hand side of a classical constraint, while in the case of Seiberg-Witten theory we require that  $\Lambda$  appear as a nontrivial modification of the classical Seiberg-Witten curve.

One can argue for the necessity of the  $\mu = G$  condition in a different way as well. We are requiring that a Seiberg-Witten theory have unbroken  $U(1)$ 's on the generic point in the moduli space. It is, however, known [25] that for  $\mu > G$  the gauge group is completely broken; thus, the  $\mu > G$  theories cannot be in the Coulomb phase everywhere on the moduli space. If  $\mu < G$ , then a dynamically generated superpotential of the form  $W_{dyn} \propto 1/(\prod_i \Phi_i^{\mu_i})^{2/(G-\mu)}$  is allowed by all symmetries of the theory, and such a superpotential is presumably generated either by instantons or by gaugino condensation. This superpotential pushes the fields to large expectation values and the theory has no stable vacuum.

<sup>1</sup>The Dynkin index is defined by  $\text{Tr } T^a T^b = \mu \delta^{ab}$ , where the  $T$ 's are the generators of the gauge group in a given representation.

TABLE I. SU theories satisfying the index constraint  $\mu = G$ . The first column gives the gauge group, the second column the field content, and the third column gives the phase of the low-energy theory. The last column gives a reference to where the low-energy solution of the given theory can be found.

Gauge group	Field content	Phase	Reference
SU( $N$ )	$N(\square + \bar{\square})$	Confining	[2]
SU( $N$ )	$\square + (N-1)\bar{\square} + 3\square$	Confining	[27]
SU( $N$ )	$\square + \bar{\square} + 2(\square + \bar{\square})$	Confining	[3]
SU( $N$ )	Adj	Coulomb phase	[1,7,8]
SU(4)	$3\square + (\square + \bar{\square})$	Confining	[3,4]
SU(4)	$4\square$	Coulomb phase	[26]
SU(5)	$2\square + \square + 3\bar{\square}$	Confining	[3]
SU(5)	$2\square + \bar{\square} + \bar{\square}$	Confining	[3]
SU(6)	$2\square + 4\bar{\square}$	Confining	[3]
SU(6)	$\square + 3(\square + \bar{\square})$	Confining	[3]
SU(6)	$\square + \square + 2\bar{\square}$	Confining	[4]
SU(6)	$2\square$	Coulomb phase	Appendix B
SU(7)	$\square + 4\bar{\square} + 2\square$	Confining	[4]

Thus we expect the  $\mu < G$  theories to be generically in the Higgs phase or, at least, to have a Higgs branch with a runaway vacuum. Thus we again conclude that the only possibility for a theory to be in the Coulomb phase everywhere on the moduli space is when the index condition  $\mu = G$  is satisfied.

We have established that a necessary condition for Seiberg-Witten theories is that the index condition  $\mu = G$  be satisfied. This restricts the number of candidate theories considerably. In the next subsection we show how to find theories which are actually in the Coulomb phase.

TABLE II. Sp theories satisfying the index constraint  $\mu = G$ . The first column gives the gauge group, the second column the field content, and the third column gives the phase of the low-energy theory. The last column gives a reference to where the low-energy solution of the given theory can be found.

Gauge group	Field content	Phase	Reference
Sp( $2N$ )	$(2N+2)\square$	Confining	[28]
Sp( $2N$ )	$\square + 4\square$	Confining	[29]
Sp( $2N$ )	$\square\square = \text{Adj}$	Coulomb phase	[12,14]
Sp(4)	$2\square + 2\square$	Confining	[4]
Sp(4)	$3\square$	Coulomb phase	[26]
Sp(6)	$2\square$	Coulomb phase	Appendix A
Sp(6)	$\square + 3\square$	Confining	[4]

TABLE III. SO theories satisfying  $\mu = G$ . The first column gives the gauge group, the second one the field content, and the third one the phase of the low-energy theory. The last column gives a reference to where the low-energy solution of the given theory is described. We use the following notation for the field content: for an SO( $N$ ) group for  $N$  odd ( $s, v$ ) denotes the number of spinors and the number of vectors, while for  $N$  even ( $s, s', v$ ) denotes the number of the two inequivalent spinor representations and the number of vectors. We do not distinguish between SO( $N$ ) and its covering group Spin( $N$ ).

Gauge group	Field content	Phase	Reference
SO( $N$ )	$\square = \text{Adj}$	Coulomb phase	[10,11,14]
SO( $N$ )	$(0, N-2)$	Coulomb phase	[26]
SO(7)	(1,4)	Confining	[31]
SO(7)	(2,3)	Confining	[32]
SO(7)	(3,2)	Confining	[3]
SO(7)	(4,1)	Confining	[3]
SO(7)	(5,0)	Confining	[3]
SO(8)	(3,2,1)	Confining	[3]
SO(8)	(2,2,2)	Confining	[3]
SO(8)	(4,2,0)	Confining	[3]
SO(8)	(4,1,1)	Confining	[32]
SO(8)	(3,3,0)	Confining	[3]
SO(8)	(5,1,0)	Confining	[32]
SO(9)	(1,5)	Confining	[32]
SO(9)	(2,3)	Confining	[3]
SO(9)	(3,1)	Confining	[3]
SO(10)	(1,0,6)	Confining	[32]
SO(10)	(2,2,0)	Confining	[3]
SO(10)	(1,1,4)	Confining	[3]
SO(10)	(2,1,2)	Confining	[3]
SO(10)	(3,1,0)	Confining	[3]
SO(10)	(2,0,4)	Confining	[3]
SO(10)	(3,0,2)	Confining	[3]
SO(10)	(4,0,0)	Confining	[3]
SO(11)	(2,1)	Confining	[3]
SO(11)	(1,5)	Confining	[3]
SO(12)	(1,1,2)	Confining	[3]
SO(12)	(2,0,2)	Confining	[3]
SO(12)	(1,0,6)	Confining	[3]
SO(13)	(1,3)	Confining	[3]
SO(14)	(1,0,4)	Confining	[3]

## B. Flows

We have seen in the previous section that a necessary condition for a theory to be in the Coulomb phase is that it satisfies the index condition  $\mu = G$ . This requirement alone reduces the number of candidate theories to a finite set. In order to decide which of these theories is in the Coulomb

phase, we have to check whether there are unbroken  $U(1)$ 's on the generic point of the moduli space.

However, in order to exclude a given candidate theory from being in the Coulomb phase, one does not always have to consider the most generic point on the moduli space. It is enough to find a flow which leads to a theory which is known not to be in the Coulomb phase. For example, consider the theory based on the exceptional group  $E_7$  with three 56-dimensional representations. This theory satisfies the  $\mu = G$  constraint. However, by giving a VEV to one of the

**56**'s, we get an  $E_6$  theory with  $2 \times (\mathbf{27} + \overline{\mathbf{27}})$ . Giving an expectation value to the **27** breaks  $E_6$  to  $F_4$ , with the remaining field content being three 26-dimensional representations. Further breaking  $F_4$  by a VEV of **26** will give  $SO(9)$  with two spinors and three vectors,  $2 \times \mathbf{16} + 3 \times \mathbf{9}$ . However, this  $SO(9)$  theory is known to be confining with a quantum-modified constraint [3] and is therefore not in the Coulomb phase. This argument implies that the whole chain of theories, with matter content such that  $\mu = G$ , given below is excluded from being in the Coulomb phase:

$$E_7 : [3 \times \mathbf{56}] \xrightarrow{\langle \mathbf{56} \rangle} E_6 : [2 \times \mathbf{27} + 2 \times \overline{\mathbf{27}}] \xrightarrow{\langle \mathbf{27} \rangle} F_4 : [3 \times \mathbf{26}] \xrightarrow{\langle \mathbf{26} \rangle} SO(9) [2 \times \mathbf{16} + 3 \times \mathbf{9}]. \quad (3.1)$$

Similarly, one could consider an  $E_6$  theory with  $n \times \mathbf{27} + (4 - n) \times \overline{\mathbf{27}}$ , where  $n = 0, 1, 2, 3, 4$ . All of these theories will also flow to  $F_4$  with  $3 \times \mathbf{26}$ , and so they are not in the Coulomb phase either. Indeed, it has been shown recently in Refs. [5,6] that all of the above theories based on exceptional groups satisfying  $\mu = G$  are confining with a quantum-deformed moduli space.

In Tables I–IV we list all theories that satisfy the  $\mu = G$  constraint and give the phase of the given theory. The first column gives the gauge group, the second column the field content, and the third column the phase of the theory. Finally, the fourth column contains a reference to where the actual low-energy solution of the given theory can be found. One can see from Tables I–IV that finding the Seiberg-Witten curves for the remaining  $N=1$  theories in the Coulomb phase completes the study of all  $\mu = G$  theories.

There are only two possibilities for the phase of the  $\mu = G$  theories: confining phase or Coulomb phase. The confining theories all have a low-energy description in terms of composite gauge invariants satisfying a quantum-modified

TABLE IV. Theories based on exceptional groups satisfying the index constraint  $\mu = G$ . The first column gives the gauge group, the second column the field content, and the third column gives the phase of the low-energy theory. The last column gives a reference to where the low-energy solution of the given theory can be found.

Gauge group	Field content	Phase	Reference
$G_2$	$4 \times \mathbf{7}$	Confining	[31]
$G_2$	$\mathbf{14} = \text{Adj}$	Coulomb phase	[15,14,17]
$F_4$	$3 \times \mathbf{26}$	Confining	[5,6]
$F_4$	$\mathbf{52} = \text{Adj}$	Coulomb phase	[14,16,17]
$E_6$	$n \times \mathbf{27} + (4 - n) \times \overline{\mathbf{27}}$	Confining	[5,6]
$E_6$	$\mathbf{78} = \text{Adj}$	Coulomb phase	[14,16,17]
$E_7$	$3 \times \mathbf{56}$	Confining	[5,6]
$E_7$	$\mathbf{133} = \text{Adj}$	Coulomb phase	[14,16]
$E_8$	$\mathbf{248} = \text{Adj}$	Coulomb phase	[14,16]

version of the classical constraints (the  $SU$ ,  $Sp$ , and  $SO$  theories in this class have been analyzed in Refs. [3–5], while the theories based on exceptional groups in Refs. [5,6]). This solution is valid everywhere on the moduli space, and there is no phase boundary between the Higgs and confining phases. This is possible because the massless fields are in a faithful representation of the gauge group; therefore, any external source can be screened by those fields.<sup>2</sup> One can also explicitly check that every  $\mu = G$  theory with chiral superfields in a faithful representation breaks the gauge group completely at generic points on the moduli space. Since at large expectation values the theory can be described entirely in terms of gauge singlet fields and there is no invariant distinction between the Higgs and confining phases, one indeed expects confinement at strong coupling. On the contrary, if the chiral superfields are not in a faithful representation of the gauge group, some external sources cannot be screened by the massless quarks and there can be points on the moduli space where additional massless fields appear. Indeed, we find that in every case where the matter fields are not in a faithful representation, at large expectation values, there are unbroken  $U(1)$  gauge factors and that the low-energy theory is in the Coulomb phase. One can see from Tables I–IV that the only theories which are in the Coulomb phase besides the pure  $N=2$  theories (which are  $N=1$  theories with a chiral superfield in the adjoint representation) are  $SO(N)$  with  $N-2$  vectors,  $SU(6)$  with  $2 \square$ , and  $Sp(6)$  with  $2 \square$ . The other two theories in the Coulomb phase belong to the  $SO(N)$  series with  $N-2$  vectors, since  $SU(4)$  with  $4 \square$  is equivalent to  $SO(6)$  with four vectors and  $Sp(4)$  with  $3 \square$  is equivalent to  $SO(5)$  with three vectors.

<sup>2</sup>In general (nonsupersymmetric) theories one needs to have scalar fields in order to have the possibility of a Higgs phase. For the possibility of a screening phase, one needs to have massless fields (either scalars or fermions) which are in the faithful representation of the gauge group. In supersymmetric theories, the presence of massless chiral superfields which are in a faithful representation satisfies both conditions.

One can easily see that these  $N=1$  theories are indeed in the Coulomb phase by considering the following flows:

$$\begin{aligned} \text{SO}(N):(N-2)\square &\rightarrow \text{SU}(2)\times\text{SU}(2):2(\square,\square), \\ \text{SU}(6):2\square &\rightarrow \text{SU}(3)\times\text{SU}(3):(\square,\bar{\square})+(\bar{\square},\square), \\ \text{Sp}(6):2\square &\rightarrow \text{SU}(2)\times\text{SU}(2)\times\text{SU}(2):(\square,\square,1)+(1,\square,\square)+(\square,1,\square). \end{aligned} \quad (3.2)$$

In the above flows the following fields have to get expectation values: in the  $\text{SO}(N)$  theories  $N-4$  vectors, in the  $\text{SU}(6)$  theory one  $\square$ , and in the  $\text{Sp}(6)$  theory one  $\square$ . Such product group theories have been shown to be in the Coulomb phase in Refs. [21,22]. The  $\text{SO}(N)$  series in (3.2) has been described in Ref. [26], while the remaining flows will enable us to find the Seiberg-Witten curves for the  $\text{SU}(6)$  and  $\text{Sp}(6)$  theories. The results are summarized in the next section, while the detailed derivations are presented in Appendixes A and B.

#### IV. $N=1$ SEIBERG-WITTEN THEORIES

In this section we give the low-energy solution of the  $N=1$  Seiberg-Witten theories. Since the solutions to the theories with one adjoint (the pure  $N=2$  theories) are well known, we refer the reader interested in these theories to the references given in the last column of Tables I–IV. Below, we give only the solution to the  $N=1$  theories considered in (3.2), and do not list all  $N=2$  examples. For these theories, we first give the high-energy field content and the unbroken global symmetries and then list the low-energy degrees of freedom which satisfy the 't Hooft anomaly matching conditions. Finally we give the curve for every theory which provides the low-energy solution for the effective  $\text{U}(1)$  gauge coupling.

##### A. $\text{SU}(6)$ with $2\square$

The field content and the invariants are				
	$\text{SU}(6)$	$\text{SU}(2)$	$\text{U}(1)_R$	$Z_{12}$
$A$	$\square$	$\square$	0	1
$S=A^2$	1	1	0	2
$T=A^4$	1	$\square\square\square$	0	4
$U=A^6$	1	1	0	6

The Seiberg-Witten curve is

$$\begin{aligned} y^2 = & \left[ x^3 - \left( \frac{9}{2} (T^2) + \frac{11}{3} S^4 - \text{SU} \right) x - 2^7 \Lambda^{12} \right. \\ & \left. - \left( 2S^3 U - \frac{107}{27} S^6 - \frac{1}{4} U^2 - \frac{3}{2} S^2 (T^2) + 9(T^3) \right) \right]^2 \\ & - 2^{14} \Lambda^{24}. \end{aligned} \quad (4.1)$$

A detailed description of this theory is given in Appendix B.

##### B. $\text{SO}(N)$ with $(N-2)\square$ [26]

The field content and the invariants are				
	$\text{SO}(N)$	$\text{SU}(N-2)$	$\text{U}(1)_R$	$Z_{2N-4}$
$Q$	$\square$	$\square$	0	1
$M=Q^2$	1	$\square\square$	0	2

Note that in this example the anomaly-matching conditions at the origin are not satisfied by the meson field  $M$  itself. The reason is that a number of monopoles become massless exactly at the origin, and their contribution to the anomalies has to be taken into account. For details see Refs. [26,30]. The Seiberg-Witten curve for this theory is

$$y^2 = [x^2 - (\det M - 8\Lambda^{2N-4})]^2 - 64\Lambda^{4N-8}.$$

##### C. $\text{Sp}(6)$ with $2\square$

The field content and the invariants are				
	$\text{Sp}(6)$	$\text{SU}(2)$	$\text{U}(1)_R$	$Z_8$
$A_1$	$\square$	$\square$	0	1
$S_{ij} = \text{Tr}(JA_i JA_j)$		$\square\square$	0	2
$T_{ijk} = \text{Tr}(JA_i JA_j JA_k)$		$\square\square\square$	0	3
$U = \text{Tr}(JA_1 JA_2 JA_1 JA_2)$		1	0	4

The Seiberg-Witten curve is given by

$$\begin{aligned} y^2 = & \left[ x^2 + 72\Lambda^8 [(S^2) + 12U] - \left( -\frac{1}{2} (T^4) + 24U^3 \right. \right. \\ & \left. \left. + \frac{3}{2} U^2 (S^2) + 3U(ST^2) + \frac{1}{4} (S^3 T^2) \right) \right]^2 - 768\Lambda^{24}. \end{aligned} \quad (4.2)$$

A detailed description of this theory is given in Appendix A.

#### V. CONCLUSIONS

We have studied  $N=1$  supersymmetric gauge theories which are in the Coulomb phase on the entire moduli space. We have shown that theories based on a simple gauge group and no tree-level superpotential must satisfy the index condition  $\mu=G$ , which is exactly the same as for theories with a quantum-deformed moduli space. One can find the theories which are actually in the Coulomb phase by studying the flows of the theory. It turns out that all  $\mu=G$  theories are either confining with a quantum-deformed moduli space if the matter content is in the faithful representation of the

gauge group or in the Coulomb phase if the matter fields are not in a faithful representation. The Seiberg-Witten curves for the new theories in the Coulomb phase can be found by studying the flows to the product group theories of Ref. [22]. This work, together with the results on confining theories with quantum-deformed moduli spaces, completes the solutions to all  $N=1$  theories with  $\mu \leq G$ .

### ACKNOWLEDGMENTS

We are grateful to Daniel Freedman for comments on the manuscript. C.C. thanks the Miller Institute for Basic Research in Science for financial support. C.C. has been supported in part by the U.S. Department of Energy under Contract No. DE-AC03-76SF00098 and in part by the National Science Foundation under Grant No. PHY-95-14797. W.S. was supported by the U.S. Department of Energy under Contract No. DE-FG03-97ER405046.

### APPENDIX A: $\text{Sp}(6)$ WITH $2\Box$

In this appendix we outline the derivation of the Seiberg-Witten curve for the  $\text{Sp}(6)$  theory with  $2\Box$ . As we already mentioned in Sec. III, giving a VEV to one of antisymmetric tensors breaks  $\text{Sp}(6)$  to  $\text{SU}(2)^3$  with precisely the field content that was considered in Ref. [22]. The curve describing the  $\text{Sp}(6)$  theory must therefore reduce to the curve for  $\text{SU}(2)^3$  in the limit of large VEVs. It turns out that considering this limit is sufficient for determining the complete curve.

Let us first describe the global symmetries of the theory and gauge-invariant operators parametrizing the moduli space:

	$\text{Sp}(6)$	$\text{SU}(2)$	$\text{U}(1)_R$	$\text{Z}_8$
$A_i$	$\Box$	$\square$	0	1
$S_{ij} = \text{Tr}(JA_i JA_j)$		$\Box\Box$	0	2
$T_{ijk} = \text{Tr}(JA_i JA_j JA_k)$		$\Box\Box\Box$	0	3
$U = \text{Tr}(JA_1 JA_2 JA_1 JA_2)$		1	0	4

(A1)

where  $J$  is the two-index antisymmetric invariant tensor of  $\text{Sp}$ . Indeed, with this choice of operators the 't Hooft anomaly-matching conditions (including the discrete anomaly-matching conditions of Ref. [30]) are satisfied at the origin, once a  $\text{U}(1)$  vector field is included in the low-energy spectrum.

We give a VEV to  $A_1$  of the form  $\langle A_1 \rangle = iv \text{diag}(\sigma_2, \omega\sigma_2, \omega^2\sigma_2)$ , where  $\omega = \exp(2\pi i/3)$ . The remaining field  $A_2$  decomposes under  $\text{SU}(2) \times \text{SU}(2) \times \text{SU}(2)$  as

$$A_2 \rightarrow Q_1(\Box, \Box, 1) + Q_2(1, \Box, \Box) + Q_3(\Box, 1, \Box) + s_{1,2}(1, 1, 1).$$

Next, we express the invariants of the  $\text{SU}(2)^3$  theory [22], that is,  $M_i = Q_i^2$  and  $T = Q_1 Q_2 Q_3$ , in terms of the  $\text{Sp}(6)$  gauge invariants, keeping only leading terms in  $1/v$ . The curve for  $\text{SU}(2)^3$ ,

$$y^2 = [x^2 - (\Lambda_1^4 M_2 + \Lambda_2^4 M_3 + \Lambda_3^4 M_1 + T^2 - M_1 M_2 M_3)]^2 - 4\Lambda_1^4 \Lambda_2^4 \Lambda_3^4,$$

contains two combinations of invariants. We will now write the two combinations  $\Lambda_1^4 M_2 + \Lambda_2^4 M_3 + \Lambda_3^4 M_1$  and  $T^2 - M_1 M_2 M_3$  in terms of  $\text{Sp}(6)$  invariants, where  $\Lambda_i$  is the characteristic scale of the  $i$ th  $\text{SU}(2)$  factor. With the above choice of the VEV for  $A_1$ , the scale-matching relations between the  $\text{Sp}(6)$  scale  $\Lambda$  and the scales of the  $\text{SU}(2)$  factors are

$$\Lambda_1^4 = \frac{\Lambda^8}{v^4(1-\omega^2)(1-\omega^4)},$$

$$\Lambda_2^4 = \frac{\Lambda^8}{v^4(\omega^2-1)(\omega^2-\omega^4)},$$

$$\Lambda_3^4 = \frac{\Lambda^8}{v^4(\omega^4-1)(\omega^4-\omega^2)}.$$

Since a curve can depend only on the field combinations invariant under global symmetries, we need to introduce invariants under the global  $\text{SU}(2)$  symmetry, which will be the variables of the full  $\text{Sp}(6)$  curve. The connection between the global  $\text{SU}(2)$  invariants and the parameters of the  $\text{SU}(2)^3$  curve are

$$\Lambda_1^4 \Lambda_2^4 \Lambda_3^4 = \frac{1}{27v^{12}} \Lambda^{24},$$

$$\Lambda_1^4 M_2 + \Lambda_2^4 M_3 + \Lambda_3^4 M_1 = -\frac{1}{72v^6} \Lambda^8 [(S^2) + 12U],$$

$$T^2 - M_1 M_2 M_3 = \frac{1}{5184v^6} \left( -\frac{1}{2} (T^4) + 24U^3 + \frac{3}{2} U^2 (S^2) + 3U(ST^2) + \frac{1}{4} (S^3 T^2) \right),$$

where the  $\text{SU}(2)$  contractions are defined as follows:

$$(S^2) = S_{i_1 i_2} S_{j_1 j_2} \epsilon^{i_1 j_1} \epsilon^{i_2 j_2},$$

$$(T^4) = T_{i_1 i_2 i_3} T_{j_1 j_2 j_3} T_{k_1 k_2 k_3} T_{l_1 l_2 l_3} \times \epsilon^{i_1 j_1} \epsilon^{i_2 j_2} \epsilon^{k_1 l_1} \epsilon^{k_2 l_2} \epsilon^{i_3 k_3} \epsilon^{j_3 l_3},$$

$$(ST^2) = T_{i_1 i_2 i_3} T_{j_1 j_2 j_3} S_{k_1 k_2} \epsilon^{i_1 j_1} \epsilon^{i_2 j_2} \epsilon^{i_3 k_1} \epsilon^{j_3 k_2},$$

$$(S^3 T^2) = T_{i_1 i_2 i_3} T_{j_1 j_2 j_3} S_{k_1 k_2} S_{l_1 l_2} S_{m_1 m_2} \times \left( \epsilon^{i_1 j_1} \epsilon^{i_2 j_2} \epsilon^{i_3 k_1} \epsilon^{k_2 l_1} \epsilon^{l_2 m_1} \epsilon^{m_2 j_3} - \frac{1}{3} \epsilon^{i_1 k_1} \epsilon^{i_2 l_1} \epsilon^{i_3 m_1} \epsilon^{j_1 k_2} \epsilon^{j_2 l_2} \epsilon^{j_3 m_2} \right).$$

Finally, we obtain the curve for  $\text{Sp}(6)$  theory in the large VEV limit by substituting the above expression into the  $\text{SU}(2)^3$  curve:

$$y^2 = \left[ x^2 + \frac{1}{72v^6} [(S^2) + 12U] \Lambda^8 - \frac{1}{5184v^6} \left( -\frac{1}{2} (T^4) + 24U^3 + \frac{3}{2} U^2 (S^2) + 3U(ST^2) + \frac{1}{4} (S^3 T^2) \right) \right]^2 - \frac{4}{27v^{12}} \Lambda^{24}. \quad (\text{A2})$$

After rescaling  $x \rightarrow x/(72v^3)$  and  $y \rightarrow y/(5184v^6)$ , we obtain

$$y^2 = \left[ x^2 + 72\Lambda^8 [(S^2) + 12U] - \left( -\frac{1}{2} (T^4) + 24U^3 + \frac{3}{2} U^2 (S^2) + 3U(ST^2) + \frac{1}{4} (S^3 T^2) \right) \right]^2 - 768\Lambda^{24}. \quad (\text{A3})$$

It is easy to check that symmetries prohibit modifications to this form of the curve. Since only  $\text{SU}(2)$  singlets can appear in the curve, any other allowed term would be of the same order in  $1/v$  and therefore a coefficient of any such terms must be zero. Thus Eq. (A3) is the final form of the full  $\text{Sp}(6)$  curve.

## APPENDIX B: $\text{SU}(6)$ WITH $2\begin{smallmatrix} \square \\ \square \end{smallmatrix}$

We now present the derivation of the curve for the  $\text{SU}(6)$  theory with  $2\begin{smallmatrix} \square \\ \square \end{smallmatrix}$ . Similarly to the derivation of the  $\text{Sp}(6)$  case, we give a VEV to one of the tensors, which breaks  $\text{SU}(6)$  to  $\text{SU}(3) \times \text{SU}(3)$  with two bifundamental fields, which is the theory considered in Ref. [22]. The field content, global symmetries, and independent gauge invariants are defined below:

	$\text{SU}(6)$	$\text{SU}(2)$	$\text{U}(1)_R$	$\text{Z}_{12}$
$A_i$	$\begin{smallmatrix} \square \\ \square \end{smallmatrix}$	$\square$	0	1
$S = A_1 A_2$		1	0	2.
$T_{ijkl} = A_i A_j A_k A_l$		$\square\square\square\square$	0	4
$U = A_1^3 A_2^3$		1	0	6

(B1)

The anomaly matching is satisfied for the low-energy spectrum including the gauge invariants and two  $\text{U}(1)$  vector multiplets. The  $\text{SU}(6)$  gauge contractions for the invariants  $S$ ,  $T$ , and  $U$  are given by

$$S = \frac{1}{6^2} A_1^{\alpha_1 \alpha_2 \alpha_3} A_2^{\beta_1 \beta_2 \beta_3} \epsilon_{\alpha_1 \alpha_2 \alpha_3 \beta_1 \beta_2 \beta_3},$$

$$T_{ijkl} = \frac{1}{6^4} \left[ A_i^{\alpha_1 \alpha_2 \alpha_3} A_j^{\beta_1 \beta_2 \beta_3} A_k^{\gamma_1 \gamma_2 \gamma_3} A_l^{\delta_1 \delta_2 \delta_3} \times \epsilon_{\alpha_1 \alpha_2 \alpha_3 \beta_1 \beta_2 \gamma_1} \epsilon_{\beta_3 \gamma_2 \gamma_3 \delta_1 \delta_2 \delta_3} - \frac{S^2}{9} (\epsilon_{ij} \epsilon_{kl} + \epsilon_{ik} \epsilon_{jl}) \right],$$

$$U = \frac{1}{144} A_1^{\alpha_1 \alpha_2 \alpha_3} A_1^{\beta_1 \beta_2 \beta_3} A_1^{\gamma_1 \gamma_2 \gamma_3} A_2^{\delta_1 \delta_2 \delta_3} A_2^{\zeta_1 \zeta_2 \zeta_3} A_2^{\eta_1 \eta_2 \eta_3} \times \epsilon_{\alpha_1 \alpha_2 \alpha_3 \beta_1 \beta_2 \gamma_1} \epsilon_{\beta_3 \gamma_2 \gamma_3 \delta_1 \delta_2 \zeta_1} \epsilon_{\zeta_3 \zeta_2 \zeta_3 \eta_1 \eta_2 \eta_3}.$$

Note the additional subtlety in the definition of  $T$ . Here  $T$  is a four-index symmetric tensor under the global  $\text{SU}(2)$ . However, the gauge contraction defined above does not yield an irreducible tensor of  $\text{SU}(2)$ . This contraction also contains an  $\text{SU}(2)$  singlet piece, which needs to be subtracted.

We give a VEV to  $A_1$  of the form  $\langle A_{123} \rangle = \langle A_{456} \rangle = v$  with zero VEVs for all other independent components. The field  $A_2$  decomposes under  $\text{SU}(3) \times \text{SU}(3)$  as

$$A_2 \rightarrow Q_1(\square, \bar{\square}) + Q_2(\bar{\square}, \square) + s_{1,2}(1, 1, 1).$$

The  $\text{SU}(3)^2$  theory has four invariants [22]  $B_i = \det Q_i$  and  $T_i = \text{Tr}(Q_1 Q_2)^i$ ,  $i = 1, 2$ , which we now express in terms of  $\text{SU}(6)$  invariants. The  $\text{SU}(3) \times \text{SU}(3)$  curve is

$$y^2 = (x^3 - u_2 x - u_3 - \Lambda_1^6 - \Lambda_2^6)^2 - 4\Lambda_1^6 \Lambda_2^6,$$

where  $\Lambda_{1,2}$  are the scales of the two  $\text{SU}(3)$  gauge groups, while  $u_2 = \frac{1}{2}(T_2 - \frac{1}{3}T_1^2)$  and  $u_3 = \frac{1}{3}(3B_1 B_2 + \frac{1}{2}T_2 T_1 - \frac{5}{18}T_1^3)$ . In terms of  $\text{SU}(6)$  invariants we have

$$u_2 = \frac{1}{16v^4} \left( \frac{9}{2} (T^2) + \frac{11}{3} S^4 - SU \right)$$

$$u_3 = \frac{1}{64v^6} \left( 2S^3 U - \frac{107}{27} S^6 - \frac{1}{4} U^2 - \frac{3}{2} S^2 (T^2) + 9(T^3) \right),$$

where the invariants under the global  $\text{SU}(2)$  are defined as follows:

$$(T^2) = T_{i_1 i_2 i_3 i_4} T_{j_1 j_2 j_3 j_4} \epsilon^{i_1 j_1} \epsilon^{i_2 j_2} \epsilon^{i_3 j_3} \epsilon^{i_4 j_4},$$

$$(T^3) = T_{i_1 i_2 i_3 i_4} T_{j_1 j_2 j_3 j_4} T_{k_1 k_2 k_3 k_4} \epsilon^{i_1 j_1} \epsilon^{i_2 j_2} \epsilon^{i_3 k_1} \epsilon^{i_4 k_2} \epsilon^{j_3 k_3} \epsilon^{j_4 k_4}.$$

Using the matching relations for the  $\text{SU}(6)$  scale  $\Lambda$  and the scales of the  $\text{SU}(3)$  groups,  $\Lambda_1^6 = \Lambda_2^6 = \Lambda^{12}/v^6$ , we obtain the  $\text{SU}(6)$  curve in the large VEV limit:

$$y^2 = \left[ x^3 - \frac{1}{16v^4} \left( \frac{9}{2} (T^2) + \frac{11}{3} S^4 - SU \right) x - 2 \frac{\Lambda^{12}}{v^6} - \frac{1}{64v^6} \left( 2S^3 U - \frac{107}{27} S^6 - \frac{1}{4} U^2 - \frac{3}{2} S^2 (T^2) + 9(T^3) \right) \right]^2 - 4 \frac{\Lambda^{24}}{v^{12}}. \quad (\text{B2})$$

After rescaling  $x \rightarrow x/(4v^2)$  and  $y \rightarrow y/(64v^6)$ , the curve takes the form



$$y^2 = \left[ x^3 - \left( \frac{9}{2} (T^2) + \frac{11}{3} S^4 - SU \right) x - 2^7 \Lambda^{12} - \left( 2S^3 U - \frac{107}{27} S^6 - \frac{1}{4} U^2 - \frac{3}{2} S^2 (T^2) + 9(T^3) \right) \right]^2$$

$$- 2^{14} \Lambda^{24}. \quad (\text{B3})$$

As before, this is the final form of the full  $SU(6)$  curve because any modification allowed by the global symmetries is of the same order in  $1/v$  as the terms already present.

- 
- [1] N. Seiberg and E. Witten, Nucl. Phys. **B426**, 19 (1994); **B431**, 484 (1994).
  - [2] N. Seiberg, Phys. Rev. D **49**, 6857 (1994); Nucl. Phys. **B435**, 129 (1995).
  - [3] C. Csáki, M. Schmaltz, and W. Skiba, Phys. Rev. Lett. **78**, 799 (1997); Phys. Rev. D **55**, 7840 (1997).
  - [4] B. Grinstein and D. Nolte, Phys. Rev. D **57**, 6471 (1998).
  - [5] B. Grinstein and D. Nolte, hep-th/9803139.
  - [6] P. Cho, Phys. Rev. D **57**, 5214 (1998).
  - [7] P. Argyres and A. Faraggi, Phys. Rev. Lett. **74**, 3931 (1995).
  - [8] A. Klemm, W. Lerche, S. Yankielowicz, and S. Theisen, Phys. Lett. B **344**, 169 (1995).
  - [9] A. Hanany and Y. Oz, Nucl. Phys. **B452**, 283 (1995); P. Argyres, R. Plesser, and A. Shapere, Phys. Rev. Lett. **75**, 1699 (1995); A. Hanany, Nucl. Phys. **B466**, 85 (1996).
  - [10] A. Brandhuber and K. Landsteiner, Phys. Lett. B **358**, 73 (1995).
  - [11] U. Danielsson and B. Sundborg, Phys. Lett. B **370**, 83 (1996).
  - [12] P. Argyres and A. Shapere, Nucl. Phys. **B461**, 437 (1996).
  - [13] A. Gorski, I. Krichever, A. Marshakov, A. Mironov, and A. Morozov, Phys. Lett. B **355**, 466 (1995).
  - [14] E. Martinec and N. Warner, Nucl. Phys. **B459**, 97 (1996).
  - [15] K. Landsteiner, J. Pierre, and S. Giddings, Phys. Rev. D **55**, 2367 (1997); M. Alishahiha, F. Ardalan, and F. Mansouri, Phys. Lett. B **381**, 446 (1996).
  - [16] M. Abolhasani, M. Alishahiha, and A. Ghezelbash, Nucl. Phys. **B480**, 279 (1996).
  - [17] U. Danielsson and B. Sundborg, Phys. Lett. B **370**, 83 (1996).
  - [18] S. Elitzur, A. Forge, A. Giveon, K. Intriligator, and E. Rabinovici, Phys. Lett. B **379**, 121 (1996); S. Terashima and S. Yang, *ibid.* **391**, 107 (1997); T. Kitao, *ibid.* **402**, 290 (1997); S. Terashima and S. Yang, hep-th/9706076.
  - [19] P. Argyres and M. Douglas, Nucl. Phys. **B448**, 93 (1995); M. Douglas and S. Shenker, *ibid.* **B447**, 271 (1995).
  - [20] E. D'Hoker, I. Krichever, and D. Phong, Nucl. Phys. **B489**, 179 (1997); **B489**, 211 (1997); **B494**, 89 (1997); E. D'Hoker and D. Phong, Phys. Lett. B **397**, 94 (1997).
  - [21] K. Intriligator and N. Seiberg, Nucl. Phys. **B431**, 551 (1994).
  - [22] C. Csáki, J. Erlich, D. Freedman, and W. Skiba, Phys. Rev. D **56**, 5209 (1997).
  - [23] A. Kapustin, Phys. Lett. B **398**, 104 (1997); T. Kitao, S. Terashima, and S. Yang, *ibid.* **399**, 75 (1997); A. Giveon, O. Pelc, and E. Rabinovici, Nucl. Phys. **B499**, 100 (1997); M. Gremm, Phys. Rev. D **57**, 2537 (1998).
  - [24] K. Intriligator, R. Leigh, and N. Seiberg, Phys. Rev. D **50**, 1092 (1994).
  - [25] G. Dotti and A. Manohar, Phys. Rev. Lett. **80**, 2758 (1998).
  - [26] K. Intriligator and N. Seiberg, Nucl. Phys. **B444**, 125 (1995).
  - [27] E. Poppitz and S. Trivedi, Phys. Lett. B **365**, 125 (1996); P. Pouliot, *ibid.* **367**, 151 (1996).
  - [28] K. Intriligator and P. Pouliot, Phys. Lett. B **353**, 471 (1995).
  - [29] P. Cho and P. Kraus, Phys. Rev. D **54**, 7640 (1996); C. Csáki, W. Skiba, and M. Schmaltz, Nucl. Phys. **B487**, 128 (1997).
  - [30] C. Csáki and H. Murayama, Nucl. Phys. **B515**, 114 (1998).
  - [31] I. Pesando, Mod. Phys. Lett. A **10**, 1871 (1995); S. Giddings and J. Pierre, Phys. Rev. D **52**, 6065 (1995); P. Pouliot, Phys. Lett. B **359**, 108 (1995).
  - [32] P. Pouliot and M. Strassler, Phys. Lett. B **370**, 76 (1996); **375**, 175 (1996).