Self-duality and soldering in odd dimensions

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Using the recently developed soldering formalism we highlight certain features of quantum mechanical models. The complete correspondence between these models and self-dual field theoretical models in odd dimensions is established. The distinction between self-duality and self-dual factorization in these dimensions is clarified. [S0556-2821(99)01718-X]

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Self-dual models in odd dimensions, characterized by the presence of Chern-Simons terms [1], have been in vogue for quite some time [2,3]. Interest in these models has been rekindled by noting their relevance in higher dimensional bosonization [4]. Some new results in this connection were reported in [5] by using the concept of soldering [6]. Interestingly, several facets of self-dual field theories in odd dimensions may be better appreciated and understood by looking at their one-dimensional counterpart—the so-called topological quantum mechanics [7]. In this paper we discuss the concepts of self-duality and soldering in the context of topological quantum mechanics. Some familiar results are explained in a different setting, leading to fresh insights. This analysis is extended to the self-dual field theoretic models in odd dimensions. Some new results are reported clarifying, in particular, the distinction between self-duality and self-dual factorization.

The quantum mechanical topological models are governed by the Lagrangian [7]

$$\mathcal{L}=\frac{m}{2}\dot{\vec{x}}^2+e\dot{\vec{x}}\cdot\vec{A}(\vec{x})-eV(\vec{x}), \tag{1}$$

implying the motion of a particle of mass m and charge e in the external electric $(-\partial_i V)$ and magnetic $(\partial_i A_j-\partial_j A_i)$ fields. For the simplest explicitly solvable model [7], the motion is two dimensional $(i=1,2)$ and rotationally symmetric in a constant magnetic field (B) and a quadratically scalar potential so that

$$A_i=-\frac{1}{2}\epsilon_{ij}x_jB,$$

$$V=\frac{k}{2}x_i^2.$$

The Lagrangian (1) therefore simplifies to (setting e=1)

$$\mathcal{L}=\frac{m}{2}\dot{x}_i{}^2+\frac{B}{2}\epsilon_{ij}x_i\dot{x}_j-\frac{k}{2}x_i^2. \tag{2}$$

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There are some interesting features of this Lagrangian. If the magnetic field is switched off (B=0), the model represents a bi-dimensional harmonic oscillator (HO):

$$\mathcal{L}_{HO}=\frac{m}{2}\dot{x}_i{}^2-\frac{k}{2}x_i^2. \tag{3}$$

Now consider the motion of the particle in the absence of the electric field so that we have

$$\mathcal{L}_+=\frac{m}{2}\dot{x}_i{}^2+\frac{B}{2}\epsilon_{ij}x_i\dot{x}_j. \tag{4}$$

Let us next illustrate the connection between Eqs. (4) and (3). Together with Eq. (4), consider the Lagrangian (5) with an independent set of coordinates y_i and where the direction of the magnetic field is reversed,

$$\mathcal{L}_-=\frac{m}{2}\dot{y}_i{}^2-\frac{B}{2}\epsilon_{ij}y_i\dot{y}_j. \tag{5}$$

It is now possible to combine Eqs. (4) and (5) by the soldering formalism [5,8]. Consider the following transformation:

$$\delta x_i=\delta y_i=\eta_i \tag{6}$$

which effects the changes,

$$\delta\mathcal{L}_\pm=J_{\pm i}\dot{\eta}_i \tag{7}$$

where

$$J_{\pm i}(z)=m\dot{z}_i\pm B\epsilon_{ji}z_j \tag{8}$$

and $z_i=x_i,y_i$. Introduce the soldering field W_i transforming as

$$\delta W_i=\dot{\eta}_i. \tag{9}$$

Then the first iterated Lagrangian,

$$\mathcal{L}^{(1)}=\mathcal{L}_++\mathcal{L}_--(J_{+i}(x)+J_{-i}(y))W_i, \tag{10}$$

transforms as

$$\delta\mathcal{L}^{(1)}=-2m\dot{\eta}_iW_i. \tag{11}$$

Including the term W_i^2 now yields an invariant Lagrangian,

$$\mathcal{L}=\mathcal{L}^{(1)}+mW_i^2; \quad \delta\mathcal{L}=0. \tag{12}$$

Since W_i is an auxiliary variable, it is possible to eliminate it by using the equation of motion,

$$W_i=\frac{1}{2m}(J_{+i}+J_{-i}). \tag{13}$$

The solution is compatible with the variations (6) and (9). Inserting Eq. (13) into Eq. (12), the final soldered Lagrangian is obtained,

$$\mathcal{L}=\frac{m}{2}\dot{q}_i^2-\frac{B^2}{2m}q_i^2, \quad q_i=\frac{1}{\sqrt{2}}(x_i-y_i), \tag{14}$$

which is no longer a function of x and y independently, but only on their difference. Identifying k with B^2/m, it is found that Eq. (14) exactly maps on to Eq. (3). This exercise shows how two identical particles moving in the presence of magnetic fields with the same magnitudes but opposite directions simulate the effect of a single particle moving in the presence of a quadratic scalar potential.

It is easy to supplement the above Lagrangian analysis by the familiar Hamiltonian formulation [7]. The Hamiltonian corresponding to Eq. (4) is given by

$$H_+=\frac{1}{2m}\left(p_i+\frac{B}{2}\epsilon_{ij}x_j\right)^2, \tag{15}$$

where p_i is the conjugate momentum,

$$p_i=\frac{\partial\mathcal{L}_+}{\partial\dot{x}_i}=m\dot{x}_i-\frac{B}{2}\epsilon_{ij}x_j. \tag{16}$$

Making a canonical transformation,

$$p_\pm=p_1\pm\frac{B}{2}x_2, \tag{17}$$

$$x_\pm=\frac{1}{2}x_1\mp\frac{1}{B}p_2, \tag{18}$$

we obtain, in the new canonical variables,

$$H_+=\frac{1}{2m}p_+^2+\frac{1}{2}B^2x_+^2. \tag{19}$$

The Hamiltonian is that of the usual HO. It is, however, expressed only in terms of (x_+,p_+) while the other canonical pair (x_-,p_-) gets eliminated. The fact that the two-dimensional Lagrangian (4) simplifies to a one-dimensional oscillator (19) is essentially tied to its symplectic structure. Likewise Eq. (5) yields the Hamiltonian for the HO expressed only in terms of the canonical set (x_-,p_-). Thus the combination of Eqs. (4) and (5) should yield a two-dimensional HO which is precisely shown by the soldering mechanism leading to Eq. (14).

It is worthwhile to mention that the massless version of Eq. (2),

$$\mathcal{L}_0=\frac{B}{2}\epsilon_{ij}x_i\dot{x}_j-\frac{k}{2}x_i^2, \tag{20}$$

also yields a one-dimensional HO. The simplest way to realize this is by eliminating either x_1 or x_2 in favor of the other. In this sense it is similar to Eq. (4). Correspondingly, a soldering scheme can be developed.

Going back to the original Lagrangian (2), it is well known [7] from a Hamiltonian analysis that the model corresponds to two decoupled one-dimensional oscillators described by the canonical pairs $(x_\pm,p_\pm)$ and frequencies $\omega_\pm$ where

$$p_\pm=\sqrt{\frac{\omega_\pm}{2m\Omega}}p_1\pm\sqrt{\frac{\omega_\pm m\Omega}{2}}x_2,$$

$$x_\pm=\sqrt{\frac{m\Omega}{2\omega_\pm}}x_1\mp\frac{1}{\sqrt{\omega_\pm m\Omega}}p_2, \tag{21}$$

$$\omega_\pm=\Omega\pm\frac{B}{2m}, \quad \Omega=\sqrt{\frac{B^2}{4m^2}+\frac{k}{m}}.$$

These are the analogues of Eqs. (17), (18). While the Hamiltonian analysis reveals the decoupling of Eq. (2) into the two one-dimensional oscillators, the soldering formalism will explicitly demonstrate the reverse process. Let us therefore consider the following *independent* Lagrangians:

$$\mathcal{L}_-=\frac{1}{2}(\omega_-\epsilon_{ij}x_i\dot{x}_j-\omega_-^2x_i^2), \tag{22}$$

$$\mathcal{L}_+=\frac{1}{2}(-\omega_+\epsilon_{ij}y_i\dot{y}_j-\omega_+^2y_i^2). \tag{23}$$

These Lagrangians are similar to the previous cases [see, for instance Eq. (20)], except that the frequencies are different $\omega_\pm$. As stated before, both of these represent one-dimensional harmonic oscillators but there are two points which ought to be stressed. The equations of motion are given by

$$x_i=\frac{1}{\omega_-}\epsilon_{ij}\dot{x}_j, \tag{24}$$

$$y_i=-\frac{1}{\omega_+}\epsilon_{ij}\dot{y}_j. \tag{25}$$

Define a dual field as

$$\tilde{x}_i=\frac{1}{\omega}\epsilon_{ij}\dot{x}_j.$$

The duality property is only on-shell because

$$\tilde{\tilde{x}}_i=\frac{1}{\omega}\epsilon_{ij}\dot{\tilde{x}}_j=x_i$$

requires the use of the equation of motion. In this sense, therefore, Eqs. (24) and (25) characterize self- and anti-self-dual solutions, respectively. Moreover, as discussed in [5,8], it is possible to interpret the Lagrangians (22) and (23) as chiral oscillators with varying frequencies $\omega_\pm$ rotating in clockwise and anticlockwise directions. Thus the ubiquitous role of self-duality and chirality becomes apparent in these models. The process of soldering will combine the dual aspects of these symmetries to yield a new model expressed in terms of the composite variable $(x-y)$. Under the transformations

$$\delta x_i = \delta y_i = \eta_i ,$$

the Lagrangians undergo the variations

$$\delta \mathcal{L}_{\mp} = \epsilon_{ij} J_{\mp j} \eta_i, \quad z = x,y, \quad J_{\mp i}(z) = \omega_{\mp}(\pm \dot{z}_i + \omega_{\mp} \epsilon_{ij} z_j).$$

Inserting the auxiliary variable W_i transforming as

$$\delta W_i = \epsilon_{ij} \eta_j$$

it is possible to construct, in analogy with Eq. (12), the following Lagrangian:

$$\mathcal{L} = \mathcal{L}_-(x) + \mathcal{L}_+(y) + W_i (J_i^-(x) + J_i^+(y)) - \frac{1}{2}(\omega_+^2 + \omega_-^2) W_i^2 .$$

This expression is on-shell invariant. Eliminating W_i by using the equation of motion, the above Lagrangian is recast in the manifestly invariant form,

$$\mathcal{L} = \frac{1}{2}\dot{X}_i{}^2 + \frac{1}{2}(\omega_+ - \omega_-)\epsilon_{ij} X_i \dot{X}_j - \frac{1}{2}\omega_+ \omega_- X_i^2 ,$$

$$X_i = \sqrt{\frac{\omega_+ \omega_-}{\omega_+^2 + \omega_-^2}}(x_i - y_i) \qquad (26)$$

where use has been made of the on-shell conditions (24), (25). Identifying the frequencies $\omega_\pm$ with those occurring in Eq. (21) we find

$$\omega_+ - \omega_- = \frac{B}{m},$$

$$\omega_- \omega_+ = \frac{k}{m}. \qquad (27)$$

After a suitable scaling it is now simple to observe that the Lagrangian in Eq. (26) exactly reproduces Eq. (2).

The above exercise therefore shows, in a precise manner, how the self- and anti-self-dual (or, alternatively, the left and right chiral) oscillators combine to yield the model (2). For identical frequencies ($\omega_+ = \omega_- = \omega$), the epsilon term in Eq. (26) vanishes so that the Lagrangian (3) is obtained, a result found earlier [5,8] in a different context. This is also expected since Eq. (3) was derived directly from a soldering of Eqs. (4) and (5), models which are equivalent to Eqs. (22) and (23) with identical frequencies.

We conclude our discussion on the topological quantum mechanics by pointing out that the equation of motion obtained from Eq. (26) factorizes into its dual (chiral) components as follows:

$$(\omega_+ \delta_{ij} + \epsilon_{ij}\partial_t)(\omega_- \delta_{jk} - \epsilon_{jk}\partial_t) X_k = 0.$$

The possibility of this factorization is ingrained in the soldering of Eqs. (22) and (23) [with equations of motions (24) and (25), respectively] to yield the final structure (26).

It is now straightforward to extend the preceding analysis to odd dimensional field theories. In this context we recall that Eq. (2) had been regarded [7] analogous to the Lagrangian density for three-dimensional topologically massive electrodynamics (Maxwell-Chern-Simons theories) in the Weyl $(A_0=0)$ gauge,

$$\mathcal{L} = \frac{1}{2}\dot{\vec{A}}^2 + \frac{\mu}{2}\dot{\vec{A}} \times \vec{A} - \frac{1}{2}(\vec{\nabla} \times \vec{A})^2.$$

In our scheme of things, however, we should interpret Eq. (2) to be the analogue of the topologically massive electrodynamics augmented by the usual mass term,

$$\mathcal{L}_S = \frac{1}{2} A_\mu A^\mu - \frac{\theta}{2m^2}\epsilon_{\mu\nu\sigma}\partial^\mu A^\nu A^\sigma - \frac{1}{4m^2} A_{\mu\nu} A^{\mu\nu}, \qquad (28)$$

$$A_{\mu\nu} = \partial_{[\mu} A_{\nu]}$$

in the limit where all spatial derivatives are neglected [1]. Correspondingly, Eqs. (22) and (23) would be interpreted as the analogues of the self- and anti-self-dual models [3]

$$\mathcal{L}_-(g) = \frac{1}{2} g_\mu g^\mu - \frac{1}{2m_-}\epsilon_{\mu\nu\lambda} g^\mu \partial^\nu g^\lambda, \qquad (29)$$

$$\mathcal{L}_+(f) = \frac{1}{2} f_\mu f^\mu + \frac{1}{2m_+}\epsilon_{\mu\nu\lambda} f^\mu \partial^\nu f^\lambda, \qquad (30)$$

once again in the limit where all spatial derivatives are ignored. Since Eqs. (22) and (23) were soldered to yield Eq. (2), it is natural to think that Eqs. (29) and (30) should be soldered to yield Eq. (28). This is indeed true as will now be shown. Indeed the soldering mechanism leads to an equivalent Lagrangian (28) with the following identifications:

$$A_\mu = f_\mu - g_\mu ,$$

$$m_+ - m_- = \theta, \qquad (31)$$

$$m_+ m_- = m^2$$

which is highly reminiscent of the quantum mechanical analysis. To begin with the soldering, consider the gauging of the following symmetry:

$$\delta f^\mu = \delta g^\mu = \epsilon^{\mu\sigma\lambda}\partial_\sigma \alpha_\lambda . \qquad (32)$$

Under these transformations, the anti-self- and self-dual Lagrangians change as

$$\delta\mathcal{L}_{\pm}=J_{\pm}^{\mu\nu}\partial_{\mu}\alpha_{\nu}, \tag{33}$$

where the currents are given by

$$J_{\pm}^{\rho\sigma}(h)=\epsilon^{\mu\rho\sigma}h_{\mu}\pm\frac{1}{m_{\pm}}\partial^{[\rho}h^{\sigma]},\quad h=f,g. \tag{34}$$

Next, the soldering field $B_{\rho\sigma}$, which is a two form gauge field transforming as

$$\delta B_{\rho\sigma}=\partial_{\rho}\alpha_{\sigma}-\partial_{\sigma}\alpha_{\rho}-\frac{1}{2M}(\partial_{\rho}\epsilon_{\sigma\eta\zeta}-\partial_{\sigma}\epsilon_{\rho\eta\zeta})\partial^{\eta}\alpha^{\zeta}, \tag{35}$$

is introduced. In analogy with the quantum mechanical analysis, it is possible to define a modified Lagrangian,

$$\mathcal{L}=\mathcal{L}_{+}+\mathcal{L}_{-}+\frac{1}{2}B^{\rho\sigma}B_{\rho\sigma}-\frac{1}{2}B^{\rho\sigma}(J_{\rho\sigma}^{+}(f)+J^{-}\rho\sigma(g)) \tag{36}$$

which transforms as

$$\delta\mathcal{L}=\frac{1}{2M}\epsilon^{\mu\nu\lambda}\left[\left(f_{\lambda}+\frac{1}{m_{+}}\epsilon_{\lambda\rho\omega}f^{\rho\omega}\right)+\left(g_{\lambda}-\frac{1}{m_{-}}\epsilon_{\lambda\rho\omega}g^{\rho\omega}\right)\right]\times[\epsilon_{\mu\sigma\beta}\partial_{\nu}\partial^{\sigma}\alpha^{\beta}] \tag{37}$$

where

$$M=\frac{m_{+}m_{-}}{m_{+}-m_{-}}. \tag{38}$$

It is useful to observe that Eq. (37) vanishes for $m_{+}=m_{-}$. In that case the Lagrangian becomes gauge invariant under the transformations (32). The auxiliary $B_{\rho\sigma}$ field can be eliminated from Eq. (36) in favor of the original variables by using the equation of motion. The final Lagrangian then turns out to be the Proca model with the basic field as $A_{\mu}=f_{\mu}-g_{\mu}$. Incidentally, following our system of ignoring spatial derivatives, the Proca model just reduces to the bidimensional harmonic oscillator (3). Likewise, Eqs. (30) and (29) with $m_{+}=m_{-}$ can be identified with Eq. (20) and its dual partner. The soldering in the latter case leads to Eq. (3) which provides another correspondence between the quantum mechanical and field theoretical models. In the same spirit it may be realized that Eq. (4) would be the analogue of the Maxwell-Chern-Simons (MCS) theory. The equivalence of Eq. (4) with Eq. (20) therefore indicates a similar connection between the MCS theory and the self-dual model (29)—a fact which has been established earlier using various approaches [3,9].

Coming back to the soldering mechanism for different masses ($m_{+}\neq m_{-}$), it is seen that the variation (37) is nonzero. It is possible to make further alterations to Eq. (36) so that the new Lagrangian is gauge invariant. Such alterations invariably require terms involving derivatives of the soldering field $B_{\rho\sigma}$. In that case a simple elimination of this field in favor of the other fields, by using the equations of motion, would not be possible. That would defeat our purpose of recasting the Lagrangian in terms of the difference ($f_{\mu}-g_{\mu}$), a form in which it would be manifestly gauge invariant leading to a new structure. It is now observed that by relaxing the requirement of gauge invariance to be only onshell, in which case

$$f_{\mu}=-\frac{1}{m_{+}}\epsilon_{\mu\nu\lambda}\partial^{\nu}f^{\lambda}, \tag{39}$$

$$g_{\mu}=+\frac{1}{m_{-}}\epsilon_{\mu\nu\lambda}\partial^{\nu}g^{\lambda}, \tag{40}$$

then $\delta\mathcal{L}$ in Eq. (37) indeed vanishes and the Lagrangian is gauge invariant. This is reminiscent of the quantum mechanical analysis.

Returning to a description in terms of the original variables is now possible by eliminating $B_{\rho\sigma}$, which acts as an auxiliary field, from Eq. (36),

$$B_{\rho\sigma}=\frac{1}{2}(J_{\rho\sigma}^{+}+J_{\rho\sigma}^{-}). \tag{41}$$

It should be mentioned that this solution is compatible with the variation (35) since

$$\frac{1}{2}\delta(J_{+}^{\rho\sigma}(f)+J_{-}^{\rho\sigma}(g))=\partial_{\rho}\alpha_{\sigma}-\partial_{\sigma}\alpha_{\rho}-\frac{1}{2M}(\partial_{\rho}\epsilon_{\sigma\eta\zeta}-\partial_{\sigma}\epsilon_{\rho\eta\zeta})\partial^{\eta}\alpha^{\zeta}=\delta B_{\rho\sigma}.$$

Inserting the solution (41) in Eq. (36) and using the on-shell conditions (39) and (40) one obtains the Chern-Simons-Proca Lagrangian (28) with the identifications (31).

A straightforward extension of the above analysis in $d=4k-1$ dimensions would lead to the soldering of the self- and anti-self-dual Lagrangians,

$$\mathcal{L}_{+}=\frac{1}{2m_{+}}\frac{1}{2k!}\epsilon_{\mu_1\cdots\mu_{2k-1}\lambda_1\cdots\lambda_{2k}}f^{\mu_1\cdots\mu_{2k-1}}\partial^{[\lambda_1}f^{\lambda_2\cdots\lambda_{2k}]}+\frac{1}{2}f_{\mu_1\cdots\mu_{2k-1}}f^{\mu_1\cdots\mu_{2k-1}}, \tag{42}$$

$$\mathcal{L}_{-}=-\frac{1}{2m_{-}}\frac{1}{2k!}\epsilon_{\mu_1\cdots\mu_{2k-1}\lambda_1\cdots\lambda_{2k}}g^{\mu_1\cdots\mu_{2k-1}}\partial^{[\lambda_1}g^{\lambda_2\cdots\lambda_{2k}]}+\frac{1}{2}g_{\mu_1\cdots\mu_{2k-1}}g^{\mu_1\cdots\mu_{2k-1}} \tag{43}$$

to yield the new Lagrangian

$$\mathcal{L}_S=\frac{1}{2}A^{\mu_1\cdots\mu_{2k-1}}A_{\mu_1\cdots\mu_{2k-1}}-\frac{1}{2M}\frac{1}{(2k-1)!}\epsilon^{\mu_1\cdots\mu_{2k-1}\sigma_1\cdots\sigma_{2k}}A_{\mu_1\cdots\mu_{2k-1}}\partial_{\sigma_1}A_{\sigma_2\cdots\sigma_{2k}}-\frac{1}{2.2k}\frac{1}{m_+m_-}F_{\sigma_1\cdots\sigma_{2k}}F^{\sigma_1\cdots\sigma_{2k}} \tag{44}$$

where

$$A_{\mu_1\cdots\mu_{2k-1}}=f_{\mu_1\cdots\mu_{2k-1}}-g_{\mu_1\cdots\mu_{2k-1}}$$

and

$$F^{\sigma_1\cdots\sigma_{2k}}=\partial^{[\sigma_1}A^{\sigma_2\cdots\sigma_{2k}]},$$

where in the latter expression antisymmetrization is done with respect to all the indices in the square bracket and M is as defined in Eq. (38). Note that the basic variables (f,g) are $(2k-1)$-form fields. For identical masses , $m_+=m_-$, the generalized Proca model is obtained.

Next let us discuss the factorizability property. As noted in [10][1] the equation of motion following from Eq. (28) factorizes as

$$\left[g^{\mu}_{\sigma}\mp\left(\frac{1}{m_\pm}\right)\epsilon^{\lambda\mu}_{\sigma}\partial_\lambda\right]\left[g^{\rho}_{\mu}\pm\left(\frac{1}{m_\mp}\right)\epsilon^{\nu\rho}_{\mu}\partial_\nu\right]A_\rho=0. \tag{45}$$

For identical masses $(m_+=m_-)$ this reduces to the Proca equation. The structure of the factorization has led to the claim that the massive modes in these models satisfy the self-duality condition. That this is not so is easily shown. Consider, for simplicity, the following generating functional for the Proca Lagrangian:

$$Z_P[j,J]=\int DA_\mu e^{-(1/2)\int[\mathcal{L}_P+\mathbf{A}_\mu j^\mu+\tilde{\mathbf{A}}_\mu J^\mu]d^3x}, \tag{46}$$

where the dual has also been introduced,

$$\tilde{A}_\mu=\frac{1}{m}\epsilon_{\mu\nu\lambda}\partial^\nu\mathbf{A}^\lambda. \tag{47}$$

The result of the Gaussian integration is

$$Z_P[j,J]=e^{-(1/2)\int[j_\mu+(1/m)\epsilon_{\mu\lambda\sigma}\partial^\lambda J^\sigma]C^{\mu\nu}[j_\nu+(1/m)\epsilon_{\nu\alpha\beta}\partial^\alpha J^\beta]}, \tag{48}$$

where

$$C^{\mu\nu}(x,y)=\frac{2}{(m^2+\Box)}\left[g^{\mu\nu}+\frac{1}{m^2}\partial^\mu\partial^\nu\right]\times\delta(x-y).$$

It is now easy to calculate the relevant correlation functions,

$$\langle A_\eta(x)A_\xi(y)\rangle=C_{\eta\xi}(x,y), \tag{49}$$

$$\langle A_\eta(x)A_\xi(y)\rangle=\langle\tilde{A}_\eta(x)\tilde{A}_\xi(y)\rangle+\frac{2}{m^2}\mathbf{g}_{\eta\xi}\delta(x-y), \tag{50}$$

$$\langle A_\eta(x)\tilde{A}_\xi(y)\rangle=\frac{2}{m}\cdot\frac{1}{(m^2+\Box)}\epsilon_{\eta\sigma\xi}\partial^\sigma\delta(x-y). \tag{51}$$

It is seen that all the correlation functions cannot be related modulo only local terms. Thus it is not possible to interpret $A_\mu=\tilde{A}_\mu$ operatorially. Hence the A_μ field cannot be regarded as self-dual.

The origin of the structure of the factorization in Eq. (45) is understood from the soldering analysis performed earlier. The two factors correspond to the self-dual and anti-self-dual modes, not in the model (28), but rather in the models (29) and (30), respectively. It is the soldering mechanism that has precisely combined these modes from distinct models with fields f_μ and g_μ to yield the new model (28) with the field $A_\mu=f_\mu-g_\mu$. This new field A_μ is altogether a separate entity which lacks the original symmetry properties.

To conclude, we have used the soldering formalism to abstract different quantum mechanical models starting from the basic harmonic oscillator. The analysis was directly extended to field theory in odd dimensions where exploiting the self-duality property, the soldering formalism was effectively employed. It was striking that all the results and interpretations found in the quantum mechanical examples had the exact analogues in the field theory.

[1]There is a sign error in this reference.

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