Scalar gluonium and instantons

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The impact of QCD instantons on scalar glueball properties is studied in the framework of an instanton-improved operator product expansion (IOPE) for the 0^{++} glueball correlation function. Direct instanton contributions are found to strongly dominate over those from perturbative fluctuations and soft vacuum fields. All IOPE sum rules, including the one involving a subtraction constant, show a high degree of stability and are, in contrast to previous glueball sum rules, consistent with the low-energy theorem for the zero-momentum correlator. The predicted glueball mass $m_G = 1.53 \pm 0.2$ GeV is less sensitive to the instanton contributions then the glueball coupling (residue) $f_G = 1.01 \pm 0.25$ GeV, which increases by about half an order of magnitude. Both glueball properties are shown to obey scaling relations as a function of the average instanton size and density.

I. INTRODUCTION

Glueballs, as the most immediate manifestation of gluonic self-interactions in the hadron spectrum, represent a key challenge for our understanding of nonperturbative Yang-Mills dynamics. Not surprisingly, therefore, theoretical interest in gluonium dates back to the early days of QCD [1] and has spurred intense research activity ever since. Estimates of glueball properties were obtained in a variety of approaches, ranging from model analyses [2] and steadily improving lattice simulations [3] to QCD sum rule calculations [4–8].

The QCD sum-rule technique, in particular, combines the advantages of an analytical approach with a firm and largely model-independent basis in QCD and should therefore be well suited for obtaining qualitative and quantitative insight into the glueball spectrum. During its early development, however, it became clear that this approach encounters conceptual and practical problems in the scalar glueball channel [4,5]. These problems, which have so far prevented fully consistent and conclusive sum-rule predictions, can be traced to the exceptionally strong coupling of the vacuum to the spin-0 glueball interpolators. The intense vacuum response generates nonperturbative violations of asymptotic freedom starting from unusually small distances $x \sim 0.02$ fm [5], and the power corrections of the conventional operator product expansion (OPE) are much too weak to account for this physics [4]. As a consequence, stability and mutual consistency of different sum rules (see below) are partially lacking, and serious difficulties are encountered in reconciling the sum rules with the low-energy theorem [4] which governs the long-distance behavior of the scalar glueball correlator.

In search of the origin for the apparently missing nonperturbative short-distance physics it is natural to expect that it may at least partly be provided by small (or "direct") instantons [4,5,9,10]. Indeed, as coherent vacuum gluon fields, instantons couple particularly strongly to the gluonic interpolators of the 0^{++} glueball channel. Nevertheless, they are neglected in the perturbatively calculated Wilson coefficients of the conventional OPE. An additional and perhaps more intuitive reason for expecting instanton physics to play a major role in the structure of scalar glueballs derives from

their exceptionally small size, $r_G \simeq 0.2$ fm, found in lattice [11] and instanton vacuum model [10] calculations. Since r_G is much smaller than the confinement scale and the size of heavier glueballs, it is rather unlikely that scalar glueballs are bound predominantly by (iterated) perturbative or by confinement forces. The attractive interactions mediated by instantons provide a suggestive alternative.

Motivated by the above considerations, our main objectives in this paper are to evaluate the direct instanton sector of the scalar glueball correlator by means of an instanton-improved OPE (IOPE) and to analyze the ensuing glueball sum rules. While analytical instanton calculations (at low energies) have long been hampered by insufficient knowledge of the instanton size distribution and notorious infrared problems, this impasse can nowadays be avoided by relying on the results of instanton vacuum model [12] and lattice [13] simulations for the required bulk features of the instanton distribution, i.e., for the average instanton size $\bar{\rho} \simeq (1/3)$ fm and density $\bar{n} \simeq (1/2)$ fm^{-4}. Despite remaining numerical discrepancies pertaining to the distribution of large-size instantons, these scales provide a solid foundation for our calculations below to which only small instantons of sizes $\rho \lesssim 0.5$ fm will contribute.

II. IOPE AND SUM RULES

Our study will be based on the correlation function

$$\Pi(-q^2) = i \int d^4x \, e^{iqx} \langle 0 | T \, O_S(x) O_S(0) | 0 \rangle \qquad (1)$$

with the interpolating field

$$O_S = \alpha_s G^a_{\mu\nu} G^{a,\mu\nu} \qquad (2)$$

carrying the quantum numbers of the scalar glueball. The standard OPE of this correlator, including perturbative Wilson coefficients up to $O(\alpha_s)$ and operator contributions up to dimension 8, is known to be [7] (up to polynomials in Q^2)

$$\Pi^{(\text{OPE})}(Q^2) = Q^4 \ln\left(\frac{Q^2}{\mu^2}\right)\left[-2\left(\frac{\alpha_s}{\pi}\right)^2\left[1 + \frac{59}{4}\frac{\alpha_s}{\pi}\right]\right.$$
$$+ b_0\left(\frac{\alpha_s}{\pi}\right)^3\ln\left(\frac{Q^2}{\mu^2}\right)\right] + 4\alpha_s\left[1 + \frac{49}{12}\frac{\alpha_s}{\pi}\right]\langle\alpha_s G^2\rangle$$
$$- \frac{\alpha_s^2 b_0}{\pi}\langle\alpha_s G^2\rangle\ln\left(\frac{Q^2}{\mu^2}\right) + \frac{1}{Q^2}\left[8\alpha_s^2\langle gG^3\rangle L^{7/11}\right.$$
$$\left.- 58\alpha_s^3\langle gG^3\rangle L^{7/11}\ln\left(\frac{Q^2}{\mu^2}\right)\right]$$
$$+ 8\pi\alpha_s\frac{1}{Q^4}\langle\alpha_s^2 G^4\rangle \tag{3}$$

$(Q^2 = -q^2)$ where $b_0 = 11N_c/3 - 2N_f/3$ is the leading-order contribution to the QCD β function, $L(Q^2) = \ln(Q/\Lambda)/\ln(\mu_0/\Lambda)$, and α_s is the QCD running coupling at one loop. We will use the parameter values $\Lambda = 0.12$ GeV, $\mu = 0.5$ GeV and the condensate values $\langle\alpha_s G^2\rangle = 0.04$ GeV^4, $\langle gG^3\rangle = -1.5\langle\alpha_s G^2\rangle^{3/2}$ of Ref. [7]. For the four-gluon condensate, finally, we adopt the standard approximation

$$\langle\alpha_s^2 G^4\rangle \equiv 14\langle(\alpha_s f_{abc}G^b_{\mu\rho}G^{\rho c}_\nu)^2\rangle - \langle(\alpha_s f_{abc}G^b_{\mu\nu}G^c_{\rho\lambda})^2\rangle$$
$$\simeq \frac{9}{16}\langle\alpha_s G^2\rangle^2. \tag{4}$$

In addition to the perturbative contributions given above, the Wilson coefficients receive nonperturbative contributions from direct instantons which have so far been neglected in the gluonium sum rules (a partial estimate was given in Ref. [9]). Analogous contributions were found to be important in several nucleon [14] and pion [15] sum rules. As noted there, effects of multi-instanton correlations can be neglected in the short-distance expansion since the relevant distances $|x| \sim |Q^{-1}| \lesssim 0.2$ fm are much smaller than the average separation $\bar{R} \sim 1$ fm between instantons in the vacuum. The diluteness of the instanton vacuum distribution, which is a consequence of $\bar{\rho}/\bar{R} \ll 1$, further reduces the impact of multi-instanton correlations and keeps the separate instantons approximately undeformed.

To leading order in the semiclassical approximation, the instanton contribution to Eq. (1) can thus be calculated by standard techniques [4] from the $O(\hbar^0)$ component of the gluon propagator in the instanton background [17] and reads

$$\Pi^{(I+\bar{I})}(Q^2) = 2^5\pi^2\bar{n}\bar{\rho}^4 Q^4 K_2^2(Q\bar{\rho}). \tag{5}$$

(K_2 is a McDonald function.) Since the average instanton size $\bar{\rho} \simeq (1/3)$ fm is small compared to $\Lambda_{\text{QCD}}^{-1}$, $O(\hbar)$ corrections to Eq. (5) are suppressed by the large instanton action $S_I(\bar{\rho}) \sim 10\hbar$. Instanton contributions to the Wilson coefficients of power corrections carry additional inverse powers of the relatively large glueball mass scale and are therefore also expected to be small (see Ref. [16] for a more detailed discussion).

Various sum rules can be constructed from the Borel transform of weighted moments of the glueball correlator:

$$\mathcal{R}_k(\tau) = \hat{B}[(-Q^2)^k\Pi(Q^2)]. \tag{6}$$

Typically, one considers $k \in \{-1,0,1,2\}$. The corresponding expressions for $\mathcal{R}_k^{(\text{OPE})}$ (together with the explicit form of the Borel operator $\hat{B}$) are given in Ref. [7]. The instanton contributions $\mathcal{R}_k^{(I+\bar{I})}$ are obtained recursively, via

$$\mathcal{R}_k^{(I+\bar{I})}(\tau) = \left(\frac{-\partial}{\partial\tau}\right)^{k+1}\mathcal{R}_{-1}^{(I+\bar{I})}(\tau) \quad (k \geq -1) \tag{7}$$

from $\mathcal{R}_{-1}^{(I+\bar{I})}$, which can be calculated in closed form $[x \equiv \bar{\rho}^2/(2\tau)]$:

$$\mathcal{R}_{-1}^{(I+\bar{I})}(\tau) = -2^6\pi^2\bar{n}x^2 e^{-x}\left[(1+x)K_0(x)\right.$$
$$\left. + \left(2 + x + \frac{2}{x}\right)K_1(x)\right] + 2^7\pi^2\bar{n}. \tag{8}$$

The sum $\mathcal{R}_k^{(\text{OPE})} + \mathcal{R}_k^{(I+\bar{I})}$ constitutes the IOPE. Note that we have removed the constant subtraction term $-\Pi^{(I+\bar{I})}(0) = -2^7\pi^2\bar{n}$ in Eq. (8) because it originates from soft instanton contributions which do not belong to the OPE coefficients. Double-counting of soft instanton physics is thereby excluded since the instanton contributions (7) do not contain powers τ^n with $n > -(k+3)$.

In order to write down the sum rules, we have to match the IOPE expressions to their "phenomenological" counterparts, which are derived from the twice subtracted dispersion relation

$$\Pi^{(\text{phen})}(Q^2) = \Pi^{(\text{phen})}(0) - \Pi^{(\text{phen})\prime}(0)Q^2$$
$$+ \frac{(Q^2)^2}{\pi}\int_0^\infty ds\frac{\text{Im}\,\Pi^{(\text{phen})}(s)}{s^2(s+Q^2)} \tag{9}$$

by parametrizing the spectral function in terms of a pole contribution and an effective continuum. Following standard procedure, the latter is obtained from the dispersive cut of the theoretical side and starts at an effective threshold s_0. Thus we have

$$\text{Im}\,\Pi^{(\text{phen})}(s) = \pi f_G^2 m_G^4\delta(s - m_G^2) + [\text{Im}\,\Pi^{(\text{OPE})}(s)$$
$$+ \text{Im}\,\Pi^{(I+\bar{I})}(s)]\theta(s - s_0). \tag{10}$$

As a consequence of the exceptional size of the instanton contributions to the scalar glueball correlator, their contributions to the continuum are an indispensable part of Eq. (10) and will turn out to play an essential role in the subsequent analysis. Explicitly, we find

$$\text{Im}\,\Pi^{(I+\bar{I})}(s) = -2^4\pi^4\bar{n}\bar{\rho}^4 s^2 J_2(\sqrt{s}\bar{\rho})Y_2(\sqrt{s}\bar{\rho}), \tag{11}$$

where the J_2 (Y_2) are Bessel (Neumann) functions. (A more detailed multipole analysis, allowing for neighboring and mixed quarkonium resonances, will be relegated to Ref. [16].)

By equating the phenomenological Borel moments, obtained from Eqs. (6) and (9), to the corresponding IOPE expressions one finally obtains the sum rules

$$\frac{\mathcal{R}_k(\tau,s_0)}{m_G^{2+2k}} = f_G^2 m_G^2 e^{-\tau m_G^2} \qquad (12)$$

with

$$\mathcal{R}_k(\tau,s_0) = \sum_{X=\text{OPE},I+\bar{I}} [\mathcal{R}_k^{(X)}(\tau) - \mathcal{R}_k^{(X\text{-cont})}(\tau,s_0)]$$
$$+ \delta_{k,-1}\Pi^{(\text{phen})}(0) \qquad (13)$$

and

$$\mathcal{R}_k^{(X\text{-cont})}(\tau,s_0) = \frac{1}{\pi}\int_{s_0}^{\infty} ds\, s^k \,\text{Im}\,\Pi^{(X)}(s)e^{-s\tau}. \qquad (14)$$

Note that the higher moments weight the higher-mass region of the spectral function more strongly and thus receive enhanced contributions from the relatively heavy (see below) glueball pole. The subtraction constant $\Pi^{(\text{phen})}(0)$ in the $\mathcal{R}_{-1}$ sum rule (regularized by removing the high-momentum contributions) can be related to the gluon condensate by the low-energy theorem (LET) [4]

$$\Pi(0) = \frac{32\pi}{b_0}\langle\alpha G^2\rangle. \qquad (15)$$

This relation provides an important consistency check for the sum-rule results, as we will discuss below.

III. SUM RULE ANALYSIS

The quantitative analysis of the sum rules amounts to determining those values of the hadronic parameters in Eq. (10) for which both sides of Eq. (12) optimally match in the fiducial Borel domain. Towards large τ this domain is bounded by keeping the contribution of the highest-dimensional operator $(\alpha_s^2 G^4)$ to $\mathcal{R}_k$ below 10% and requiring multi-instanton contributions to be negligible. The latter requirement will be (conservatively) implemented by demanding $\tau \lesssim 1$ GeV^{-2}. Towards small τ we prescribe that the continuum contributions do not exceed 50% of the $\mathcal{R}_k$.

The standard optimization procedure followed in previous analyses determined only the glueball mass m_G and coupling f_G from matching the sum rules, while the threshold s_0 had to be found by other means (e.g., by finite-energy sum rules [7] or stability criteria [8]). The IOPE sum rules turn out to be stable enough, however, to determine s_0 together with the resonance parameters m_G and f_G from the same sum rule. This is the procedure which we will adopt below.

We start by analyzing the $\mathcal{R}_0$ sum rule [i.e., Eq. (12) with $k=0$]. Figure 1 shows both sides of the optimized sum rule and separately the three components (OPE with subtracted OPE continuum, instanton contribution, and instanton continuum) which make up its left-hand side. The matching between both sides of the sum rule is almost perfect over the whole fiducial region. Comparing standard OPE and instan-
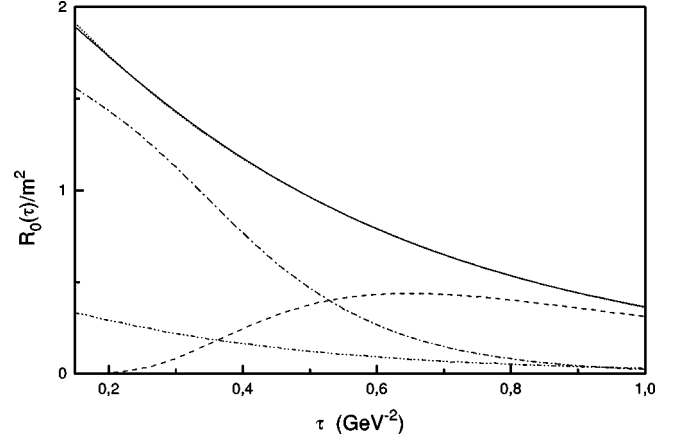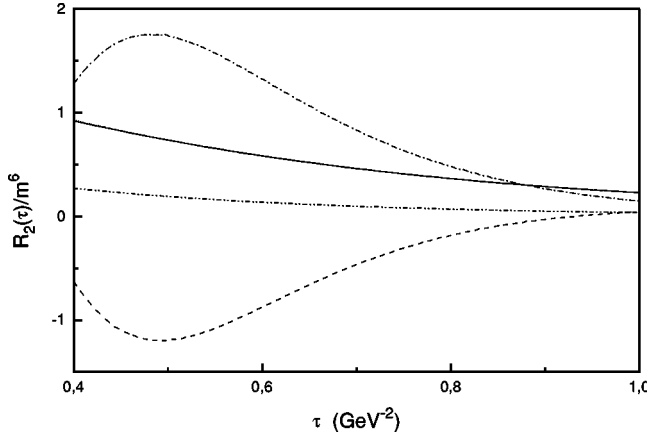

FIG. 1. The right-hand side $f_G^2 m_G^2 \exp(-m_G^2\tau)$ of the $\mathcal{R}_0$ sum rule (dotted), compared with the optimized left-hand side $\mathcal{R}_0(\tau,s_0)/m_G^2$ (solid line) and its three components (all in units of GeV^4): the conventional OPE $\mathcal{R}_0^{(\text{OPE})}(\tau,s_0)/m_G^2$ (dash-double-dotted), the instanton contribution $\mathcal{R}_0^{(I+\bar{I})}(\tau)/m_G^2$ (dashed), and the instanton continuum part $-\mathcal{R}_0^{(I\text{-cont})}(\tau,s_0)/m_G^2$ (dash-dotted).

ton (including continuum) contributions shows that the latter are about 5 times larger. Thus, the instanton contributions strongly dominate over the whole fiducial region and increase the predictions for f_G^2 by about a factor of 5, resulting in $f_G = 1.14$ GeV. A similarly strong enhancement of f_G was found in instanton vacuum model calculations [10]. The prediction for the glueball mass, $m_G = 1.40$ GeV, on the other hand, differs surprisingly little from what is obtained without the instanton part. The continuum threshold becomes $s_0 = 5.1$ GeV^2.

Figure 1 furthermore reveals that the instanton contributions to the unitarity cut are indispensable for generating an exponential τ behavior below $\tau \simeq 0.8$ GeV^{-2}, and thus for an acceptable fit to the pole contribution. (For larger values of τ the continuum contributions are practically negligible and a fit to the instanton contribution alone would become possible, although mostly outside of the fiducial domain and with about 20% smaller values for m_G and f_G^2.) It should also be noted that the hard nonperturbative instanton physics begins to enter $\mathcal{R}_0$ at much smaller τ than the soft condensate contributions, thereby confirming the existence of an exceptionally large mass scale in the scalar glueball channel [5].

The analysis of the remaining sum rules (which all show a high degree of stability) confirms the above observations about the role of the instanton contributions. As an additional example, we plot in Fig. 2 the separate contributions to the $\mathcal{R}_2$ sum rule and the fit of both sides, which is again excellent. Since the instanton contribution is somewhat less pronounced than in $\mathcal{R}_0$, we find a smaller value for the coupling: $f_G = 1.01$ GeV. The result for the glueball mass increases to $m_G = 1.53$ GeV, and the threshold $s_0 = 4.89$ GeV^2 is slightly reduced. The results of the $\mathcal{R}_{-1}$ and $\mathcal{R}_1$ sum rules confirm the tendency of lower moments to predict somewhat smaller masses and somewhat larger couplings [while maintaining consistency with the low-energy theorem (15)]. The predictions of the higher moments should be more reliable,

FIG. 2. Same as Fig. 1 for the $\mathcal{R}_2$ sum rule.

FIG. 3. The square root of the ratio $\mathcal{R}_2(\tau)/\mathcal{R}_1(\tau)$. The weak τ dependence confirms the high consistency between the $\mathcal{R}_1(\tau)$ and $\mathcal{R}_2(\tau)$ sum rules.

however, because they receive stronger pole contributions.

The $\mathcal{R}_{-1}$ sum rule has played both conceptually and practically a special role since it contains the subtraction constant $\Pi(0)$ which dominates the power corrections of the conventional OPE (in the fiducial region). Attempts to fit the resulting, almost flat τ behavior to the exponential pole contribution inevitably generate very small pole masses, at least half an order of magnitude smaller than those predicted by the other sum rules. This well-known inconsistency (and the need to abandon the $\mathcal{R}_{-1}$ sum rule in practice) is largely resolved by the massive instanton contributions. Their strong decay yields excellent fits and pole masses of the same order as those obtained from the higher moments. Still, the $\mathcal{R}_{-1}$ sum rule remains probably least suited for quantitative predictions since it is most sensitive to the inaccurately known value of the gluon condensate and least sensitive to the pole contribution. The mutual agreement of all four IOPE sum rules (in the typical range of uncertainty for sum rule results) and their consistency with the low-energy theorem,[1] however, is reassuring and of considerable conceptual importance.
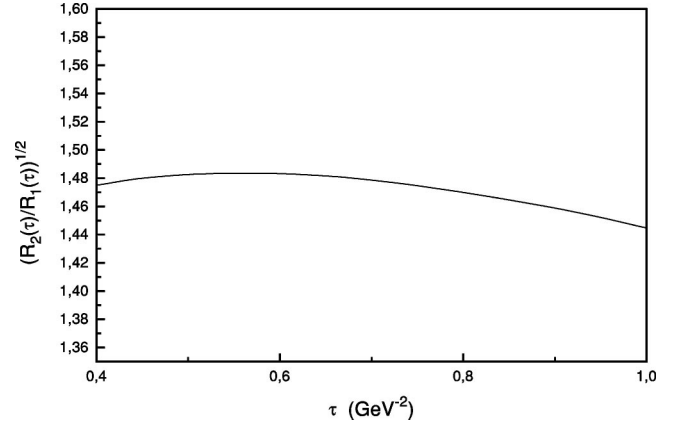
For a quantitative consistency check between the predictions of different sum rules, we have evaluated the τ-dependent mass function

$$m_G^{(1,2)}(\tau) \equiv \sqrt{\frac{\mathcal{R}_2(\tau,s_0)}{\mathcal{R}_1(\tau,s_0)}}. \qquad (16)$$

Figure 3 shows that it deviates less than 2% from the constant m_G over the whole fiducial region, indicating a high degree of compatibility between the sum rules.

We have also found that the instanton contributions to the $\mathcal{R}_k$ by themselves can generate stable sum rules. Their approximately exponential τ behavior matches very well to the

pole term, although with about 20% smaller glueball masses than those obtained from the full sum rules. This indicates that instantons alone can (over) bind the scalar glueball, in agreement with the findings of instanton vacuum models [10] (which also show a tendency towards smaller glueball masses).

IV. DISCUSSION AND CONCLUSIONS

We evaluated and analyzed the instanton contributions both to the OPE of the scalar glueball correlator (or, more precisely, to the Wilson coefficient of the unit operator) and to the continuum part of its phenomenological spectral-function model, and we solved the corresponding QCD sum rules. The previously neglected instanton contributions turn out to be dominant and render the IOPE sum rules the first overall consistent set in the scalar glueball channel.

In particular, the IOPE resolves two long-standing flaws of the earlier sum rules: the mutual inconsistency between different Borel moments and the inconsistency with the low-energy theorem for the zero-momentum correlator. Even the previously deficient and usually discarded lowest-moment ($\mathcal{R}_{-1}$) sum rule becomes consistent both with those from the higher moments and with the low-energy theorem. Any evidence for a low-lying ($m \ll 1$ GeV) gluonium state (or a state strongly coupled to gluonic interpolators), sometimes argued for on the basis of this sum rule [4,5], is thereby rendered obsolete.

The most dramatic phenomenological impact of the direct instanton contributions is associated with f_G^2, the residuum of the glueball pole. Due to the exceptional size of the instanton contributions, its value increases by about a factor of 5. Taking the quantitative predictions of the $\mathcal{R}_2$ sum rule to be the most reliable ones, we obtain $m_G = 1.53 \pm 0.2$ GeV (in accord with recent lattice results [3]) and $f_G = 1.01 \pm 0.25$ GeV, where the errors are estimated from the uncertainties of the input parameters and the spread between the individual sum rules. Potential ramifications for experimental glueball searches will be considered in a forthcoming publication.

All four IOPE sum rules show an unprecedented degree

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[1]We have checked that the zero-momentum correlator $\Pi^{(\mathrm{reg})}(0)$, obtained from the UV-regularized, unsubtracted dispersion relation with the spectral density (10) (where m_G, f_G, and s_0 have the predicted values) satisfies the low-energy theorem (15) for all four sum rules in the range of uncertainty introduced by the inaccurately known value of the gluon condensate [16].

of stability and allow for a simultaneous 3-parameter fit to the glueball mass, its coupling, and the continuum threshold. The stability region extends far beyond the fiducial τ interval and renders, as a side effect, the IOPE sum rule results almost insensitive to the precise boundaries of the fiducial domain. Most importantly, however, the high stability indicates that the IOPE provides a rather complete description of the short-distance glueball correlator.

A crucial contribution to the IOPE sum rules arises from the discontinuity of the instanton terms in the extended continuum part of the spectral functions. (The rough estimate of instanton contributions to the $\mathcal{R}_0$ sum rule in Ref. [9] missed this contribution.) In addition to substantially improving the overall consistency and stability of the sum rules, the richer structure on the phenomenological sides also sheds new light on the spectral content of the scalar glueball correlator. For once, the quantitative sum-rule analysis reveals that the instanton contributions, together with the weaker perturbative terms, counterbalance the pole contribution. This leads to an improved description of the correlator towards low momenta and thereby reconciles the sum rules with the low-energy theorem (15), a stringent consistency check[2] in the $Q \to 0$ limit.

A remarkable interplay between perturbative and nonperturbative physics can also be seen in the opposite limit, i.e., at short distances. As a consequence of the improved continuum description, the instanton contributions to the correlator remain effective at small τ and stay finite even for $\tau \to 0$ (in contrast to their contributions to the IOPE in the same limit, which suffer the expected suppression associated with funnelling a sizeable momentum through the coherent instanton field). This indicates that small-instanton physics accounts not only for much of the ground-state contribution but also for part of the higher-lying spectral strength (in the sense of a generalized quark-hadron duality) in the scalar glueball correlator. Thus, the spectral distribution favored by the IOPE sum rules seems to imply a rather prominent role for instanton-induced effects in excited glueball states (or in multiparticle states with strong coupling to the energy-momentum tensor). The phenomenological impact of these results, which do not depend on details of the IOPE and should therefore be rather robust, deserves further study.

In contrast to previously studied IOPE sum rules for quark-based correlators [14,15], those for the scalar glueball are the first where (i) the instanton contributions do not enter via topological quark zero-modes and (ii) the sum rules reach a satisfactory (though not excellent) level of consistency even without any perturbative and soft contributions, i.e., with the instanton terms alone. The latter result explains why the instanton liquid model yields scalar glueball properties similar to those obtained above [10], and can be traced to both the exceptional strength and the particular shape (mainly the curvature) of the instanton contributions. In combination, those properties produce an approximately expo-

nential τ dependence which extends far beyond the fiducial region and fairly well matches the ground-state signal without any perturbative and condensate contributions.

The predominance and approximate self-sufficiency of the instanton contributions has a suggestive physical interpretation: it indicates that instantons may generate the bulk of the attractive forces which bind the scalar glueball. Moreover, and in contrast to the instanton liquid model of Ref. [10], the IOPE allows to consistently compare the instanton contributions with those of the remaining soft and perturbative fields, and to thereby judge their relative importance. At present, the IOPE seems to be the only controlled and analytical framework in which this can be achieved. The combined effect of the soft and perturbative contributions turns out to be repulsive and increases, consistently in all sum rules, the glueball mass by about 20%.

Finally, closer inspection of the IOPE sum rules reveals another, quite striking instanton effect: the scales of the predicted 0^{++} glueball properties turn out to be approximately set by the bulk features of the instanton size distribution. Indeed, neglecting the standard OPE contributions one finds that the glueball parameters scale as[3]

$$m_G \sim \bar{\rho}^{-1}, \tag{17}$$

$$f_G^2 \sim \bar{n}\bar{\rho}^2. \tag{18}$$

In the case of proportionality between the glueball size r_G and its Compton wavelength, one would obtain another scaling relation

$$r_G \sim \bar{\rho}, \tag{19}$$

which could explain the small values $r_G \sim 0.2$ fm found on the lattice [11].

Conceptually, the main virtue of the above scaling relations lies in establishing an explicit link between fundamental vacuum and hadron properties. Although strong interdependences between QCD vacuum and hadron structure are expected on general grounds, such scaling relations seem to have not been encountered previously. They could be of practical use, e.g., for the test of instanton vacuum models, to provide constraints for glueball model building, or generalized to finite temperature and baryon density, where the scales of the instanton distribution change.

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[2]On the lattice, the feasibility of this check is compromised by finite-size effects.

[3]It is worth noting that the arguments leading to these scaling relations take advantage of the analytical character of the IOPE sum rules. The scaling would be more difficult to uncover in numerical approaches (as in lattice simulations, where in addition the instanton distribution parameters cannot easily be varied).

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