Quintessence and Born-Infeld cosmology

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Recent observations suggest that the universe is in a state of accelerated cosmic expansion. Herewith we investigate this scenario within the Born-Infeld theory, which has been employed to describe open strings ending on D-branes. A multidimensional model with a topology $R\times S^3\times S^d$, a cosmological constant, dust matter, and gauge fields is considered for that purpose. Two situations are subsequently discussed, according to whether string effects are (i) dominant or (ii) induce perturbations in the gauge field sector. Studying the set of equations governing the cosmological dynamics, we find that Born-Infeld cosmology can be compatible with the presently measured acceleration, together with a compactified internal space. This is shown to depend on the gauge field components in the internal dimensions as well as string modifications to the gauge matter sector. Furthermore, we argue regarding situation (i) that quintessence could constitute a transient stage.

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A number of recent measurements point to the growing evidence that the expanding universe is currently accelerating. Those observations comprise the luminosity-redshift relation observed from type Ia supernovae [1]. Such a conclusion has been strengthened by the study of the accoustic peak in the cosmic microwave background [2] and the mass power spectrum [3]. One obvious candidate to explain this effect is the vacuum energy density or cosmological constant. Another interesting alternative is constituted by a dynamical vacuum energy or quintessence model [4,5]. These include a scalar field slowly evolving down its shallow potential whose cosmological influence was until recently overdamped by the expansion of the universe. With potentials of the exponential type, it was possible to identify conditions under which the present deceleration parameter $q_0\equiv -\ddot{a}a/\dot{a}^2$ is negative [5,6]. However, the vast majority of these quintessence models usually lead to a fine-tuning problem [7]. Other models involving scalar fields coupled to gravity without a potential were investigated in Ref. [8]. From the point of view of particle physics, other suggestions have also been advanced [9], namely within Einstein-Yang-Mills (EYM) cosmologies [10].

Our knowledge of cosmologies with the above type of acceleration can be enlarged with alternative scenarios retrieved from M or string theory [11]. In this context, several models have been investigated [9,12,13], although Born-Infeld (BI) cosmologies have not yet been examined. Actions of the BI type have recently been the subject of wide interest [14]. This comes from the result that the effective action for the open string ending on D-branes can be written in a BI form. In that respect, BI cosmological solutions could assist in the understanding of brane dynamics regarding the universe evolution. Papers discussing cosmological models with $U(1)$ BI matter can be found in Ref. [15]. Moreover, non-Abelian BI cosmologies were investigated in Ref. [16]. Hence, admitting that the conditions for the present accelerated cosmic expansion could have been determined by brane or strings, gauge, and gravity field effects at prior stages, would a BI cosmological framework be compatible with recent observational data?

In order to address this issue, we consider a D-dimensional space-time M^D, whose action includes a BI matter sector [14–16] as

$$S[\hat{g}_{\hat{\mu}\hat{\nu}},\hat{A}_{\hat{\mu}}]=-\int_{\mathcal{M}^D} d\hat{x}\sqrt{-\hat{g}}\left[\frac{\hat{R}-2\hat{\Lambda}}{16\pi\hat{k}}+\frac{\hat{\beta}^2}{4\pi}(\hat{\mathfrak{R}}-1)\right], \tag{1}$$

with

$$\hat{\mathfrak{R}}=\left[1+\frac{1}{2\hat{\beta}^2\hat{\epsilon}^2}\mathrm{Tr}\hat{F}_{\hat{\mu}\hat{\nu}}\hat{F}^{\hat{\mu}\hat{\nu}}-\frac{1}{16\hat{\beta}^4\hat{\epsilon}^4}(\mathrm{Tr}\tilde{F}_{\hat{\mu}\hat{\nu}}\hat{F}^{\hat{\mu}\hat{\nu}})^2\right]^{1/2}. \tag{2}$$

Notice that $\hat{g}_{\hat{\mu}\hat{\nu}}$ is the $D=(4+d)$-dimensional metric, and $\hat{R}$, $\hat{\epsilon}$, $\hat{k}$, and $\hat{\Lambda}$ are, respectively, the scalar curvature, gauge coupling, gravitational, and cosmological constants in D dimensions. In addition, $\hat{F}_{\hat{\mu}\hat{\nu}}=\partial_{\hat{\mu}}\hat{A}_{\hat{\nu}}-\partial_{\hat{\nu}}\hat{A}_{\hat{\mu}}+[\hat{A}_{\hat{\mu}},\hat{A}_{\hat{\nu}}]$, $\hat{A}_{\mu}$ denotes the components of the gauge field, $\hat{\beta}$ is the BI maximal field strength [16], and $\tilde{F}^{\mu\nu}$ is the dual of $F_{\mu\nu}$. We write $\mathcal{M}^D=M^4\times I^d$, with M^4 being the four-dimensional space-time and I^d a d-dimensional compact space. For definiteness, let us choose $\mathcal{M}^{4+d}=\mathbf{R}\times S^3\times S^d$, where $S^3(S^d)$ is the three-(d-) dimensional sphere. The group of spatial homogeneity and isotropy is, in this case, $G^{\mathrm{HI}}=SO(4)\times SO(d+1)$, while the group of spatial isotropy is $H^{\mathrm{I}}=SO(3)\times SO(d)$. The most general form of an $SO(4)\times SO(d+1)$-invariant metric is

$$\hat{g}=-\tilde{N}^2(t)dt^2+\tilde{a}^2(t)\sum_{i=1}^{3}\omega^i\omega^i+b^2(t)\sum_{m=4}^{d+3}\omega^m\omega^m, \tag{3}$$

where the scale factors $\tilde{a}(t),b(t)$ and the lapse function $\tilde{N}(t)$ are arbitrary nonvanishing functions of time.

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It is further needed to find gauge fields which are consistent with the above spatial homogeneity and isotropy. A most useful class are the so-called *symmetric* fields, i.e., fields invariant up to a gauge transformation. These have been thoroughly employed before (see Refs. [10,17]). The *Ansatz* for the $SO(4)\times SO(d+1)$-symmetric gauge field within a gauge group $\hat{K}=SO(N)$, $N\leqslant 3+d$, is

$$\hat{A}=\frac{1}{2}\sum_{p,q=1}^{N-3-d} B^{pq}(t)\mathcal{T}^{(N)}_{3+d+p\;3+d+q}dt+\frac{1}{2}\sum_{1\leqslant i<j\leqslant 3}\mathcal{T}^{(N)}_{ij}\omega^{ij}+\frac{1}{2}\sum_{4\leqslant m<n\leqslant 3}\mathcal{T}^{(N)}_{mn}\tilde{\omega}^{m-3\;n-3}+\sum_{i=1}^{3}\left[\frac{1}{4}f_0(t)\sum_{j,k=1}^{3}\epsilon_{jik}\mathcal{T}^{(N)}_{jk}+\frac{1}{2}\sum_{p=1}^{N-3-d}f_p(t)\mathcal{T}^{(N)}_{i\;d+3+p}\right]\omega^i+\sum_{m=4}^{d+3}\left[\frac{1}{2}\sum_{q=1}^{N-3-d}g_q(t)\mathcal{T}^{(N)}_{m\;d+3+q}\right]\omega^m, \tag{4}$$

where $f_0(t)$, $f_p(t)$, $g_q(t)$, $B^{pq}(t)$ are arbitrary functions and $\mathcal{T}^{(N)}_{pq}$ are the generators of $\hat{K}$.

Notice that $f_0(t),\mathbf{f}=\{f_p\}$ represent the components in the four-dimensional space-time, while $\mathbf{g}=\{g_q\}$ denotes the components in the space I^d and $\hat{B}$ is an antisymmetric matrix. It is important to stress that we consider gauge field configurations with *nonvanishing* time-dependent components in *both* external and internal S^3 and S^d spaces.

The next step is to substitute the above *Ansätze* into the complete action, together with performing the conformal changes $\tilde{N}^2(t)=b_0^dN^2(t)/b^d(t)$, $\tilde{a}^2(t)=b_0^da^2(t)/b^d(t)$, where b_0 denotes the equilibrium value of b. Aiming to study the present dynamics, we focus on the variables a, b and the contributions to the effective potential from the BI term for the gauge fields. In what follows, we restrict ourselves to a static vacuum configuration of the gauge fields: $f_0=f_0^v$, $\mathbf{f}=\mathbf{f}^v$, $\mathbf{g}=\mathbf{g}^v$, with $\mathbf{f}$ and $\mathbf{g}$ being orthogonal. The notation $v_1\equiv V_1(f_0^v,\mathbf{f}^v)$ and $v_2\equiv V_2(\mathbf{g}^v)$ will be used throughout this paper. Complying to the choices above, we subsequently obtain the effective reduced action

$$S_{\rm eff}=-16\pi^2\int dtNa^3\left\{-\frac{3}{8\pi k}\frac{1}{a^2}\left[\frac{\dot{a}}{N}\right]^2+\frac{3}{32\pi k}\frac{1}{a^2}+\frac{1}{2}\left[\frac{\dot{\psi}}{N}\right]^2-\Omega\right\}, \tag{5}$$

where the potential includes exponential functions of ψ, which has been pointed to as a common feature in string or M theory [12],

$$\Omega(a,\psi)=\frac{1}{4\pi}v_db_0^de^{d\psi\gamma}\hat{\beta}^2e^{-2d\gamma\psi}\left(\left[1+e^{2d\gamma\psi}\frac{3}{\hat{\beta}^2\hat{\epsilon}^2a^4}v_1+e^{-4\gamma\psi}\frac{4}{b_0^4}\frac{d(d-1)}{8\hat{\epsilon}^2\hat{\beta}^2}v_2\right]^{1/2}-1\right)+e^{-d\gamma\psi}\left[-e^{-2\gamma\psi}\frac{1}{16\pi k}\frac{d(d-1)}{4}\frac{1}{b_0^2}+\frac{\Lambda}{8\pi k}\right], \tag{6}$$

with $k=\hat{k}/v_db_0^d$, $\epsilon^2=\hat{\epsilon}^2/v_db_0^d$, $\gamma=\sqrt{16\pi k/d(d+2)}$,$v_d$ is the volume of S^d for $b=1$, $\psi=\gamma^{-1}\ln(b/b_0)$ is the dilaton, dots denote time derivatives, and $\Lambda=v_db_0^d\hat{\Lambda}$.

The Friedmann equation is

$$\left(\frac{\dot{a}}{a}\right)^2=-\frac{1}{4a^2}+\frac{8\pi k}{3}\left(\frac{\dot{\psi}^2}{2}+\Omega(a,\psi)+\rho\right), \tag{7}$$

where we added $\rho=\rho_0(a_0^3/a^3)$ corresponding to dust energy density, with ρ_0 and a_0 being the energy density and scale factor at present, respectively. The equation for the dilaton is

$$\ddot{\psi}+\frac{\dot{a}}{a}\dot{\psi}=-\frac{\partial\Omega(a,\psi)}{\partial\psi}, \tag{8}$$

and the Raychaudhuri equation is written as

$$\ddot{a}=-\frac{8\pi k}{3}a\dot{\psi}^2-\frac{4\pi k}{3}a\left(\frac{\rho_0a_0^3}{a^3}\right)+\frac{8\pi k}{3}a\Omega+\frac{4\pi k}{3}a^2\frac{\partial\Omega}{\partial a}. \tag{9}$$

Furthermore, the deceleration parameter is given by

$$q=\frac{\frac{8\pi k}{3}a^2\dot{\psi}^2+\frac{4\pi k}{3}\frac{\rho_0a_0^3}{a^3}-\frac{8\pi k}{3}a^2\Omega}{-\frac{1}{4}+\frac{8\pi k}{3}a^2\dot{\psi}^2+\frac{8\pi k}{3}\frac{\rho_0a_0^3}{a^3}+\frac{8\pi k}{3}a^2\Omega}. \tag{10}$$

The presence of BI modifications to the gauge sector is manifest in the square root term in Ω.

The form of Ω implies an intricate analysis concerning whether an accelerated expansion exists in M^4 for large values of a. This state would be induced by the value of $\Omega(a\rightarrow\infty,\psi_{\rm ext})$, associated to a stationary compactification of I^d, where $\psi\simeq\psi_{\rm extremum}$. We are required to solve both a generic quartic and cubic equation (see Ref. [18]). The corresponding coefficients are all nontrivial functions of the parameters introduced in Eqs. (5),(6). In the limit $a\rightarrow\infty$ we may neglect the term in a^{-4} associated with v_1 as well as the term $a^2(\partial\Omega/\partial a)$ in Eqs. (7)–(10). However, the relevant information required to establish the existence of compactification and acceleration can be retrieved by analyzing the two regimes that will influence the cosmology with actions (5), (6).

We shall therefore consider the following limiting cases, between which the dynamics governed by Eqs. (7)–(9) interpolate.

Let us take the limit where

$$e^{-4\gamma\psi}\frac{4}{b_0^4}\frac{d(d-1)}{8\hat{\epsilon}^2\hat{\beta}^2}v_2 \gg 1,$$

corresponding to a situation where the internal dimensions are small. In this regime, BI effects clearly dominate over standard gauge (radiation) dynamics (see Refs. [10,16,17]). The potential Ω is approximated by

$$\Omega(a,\psi)\simeq\frac{1}{4\pi}\frac{\hat{\epsilon}}{\hat{\beta}}e^{-d\psi\gamma}e^{-2\gamma\psi}\frac{1}{b_0^2}\frac{\sqrt{d(d-1)}}{\sqrt{2}\epsilon^2}\sqrt{v_2} - e^{-d\gamma\psi}\left[e^{-2\gamma\psi}\frac{1}{16\pi k}\frac{d(d-1)}{4}\frac{1}{b_0^2}-\frac{\Lambda}{8\pi k}\right]. \tag{11}$$

Solutions where the internal dimensions are compactified occur if $e^{2\gamma\psi}=(A-C)(d+2)/B$, where

$$A\equiv\frac{1}{16\pi k}\frac{d(d-1)}{4}\frac{1}{b_0^2},\quad B\equiv\frac{\Lambda}{8\pi k},$$

$$C\equiv\frac{1}{4\pi}\frac{\hat{\epsilon}}{\hat{\beta}}\frac{1}{b_0^2}\frac{\sqrt{d(d-1)}}{\sqrt{2}\epsilon^2}\sqrt{v_2}.$$

If we write the solutions such that $\psi_{\rm ext}\simeq 0$, this will determine the cosmological constant to be fixed by

$$\Lambda=\frac{8\pi k(d+2)}{b_0^2}\left(\frac{d(d-1)}{8}-\sqrt{2}\frac{\hat{\epsilon}}{\hat{\beta}}\frac{\sqrt{d(d-1)}}{\epsilon^2}\sqrt{v_2}\right) \tag{12}$$

and Ω is given by $\Omega_{\rm ext}=\Lambda(d-1)/(d+2)8\pi k$ at this extremum. Two possibilities can then be identified: when (a) $A-C>0$, $B\sim\Lambda>0$ or (b) $A-C<0$, $B<0$. Concerning (a), the potential has a local positive maximum that may be located in a suitably flat region and asymptotically approaches zero at large values of the dilaton field. This implies that the extra dimensions may have evolved from $b\simeq 0$ towards a vacuum state at $b\simeq b_0$. A universe in this state could have an energy density (provided by $\Omega_{\rm ext}$ through the stationary compactification of I^d) larger than that of dust matter. However, quantum fluctuations may imply the dilaton will proceed either to $b\sim 0$ or $b\sim\infty$, with $\Omega\to-\infty$ (the universe will initiate a contraction) or $\Omega\to 0^+$ (expansion of the internal space), respectively. Therefore, scenario (a) may constitute an unstable cosmological stage. In possibility (b), we have instead a stable compactification but the minimum is negative and the potential behaves as $\Omega\to 0^-$ at $\psi\to\infty$. At this minimum, the universe will eventually contract as the effective four-dimensional cosmological constant $\Lambda^{(4)}\equiv 8\pi k\Omega_{\rm min}$ is negative. This is in contrast with current observational data.

Taking $\dot{\psi}\approx 0$ and assuming that ψ has had enough time to settle at these extrema with negligible oscillations, the current deceleration parameter is written as

$$q_0\simeq\frac{\dfrac{\Omega_0^m H_0^2 a_0^2}{2}-a_0^2\dfrac{\Lambda}{3}\left(\dfrac{d-1}{d+2}\right)}{-\dfrac{1}{4}+\Omega_0^m H_0^2 a_0^2+a_0^2\dfrac{\Lambda}{3}\left(\dfrac{d-1}{d+2}\right)}, \tag{13}$$

where $\rho_0\equiv\Omega_0^m(3H_0^2/8\pi k)$, with H_0 as the current value of the Hubble parameter and $\Omega_0^m\equiv\rho_0/\rho_C$, ρ_C being the critical density. Negative values for q_0 can be obtained when, e.g.,

$$a_0^2\frac{\Lambda}{3}\left(\frac{d-1}{d+2}\right)>2\Omega_0^m H_0^2 a_0^2$$

and

$$a_0^2\frac{\Lambda}{3}\left(\frac{d-1}{d+2}\right)>\frac{1}{4}-\Omega_0^m H_0^2 a_0^2.$$

In particular, for $\Lambda>0$ we can find parameters for which q_0 agrees with current data. Employing $a_0^2=10^{120}b_0^2$, $H_0^2=10^4 h_0^2\,{\rm km}^2\,{\rm s}^{-2}\,{\rm Mpc}^{-2}$, $h_0\in[0.4,0.7]$, $\Omega_0^m\simeq 0.3$, b_0^2 of the order of the Planck size, it is possible to identify values for v_2, ϵ, $\hat{\epsilon}$, $\hat{\beta}$, d that provide $q_0\in[-0.6,-0.4]$, satisfying the observational bound $|\Lambda^{(4)}|<10^{-120}/16\pi k$. Hence, BI cosmologies can be found in a state of accelerated expansion in agreement with present measurements. This situation is nevertheless afflicted by a fine tuning of parameters. Moreover, it may be unstable (if oscillations in ψ increase), therefore suggesting that quintessence could be a transient effect, with dust energy density eventually overdamping Ω.

Another limit needs to be investigated, namely when

$$e^{-4\gamma\psi}\frac{4}{b_0^4}\frac{d(d-1)}{8\hat{\epsilon}^2\hat{\beta}^2}v_2 \ll 1.$$

This corresponds to a situation with internal dimensions larger than in Eq. (11), where BI corrections will constitute perturbations to the standard gauge (radiation) dynamics (see Refs. [10,16,17]). The potential Ω (with $a\to\infty$) is different from Eq. (11) and approximated by

$$\Omega\simeq e^{-d\gamma\psi}[Ae^{-4\gamma\psi}+Be^{-8\gamma\psi}-Ce^{-2\gamma\psi}+D], \tag{14}$$

where

$$A=\frac{1}{16\pi}\frac{1}{b_0^4}\frac{d(d-1)}{\hat{\epsilon}^2\hat{\beta}^2}v_2,\quad B=\frac{1}{4\pi}\frac{1}{32}\frac{1}{b_0^8}\frac{d^2(d-1)^2}{\hat{\epsilon}^2\hat{\beta}^2}\frac{v_2^2}{\varepsilon^2},$$

$$C=\frac{1}{16\pi k}\frac{d(d-1)}{4}\frac{1}{b_0^2},\quad D=\frac{\Lambda}{8\pi k}.$$

We employ the following argument to simplify the quantitative analysis. Using $y\equiv e^{-2\gamma\psi}>0$, the presence of the quartic term y^4 in Eq. (14) amounts to shifting any minimum of the quadratic potential towards the origin, decreasing its magnitude. A similar effect is obtained with $Ay^2-(C/\xi)y+D$, $\xi>1$ being a parameter that represents the influence of $e^{-8\gamma\psi}$ at the minimum. The potential Ω is then approximated by $\Omega\simeq y^{d/2}(Ay^2-Cy/\xi+D)$. Stable compactification at a minimum occurs as long as

$$\Lambda<\frac{d-1}{16k}\,\frac{(d+2)^2}{d+4}\,\frac{\epsilon^2 v_2}{64}\,\frac{1}{\xi^2}. \tag{15}$$

We further note that $y>0\Rightarrow\Lambda>0$. Seeking that the solution occurs for $\psi\simeq 0$, we have to fine-tune Λ as $\Lambda\geqslant d(d-1)/16b_0^2\xi$, which further corresponds to $v_2/\epsilon^2=b_0^2/64k\xi$. In order that the effective four-dimensional effective constant be positive, we take

$$\Lambda\simeq\frac{d(d-1)}{16b_0^2}\,\frac{(1+\delta)}{\xi}, \tag{16}$$

where δ is an additional parameter used to comply to the above bounds. Note that Λ is now fixed through d, b_0, ξ, and δ. Subsequently, the current deceleration parameter will be given by

$$q_0\simeq\frac{\dfrac{\Omega_0^m H_0^2 a_0^2}{2}-\dfrac{a_0^2}{b_0^2}\,\dfrac{d(d-1)}{48}\,\dfrac{\delta}{\xi}}{-\dfrac{1}{4}+\Omega_0^m H_0^2 a_0^2+\dfrac{a_0^2}{b_0^2}\,\dfrac{d(d-1)}{48}\,\dfrac{\delta}{\xi}}. \tag{17}$$

Values for d, b_0, v_2, $\hat{\epsilon}$, $\hat{\beta}$, ξ, δ, ϵ can be chosen such that we can again obtain $q_0\in[-0.6,-0.4]$. This constitutes an additional scenario where BI cosmologies can be found in an accelerated expansion consistent with observations. Nevertheless, it is also afflicted by a fine tuning of parameters.

We summarize this paper by pointing out that BI cosmological models can indeed be compatible with the recently observed state of accelerated expansion. In particular, quintessence can constitute a possible transient stage, afflicted by a fine tuning of the parameters introduced with the multidimensional framework (1)–(4). However, this is a problem common to phenomenological approaches of fundamental descriptions. Further research is surely needed and is well motivated. Subsequent directions to investigate could include (i) a thorough numerical analysis, (ii) discussing fluctuations in the gauge field vacuum states, (iii) analyzing the stability of compatification in BI theories after inflation, or (iv) considering corrections to the curvature as well as to the gauge fields (in a BI matter sector) brought in from M or string theory [11]. Finally, it was pointed out that exponential potentials $V\sim V_0 e^{-\lambda\kappa\psi}$ may induce a power-law accelerated expansion ($a\sim t^p, p>1/3$) with $\lambda^2<6$, $V_0>0$ or $\lambda^2>6$, $V_0<0$ [6]. The dilaton in Eqs. (5),(6) could induce a decaying behavior for $\Omega(a\to\infty,\psi)$ as ψ rolls down, possibly constituting an attractor in a (sub)set of solutions. This could also be investigated, namely for Eqs. (11),(12).

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