Particle velocity in noncommutative space-time

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We investigate particle velocity in κ-Minkowski space-time, which is one of the realizations of noncommutative space-time. We emphasize that arrival time analyses by high-energy γ rays or neutrinos, which have been considered as powerful tools to restrict the violation of Lorentz invariance, are not effective in detecting space-time noncommutativity. In contrast with these examples, we point out the possibility that *low-energy massive particles* play an important role in detecting it.

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I. INTRODUCTION

It is believed that general relativity describes the large-scale structure of space-time, and it has revealed the history and present state of our universe. The direct evidence for the validity of general relativity in a strong gravitational regime will be obtained by the observations of gravitational waves from inspiraling binaries in the near future. Little is known about the small scale structure of space-time because gravity also should be quantized consistently in such a regime, which has not been completed yet. The physics of small-scale structure is important because there are some phenomena in our universe for which such physics may be needed to describe them, e.g., the birth of the universe, space-time singularity, and ultrahigh-energy cosmic rays. The last one is one of the main topics of this paper.

Although we do not have the coherent theory of small-scale structure, several attempts have been made to extract its information and effects. Among them, the simplest way is modifying the dispersion relation that leads to the violation of Lorentz invariance. We call the theories obtained by this method modified dispersion relation (MDR) models.

The violation of Lorentz invariance appears in the context of string or *M* theories, where the space-time structure is modified to include the space-time noncommutativity [1]. Space-time noncommutativity also arises as a result of deformation quantization [2]. By using the MDR models, the robustness of the spectrum of black hole evaporation and of the fluctuation generated in an inflationary cosmology were discussed [3,4]. Remarkably enough, it was discussed that the anomalous detection of the ultrahigh-energy cosmic rays beyond the Greisen-Zatsepin-Kuzmin cutoff may be observational evidence for the violation of Lorentz invariance [5,6].

It has been discussed that there is a severe constraint on the energy scale where the Lorentz invariance might be violated, i.e., the scale of quantum gravity (QG) as $E_{\rm QG}\gtrsim 7.2\times 10^{16}$ GeV by the detection of γ rays from Markarian (Mk) 421 as pointed out in Refs. [7–10]. They examined the energy dependence of the arrival time of photons and compared it with the observations of Mk 421 to obtain this constraint.

In general, however, it is plausible that not only a dispersion relation but also other relations such as energy-momentum conservation laws might be altered in Planck scale physics. A particle velocity in such models may be qualitatively different from that in the MDR models. To investigate these features, we employ a model called κ-Minkowski space-time, where noncommutativity is introduced as $[x^i,t]=i\lambda x^i$ [11–13], and we compare a group velocity in the κ-Minkowski space-time evaluated in our previous paper [6] with that in the MDR models. The properties of this group velocity were also investigated in Ref. [14].

This paper is organized as follows. In Sec. II, we review the previous discussion in the MDR models. After introducing the κ-Minkowski space-time in Sec. III, we discuss the particle velocity in this model in Sec. IV. In Sec. V, we consider the observational possibilities of time delay by comparing a particle velocity in the usual Minkowski space-time with that in the κ-Minkowski space-time and MDR models. We show that the space-time noncommutativity does *not* affect the velocity of massless particles, which implies that the arrival time analysis by γ rays is *not* useful to detect the space-time noncommutativity. We also discuss a possibility that the space-time noncommutativity might be detected by using low-energy massive particles. In Sec. VI, we summarize our results and mention future work. We use the signature $(-,+,+,+)$ and units in which $c=\hbar=1$ below.

II. MODIFIED DISPERSION RELATION MODELS

Although there are various ways to modify the dispersion relation, we consider here the form in Ref. [8] as $p^2+m^2=E^2[1+f(E/E_{\rm QG})]$, where f is a model-dependent function and $E_{\rm QG}$ is the effective energy scale of quantum gravity. For simplicity, we assume that f is an analytic function. Although, in general, f and $E_{\rm QG}$ may depend on the species and properties of the particles [5], we do not consider this possibility, which implies that the effects of quantum gravity originate from the space-time structure. In the low-energy

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limit, $E \ll E_{\rm QG}$, the above dispersion relation becomes

$$p^2+m^2=E^2+\frac{\xi E^n}{E_{\rm QG}^{n-2}}, \tag{2.1}$$

up to the lowest correction. We have chosen $\xi=\pm 1$ and $n \geqslant 3$ is the integer, which is determined by the form of the function *f*. Note that $E<m$ for $\xi=1$ in the low-momentum limit. This type of dispersion relation also appears in the Liouville string approach to quantum gravity [15].

The velocity $v_{\rm MDR}$ in this model is obtained by differentiating the dispersion relation (2.1) with respect to *p*,

$$v_{\rm MDR}:=\frac{dE}{dp}=\frac{2\sqrt{E^2-m^2+\xi E^n/E_{\rm QG}^{n-2}}}{2E+n\xi E^{n-1}/E_{\rm QG}^{n-2}}. \tag{2.2}$$

It should be noted that $v_{\rm MDR}$ depends on the energy even for massless particles because of the correction term. We can make use of the energy dependence to restrict $E_{\rm QG}$.

Let us consider a γ ray from the distant source. We approximate the velocity of the γ ray by expanding Eq. (2.2) by $E/E_{\rm QG}$ to

$$v_{\rm MDR}\approx 1-\frac{\xi(n-1)}{2}\left(\frac{E}{E_{\rm QG}}\right)^{n-2}. \tag{2.3}$$

Although the correction term may be very small, the difference of arrival time depending on the energy of the photons may become large enough to measure if the γ rays travel a very long distance [7–10]. The time delay is evaluated as

$$\begin{aligned}\delta t&=\frac{L}{v_{\rm MDR}(E_1)}-\frac{L}{v_{\rm MDR}(E_2)}\\ &\approx\frac{(n-1)\xi L}{2E_{\rm QG}{}^{n-2}}(E_1^{n-2}-E_2^{n-2}),\end{aligned} \tag{2.4}$$

where L, E_1, and E_2 are the distance from the source to the Earth and the amounts of the energy of particles 1 and 2, respectively.

One of the examples of this kind of analysis is the arrival time analysis by γ rays from Mk 421 ($\sim$150 Mpc from the Earth). It was reported that γ rays in the energy range between 1 and 2 TeV arrived at the Earth within the time difference $\sim$200 s [7]. Then, $E_{\rm QG}$ is constrained to $E_{\rm QG}\gtrsim[3.6\times(n-1)(n-2)\times 10^{13}]^{1/(n-2)}\times 10^3$ GeV. Since the value of *n* has been assumed to be 3 in most of the previous works, it has been concluded that $E_{\rm QG}\gtrsim 7.2\times 10^{16}$ GeV. We should note, however, that *n* may be 4 or larger. In this case, the constraint becomes $E_{\rm QG}\gtrsim 1.5\times 10^{10}$ GeV for $n=4$ and $E_{\rm QG}\gtrsim 7.6\times 10^7$ GeV for $n=5$. Hence the constraint may become quite loose compared with the previous reports.

III. κ-POINCARÉ ALGEBRA AND κ-MINKOWSKI SPACE-TIME

Here, we review the κ-Poincaré algebra [16], which has the structure of a Hopf algebra (quantum group) [17]. The generators of the κ-Poincaré algebra $\mathcal{P}_\kappa$ satisfy the following commutation relations:

$$\begin{aligned}[M_{\mu\nu},M_{\rho\sigma}]=i(&\eta_{\mu\sigma}M_{\nu\rho}-\eta_{\mu\rho}M_{\nu\sigma}\\ &+\eta_{\nu\rho}M_{\mu\sigma}-\eta_{\nu\sigma}M_{\mu\rho}),\end{aligned} \tag{3.1}$$

$$[M_i,p_0]=0, \tag{3.2}$$

$$[M_i,p_j]=i\epsilon_{ijk}p_k, \tag{3.3}$$

$$[N_i,p_0]=ip_i, \tag{3.4}$$

$$[N_i,p_j]=-i\,\delta_{ij}\left[\frac{1}{2\lambda}(1-e^{2p_0\lambda})+\frac{\lambda}{2}\boldsymbol{p}^2\right]+i\lambda p_i p_j, \tag{3.5}$$

$$[p_\mu,p_\nu]=0, \tag{3.6}$$

where $M_i\equiv\frac{1}{2}\epsilon_{ijk}M_{jk}$, $N_i\equiv M_{0i}$, and p_μ are generators of rotation, boost, and translation, respectively. The Greek and Roman indices take the values from 0 to 3 and from 1 to 3, respectively. We abbreviate $\Sigma_i p_i^2$ as $\mathbf{p}^2$. We can recover the ordinary commutation relations of the Poincaré algebra in the limit $\lambda\to 0$. The dispersion relation is determined by the eigenvalue of the Casimir operator that commutes with all elements in $\mathcal{P}_\kappa$:

$$\frac{2\cosh(\lambda p_0)}{\lambda^2}-\boldsymbol{p}^2 e^{-\lambda p_0}=\frac{2\cosh(\lambda m)}{\lambda^2}, \tag{3.7}$$

where the rest mass m is defined as the energy with $p_i=0$. The coproducts $\Delta:\mathcal{P}_\kappa\to\mathcal{P}_\kappa\otimes\mathcal{P}_\kappa$ of the basic generators are

$$\Delta(M_i)=M_i\otimes 1+1\otimes M_i, \tag{3.8}$$

$$\Delta(N_i)=N_i\otimes 1+e^{p_0\lambda}\otimes N_i-\lambda\epsilon_{ijk}p_j\otimes M_k, \tag{3.9}$$

$$\Delta(p_0)=p_0\otimes 1+1\otimes p_0, \tag{3.10}$$

$$\Delta(p_i)=p_i\otimes 1+e^{p_0\lambda}\otimes p_i. \tag{3.11}$$

The above coproducts of p_μ, Eqs. (3.10) and (3.11), are interpreted as the non-Abelian addition law of energy momenta for particles 1 and 2 as

$$(E_1,\boldsymbol{p}_1)\hat{+}(E_2,\boldsymbol{p}_2):=(E_1+E_2,\boldsymbol{p}_1+e^{\lambda E_1}\boldsymbol{p}_2), \tag{3.12}$$

where we identify p_0 with energy E. Note that the associativity of the addition law is given by the coassociativity $(\Delta\otimes id)\bigcirc\Delta=(id\otimes\Delta)\bigcirc\Delta$. The coproducts of other elements in $\mathcal{P}_\kappa$ are extended as $\Delta(1)=1\otimes 1$ and $\Delta(MM')=\Delta(M)\Delta(M'),\forall M,M'\in\mathcal{P}_\kappa$. We can check the consistency between this extension of the coproducts as an algebra homomorphism and the commutation relation, i.e., $\Delta[M,\,M']=[\Delta M,\Delta M']$. This consistency guarantees that the κ-Poincaré algebra is form-invariant for multiparticle systems.

The asymmetry of the coproducts for the permutation of particles is called noncocommutativity. The noncocommutativity of the coproducts for the translation sector $T \subset \mathcal{P}_\kappa$ has two important meanings. One is that the noncommutativity of the κ-Minkowski space-time is a direct consequence of the noncocommutativity. Elements in the κ-Minkowski space-time are defined as linear functionals on the translation sector, $T^*: T \to \mathbf{C}$. The products in T^* are defined in terms of coproducts in T, i.e., $\forall x,y \in T^*$ and $\forall p \in T$,

$$\langle xy,p\rangle := \langle x\otimes y,\Delta p\rangle \tag{3.13}$$

$$=\sum_a \langle x,p_{a(1)}\rangle\langle y,p_{a(2)}\rangle, \tag{3.14}$$

where we write the coproducts as $\Delta(p)=\Sigma_a p_{a(1)}\otimes p_{a(2)}$. With the duality relations $\langle x^\mu,p_\nu\rangle=-i\delta^\mu_\nu$, this leads to the following commutation relations [11]:

$$[x^i,x^0]=i\lambda x^i, \tag{3.15}$$

$$[x^0,x^0]=0, \tag{3.16}$$

$$[x^i,x^j]=0. \tag{3.17}$$

The other is that the noncocommutativity leads to a deformed group velocity formula [6], which is different from the usual velocity formula $dE/d\boldsymbol{p}$, as will be shown in the next section.

We can also define differentiation, integration, and Fourier transformation [18]. The plane wave $\psi_{(E,\boldsymbol{p})}=e^{i\boldsymbol{p}\cdot\boldsymbol{x}}e^{iEt}$ in the κ-Minkowski space-time introduced in [19,20] respects the non-Abelian addition law of energy-momenta in the sense

$$\psi_{(E_1,\boldsymbol{p}_1)}\psi_{(E_2,\boldsymbol{p}_2)}=e^{i\boldsymbol{p}_1\cdot\boldsymbol{x}}e^{iE_1t}e^{i\boldsymbol{p}_2\cdot\boldsymbol{x}}e^{iE_2t} \tag{3.18}$$

$$=\psi_{(E_1+E_2,\boldsymbol{p}_1+e^{\lambda E_1}\boldsymbol{p}_2)}. \tag{3.19}$$

IV. VELOCITY FORMULA

From the properties in the κ-Minkowski space-time in Sec. III, we can establish group velocity formulas. For this purpose, we consider infinitesimal changes ΔE and $\Delta\boldsymbol{p}$ in E and $\boldsymbol{p}$, respectively, as a result of adding $(\Delta E',\Delta\boldsymbol{p}')$ as

$$(E,\boldsymbol{p})\hat{+}(\Delta E',\Delta\boldsymbol{p}')=(E+\Delta E,\boldsymbol{p}+\Delta\boldsymbol{p}). \tag{4.1}$$

By the addition law (3.12), we have

$$(\Delta E',\Delta\boldsymbol{p}')=\left(\Delta E,\frac{\Delta\boldsymbol{p}}{e^{\lambda E}}\right). \tag{4.2}$$

Next, we construct a wave packet by superposing plane waves. Here we only consider two waves for simplicity, whose momenta and amounts of energy are different infinitesimally from each other [21]:

$$I=\psi_{(E-\Delta E,\boldsymbol{p}-\Delta\boldsymbol{p})}+\psi_{(E+\Delta E,\boldsymbol{p}+\Delta\boldsymbol{p})}$$
$$\cong 2e^{i\boldsymbol{p}\cdot\boldsymbol{x}}e^{iEt}\cos\left[\frac{\Delta\boldsymbol{p}}{e^{\lambda E}}\cdot\left(\boldsymbol{x}+\frac{e^{\lambda E}\Delta Et}{\Delta\boldsymbol{p}}\right)\right], \tag{4.3}$$

where we neglected the terms that vanish in the limit $\Delta\boldsymbol{p}\to 0$. The group velocity $\boldsymbol{v}_l$ of this wave packet can be written as

$$\boldsymbol{v}_l := e^{\lambda E}\frac{dE}{d\boldsymbol{p}}. \tag{4.4}$$

There remains ambiguity in constructing the wave packet because of the noncommutativity of the space-time. Another possibility is

$$(\Delta E',\Delta\boldsymbol{p}')\hat{+}(E,\boldsymbol{p})=(E+\Delta E,\boldsymbol{p}+\Delta\boldsymbol{p}). \tag{4.5}$$

In this case, the corresponding group velocity $\boldsymbol{v}_r$ is

$$\boldsymbol{v}_r := \left(1-\lambda\boldsymbol{p}\cdot\frac{dE}{d\boldsymbol{p}}\right)^{-1}\frac{dE}{d\boldsymbol{p}}. \tag{4.6}$$

These velocities can be expressed explicitly in terms of the functions of E and m by using the dispersion relation. By the definitions of $\boldsymbol{v}_l$ and $\boldsymbol{v}_r$, we find

$$\boldsymbol{v}_l=\frac{e^{\lambda E/2}\sqrt{2[\cosh(\lambda E)-\cosh(\lambda m)]}}{|e^{\lambda E}-\cosh(\lambda m)|}\boldsymbol{e}, \tag{4.7}$$

$$\boldsymbol{v}_r=\frac{e^{-\lambda E/2}\sqrt{2[\cosh(\lambda E)-\cosh(\lambda m)]}}{|e^{-\lambda E}-\cosh(\lambda m)|}\boldsymbol{e}, \tag{4.8}$$

where $\boldsymbol{e}:=\boldsymbol{p}/|\boldsymbol{p}|$. We find that the velocities have the same direction as that of the momenta. Note also that there is a correspondence between the transformations $\lambda\to-\lambda$ and $\boldsymbol{v}_l\to\boldsymbol{v}_r$.

These velocities were also investigated by Lukierski and Nowicki and the following facts were pointed out in Ref. [14]: (i) $v_l:=|\boldsymbol{v}_l|$, $v_r:=|\boldsymbol{v}_r|\leqslant 1$ for all energies; (ii) $dv_l/dE>0$, $dv_r/dE>0$; and (iii) v_r has a classical velocity addition law, i.e., the addition of parallel velocities v_{r1} and v_{r2} becomes

$$v_{r12}=\frac{v_{r1}+v_{r2}}{1+v_{r1}v_{r2}}. \tag{4.9}$$

If the boost generator N_i were an even function for λ, this addition law would hold even for v_l because of the correspondence mentioned above. However, this is not the case. We postpone the interpretation of this asymmetry to future work.

Next, we discuss the application of the above velocity formulas. In the MDR models, since the energy scale of quantum gravity E_{QG} is introduced perturbatively [see Eq. (2.3)], it is reasonable to apply the velocity formulas under

TABLE I. Approximation of the velocity v_l in the case $|\lambda E| \gg 1$.

	$E/m \gg 1$	$\lvert\lambda(E-m)\rvert \ll 1$
$\lambda > 0$	$1+e^{-2\lambda E}[\frac{1}{2}-\cosh^2(\lambda m)]$	$2\sqrt{\lambda(E-m)}$
$\lambda < 0$	$1/\cosh(\lambda m)$	$2e^{\lambda m}\sqrt{\lambda(m-E)}$

the condition $E \ll E_{\mathrm{QG}}$, while if we apply the velocity formulas in the κ-Minkowski space-time, the energy range is not restricted.

Let us examine the case beyond the quantum gravity scale, i.e., $|\lambda E| \gg 1$. Since we can obtain the information about v_r by using the transformation $\lambda \to -\lambda$ to v_l, we only examine v_l below. We evaluate the velocity v_l in the following limits (see Table I). When $\lambda > 0$ and $E/m \gg 1$, we can find that the velocity of massive particles approaches 1 much faster than that in the Minkowski space-time as the energy of the particle increases. However, for $\lambda < 0$ and $E/m \gg 1$, the difference of the velocity from 1 becomes large as the mass of the particle increases. Note that if $|\lambda(E-m)| \ll 1$, we obtain $|\lambda m| \gg 1$ by using the condition $|\lambda E| \gg 1$. Since $E-m$ is written as $m(1/\sqrt{1-v_M^2}-1)$ in the Minkowski space-time, where v_M is the velocity in the Minkowski space-time, we can rewrite the condition $|\lambda(E-m)| \ll 1$ as $|\lambda m(1/\sqrt{1-v_M^2}-1)| \ll 1$, which leads to $v_M \ll 1$ because of $|\lambda m| \gg 1$. Then, we find $v_l \simeq v_M\sqrt{2\lambda m}$ and $v_l \simeq e^{\lambda m} v_M \sqrt{-2\lambda m}$ for $\lambda > 0$ and for $\lambda < 0$, respectively. Thus, we find that v_l for the case $|\lambda m| \gg 1$ is quite different from v_M, which is a good approximation for describing a velocity of macroscopic bodies in our world under the conditions we are considering. To describe a velocity of macroscopic bodies in the κ-Minkowski space-time, we must consider carefully what are the energy and the momentum, since these quantities are obtained by a total sum of those elementary particles according to the addition law (3.12). The above discrepancy may be explained by this reason. Below, we only consider elementary particles and restrict the discussion to the case $|\lambda m| \ll 1$.

V. MEASUREMENTS OF THE EFFECTIVE SCALE OF "QUANTUM GRAVITY" BY MASSIVE PARTICLES

In this section, we compare v_l with v_{MDR} and discuss the possibility of detection of an effective scale of quantum gravity by observations and experiments. The behavior of the velocities is quite different depending on the mass and energy of the particle. Hence, we consider two limiting cases: (i) the "relativistic" case ($m \ll E$) and (ii) the "nonrelativistic" case ($m \approx E$) [22].

In the relativistic case, $m \ll E$, and under the assumptions $E \ll E_{\mathrm{QG}}$ and $E \ll |\lambda^{-1}|$, v_{MDR} and v_l are

$$v_{\mathrm{MDR}} \approx 1-\frac{1}{2}\left(\frac{m}{E}\right)^2-\frac{\xi(n-1)}{2}\left(\frac{E}{E_{\mathrm{QG}}}\right)^{n-2}, \tag{5.1}$$

$$v_l \approx 1-\frac{1}{2}\left(\frac{m}{E}\right)^2+\frac{\lambda m^2}{2E}, \tag{5.2}$$

at the lowest order of m/E and E/E_{QG} in the MDR models and λE in the κ-Minkowski space-time, respectively. When $m=0$ in the MDR models, E_{QG} can be constrained by the γ rays from the Mk 421, as mentioned in Sec. II. However, since $v_l=1$ for massless particles (we can confirm this is also true for all orders of λm and λE), λ is not constrained by massless particles. This is an important result since it shows that there are a wide variety of candidates for the theory of quantum gravity, for some of which the scale of quantum gravity is not constrained by present observations. The situation changes for massive particles since the lowest-order correction appears in the coupled form with the mass of the particle in the κ-Minkowski space-time, while that of the MDR models does not depend on the mass of the particle.

First, we consider neutrinos from supernovae with energy $E_\nu \sim 10^{10}$ eV to detect space-time noncommutativity. We assume that the mass of an electron neutrino and all the parameters necessary to describe neutrino physics are determined by other experiments and observations, and use the delay of the arrival time between the neutrinos and gravitational waves to evaluate the scale of quantum gravity. In this case, the delay of the arrival time is

$$\delta t \approx \frac{L m_\nu^2}{2E_\nu}\left(\frac{1}{E_\nu}+\lambda\right). \tag{5.3}$$

Since neutrinos are emitted continuously during about 10 s, it is impossible to determine the time when the neutrino is emitted more accurately than that time scale. For this reason, $\delta t \gtrsim 10$ s is necessary to detect the effect of quantum gravity. As for λ, since there is no restriction from the arrival time analysis of γ rays, λ may take a large value. However, by considering reaction processes by collider experiments, we can restrict $|\lambda| \lesssim 10^{-12}\ \mathrm{eV}^{-1}$ since the threshold of the reaction will change drastically for $|\lambda| > 1/E_{\mathrm{th}}$, where E_{th} is the threshold energy in the Minkowski space-time [6]. Then, L becomes far longer than the horizon scale in the present universe even if $|\lambda| = 10^{-12}\ \mathrm{eV}^{-1}$. Thus, it is difficult to detect this effect in this phenomenon.

Neutrinos from γ-ray bursts in fireball models have a different energy scale. In the bursts, neutrinos with energy $\sim 10^{14}$ eV and γ rays are expected to be radiated away in ~ 1 s [23]. We show that we cannot detect space-time noncommutativity even if we neglect the dissipation of the γ ray. In the $E \gg 1/|\lambda|$ case, we can evaluate the delay of the arrival time of neutrinos compared with the γ rays from Table I as

$$\delta t \approx \frac{L}{2} e^{-2\lambda E}[1+2(\lambda m_\nu)^2] \quad \text{for} \quad \lambda > 0, \tag{5.4}$$

$$\delta t \approx \frac{L}{2}(\lambda m_\nu)^2 \quad \text{for} \quad \lambda < 0, \tag{5.5}$$

where we have used the conditions $E/m \gg 1$ and $|\lambda m| \ll 1$. If we assume $\delta t \sim 1$ s and $|\lambda| = 10^{-12}\ \mathrm{eV}^{-1}$, the path of the particle's travel becomes far longer than the horizon scale in the present universe. In the $E \sim 1/|\lambda|$ case, the arrival time delay cannot be described in a simple way. There is, however, no qualitative difference from the above case. Hence, it is difficult to detect space-time noncommutativity by this method.

Next, we examine the nonrelativistic case, $m \sim E \ll E_{\mathrm{QG}}$ (or $|\lambda^{-1}|$). The velocity in each model is

$$v_{\mathrm{MDR}} \approx \sqrt{1-\left(\frac{m}{E}\right)^2} \times \left[1+\frac{\xi}{2}\frac{E^2(1-n)+nm^2}{E^2-m^2}\left(\frac{E}{E_{\mathrm{QG}}}\right)^{n-2}\right], \quad (5.6)$$

$$v_l \approx \sqrt{1-\left(\frac{m}{E}\right)^2}\left(1+\frac{\lambda m^2}{2E}\right). \quad (5.7)$$

Note that the absolute value of the correction for the velocity in the κ-Minkowski space-time decreases with energy, while that in the MDR model increases. Although in the low-energy limit the dispersion relation in the κ-Minkowski space-time has the same form as that in the MDR models, the correction for the velocity is quite different.

Because of the above difference in the correction terms, there is a possibility that the evidence for space-time noncommutativity can be detected by the use of the low-energy massive particles. Here, we consider the ultracold neutrons with energy $E_n - m_n \sim 10^{-2}$ eV [24]. Since the mass of a neutron m_n is measured with high accuracy, we can estimate the time interval in which the neutron travels the interval L in the Minkowski space-time. If a time lag is obtained in an experiment, it can be interpreted as the effect of space-time noncommutativity. This time lag is calculated in the κ-Minkowski space-time as

$$\delta t = \frac{L}{v_l} - \frac{L}{v_M} \approx \frac{L}{v_M}\frac{\lambda m_n^2}{2E_n}. \quad (5.8)$$

By substituting the value of the apparatus [25], $L \sim 100$ m, we have

$$\delta t \sim 10^{-1} \lambda m_n. \quad (5.9)$$

If the resolution for the measurement of the time lag is $\sim 10^{-10}$ s and $|\lambda| \gtrsim 10^{-18}$ eV^{-1}, we can detect space-time noncommutativity.

VI. CONCLUSION

We have investigated what are the qualitative differences of the velocity formula in the κ-Minkowski space-time from that in the MDR models. Most of the previous papers had adopted the MDR models since they are among the simplest models of quantum gravity. However, many of the MDR models do not have a physical foundation in how the correction terms naturally arise in the dispersion relation. For example, since the usual Lorentz transformation had been used in the previous work, one could not have avoided the existence of a preferred frame as a result. Since we have taken the position that the existence of a preferred frame is not favorable, we have considered the κ-Minkowski space-time where the deformed Lorentz transformation and the deformed dispersion relation arise as a result of the deformation quantization.

We have found that since massless particles move in a constant speed in the κ-Minkowski space-time, the arrival time analyses by γ rays are not capable of detecting the difference from the Minkowski space-time. This example shows that it is difficult to constrain all kinds of Lorentz invariance by a single experiment. Therefore, we need to investigate specific models individually. We have also considered the possibility to detect space-time noncommutativity by low-energy massive particles. In our model, if the resolution for the measurement of the time lag is given by $\sim 10^{-10}$ s, it *is* possible to constrain λ to $|\lambda| \gtrsim 10^{-18}$ eV^{-1}. Although these features have not been investigated so far, they may be important.

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