CP violation in supersymmetric $U(1)'$ models

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The supersymmetric *CP* problem is studied within superstring-motivated extensions of the minimal supersymmetric standard model (MSSM) with an additional $U(1)'$ gauge symmetry broken at the TeV scale. This class of models offers an attractive solution to the μ problem of the MSSM, in which $U(1)'$ gauge invariance forbids the bare μ term, but an effective μ parameter is generated by the vacuum expectation value of a standard model singlet S which has a superpotential coupling of the form SH_uH_d to the electroweak Higgs doublets. The effective μ parameter is thus dynamically determined as a function of the soft supersymmetry breaking parameters, and can be complex if the soft parameters have nontrivial *CP*-violating phases. We examine the phenomenological constraints on the reparametrization invariant phase combinations within this framework, and find that the supersymmetric *CP* problem can be greatly alleviated in models in which the phase of the $SU(2)$ gaugino mass parameter is aligned with the soft trilinear scalar mass parameter associated with the SH_uH_d coupling. We also study how the phases filter into the Higgs sector, including only the dominant top quark and top squark loops. We find that while the Higgs sector conserves *CP* at the renormalizable level to all orders of perturbation theory, *CP* violation can enter at the nonrenormalizable level at one-loop order. In the majority of the parameter space, the lightest Higgs boson remains essentially *CP* even but the heavier Higgs bosons can exhibit large *CP*-violating mixings, similar to the *CP*-violating MSSM with large μ parameter.

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I. INTRODUCTION

While phenomenological models with low energy supersymmetry (SUSY) are arguably the best candidates for physics beyond the standard model (SM), they typically include a large number of parameters associated with the soft supersymmetry breaking sector. For example, the minimal supersymmetric standard model (MSSM), which has two Higgs doublets and conserved R parity, contains 105 new parameters [1], including the bilinear Higgs superpotential parameter μ and the soft SUSY breaking parameters (this counting does not include the gravitino mass and coupling). The parameter count generically increases if such SUSY models are extended beyond the minimal gauge structure and particle content of the MSSM, unless symmetry relations exist in the theory which relate subsets of parameters. Many of these new parameters are phases, which both provide new sources of *CP* violation and modify the amplitudes for *CP*-conserving processes. Even if certain sectors of the theory exhibit no *CP* violation at tree level (e.g., if the relevant phases can be eliminated by global phase rotations), the phases can leak into such sectors of the theory at the loop level and have an impact on collider phenomenology and cosmology.

In contrast with the SM, in which the only source of *CP* violation is present in the Cabibbo-Kobayashi-Maskawa (CKM) matrix and thus is intimately tied to flavor physics, *CP*-violating phases within SUSY models can occur in both flavor-conserving and flavor-changing couplings. The phases of the flavor-conserving couplings (which have no analogue in the SM) are of particular interest because they can have significant phenomenological implications which can be studied without knowledge of the origin of intergenerational mixing. In the MSSM, these phases are given by reparametrization invariant combinations of the phases of the gaugino mass parameters $M_a(a=1,2,3)$, the trilinear couplings A_f, the μ parameter, and the Higgs bilinear coupling $b\equiv\mu B$. However, not all of these phases are physical; after utilizing the $U(1)_{PQ}$ and $U(1)_R$ global symmetries of the MSSM, the reparametrization invariant phase combinations are $\theta_f=\phi_\mu+\phi_{A_f}-\phi_b$ and $\theta_a=\phi_\mu+\phi_{M_a}-\phi_b$ (in self-evident notation).

Such phases have traditionally been assumed to be small due to what is known as the supersymmetric *CP* problem: the experimental upper limits on the electric dipole moments (EDMs) of the electron, neutron, and certain atoms individually constrain the phases to be less than $O(10^{-2})$ for sparticle masses consistent with naturalness [2–4]. However, recent studies have shown that EDM bounds can be satisfied without requiring all reparametrization invariant phase combinations to be small, if either (i) certain cancellations exist between different EDM contributions [5–9], or (ii) the sparticles of the first and second families have multi-TeV masses [10].[1] Even within each of these scenarios, the particularly

[1]The EDM bounds are more difficult to satisfy in both of these scenarios when $\tan\beta$ (the ratio of electroweak Higgs vacuum expectation values) is large. Not only are cancellations in the one-loop EDMs more difficult to achieve, but certain two-loop contributions are then enhanced [11,12] which do not decouple when the sfermions are heavy. In part for this reason, we will restrict our attention in this paper to the small $\tan\beta$ regime.

strong constraints arising from the atomic EDMs [13] lead to a general upper bound of $\lesssim O(10^{-3})$ on the reparametrization invariant phase present in the chargino sector (θ_2 in our notation), while the other phases are comparatively unconstrained [8]. These constraints will be discussed in detail later in the paper; for now, it is worth noting that this ''*CP* hierarchy problem'' is an intriguing issue to be addressed within models of the soft parameters which include *CP* violation.[2]

Of course, if the reparametrization invariant phases are sizeable, they can have important phenomenological consequences. Within the MSSM, one of the examples in which these phases can have a significant impact is the Higgs sector. As is well known, the MSSM Higgs sector conserves *CP* at tree level. However, radiative corrections involving the SM fields and their superpartners, with the dominant effects typically due to top quark and top squark loops, have a substantial impact on Higgs masses and mixings. For example, the one-loop radiative corrections substantially elevate the tree-level theoretical upper bound of M_Z on the mass of the lightest Higgs boson [16]; these results have been improved by utilizing complete one-loop on-shell renormalization [17], renormalization group methods [18], diagrammatic methods with leading order QCD corrections [19], two-loop on-shell renormalization [20], and complete two-loop effective potential [21]. If the radiative corrections include a nontrivial dependence on phases, the Higgs potential violates *CP* explicitly at one-loop. The Higgs mass eigenstates then no longer have definite *CP* properties, which leads to important implications for Higgs production and decay [22–25].

The MSSM offers a minimal framework for stabilizing the Higgs sector against quadratic divergences. However, it is well known that the MSSM has a hierarchy problem with respect to the scale of the superpotential μ parameter [26], which has a natural scale of $O(M_{GUT})$, and the electroweak scale. An elegant framework in which to address this ''μ problem'' is to generate the μ parameter via the vacuum expectation value (VEV) of a SM singlet S. One simple possibility[3] [28] is to invoke an additional nonanomalous $U(1)'$ gauge symmetry broken at the TeV scale, as expected in many string models. For suitable $U(1)'$ charges, the bare μ parameter is forbidden but the operator $h_s S H_u \cdot H_d$ is allowed, such that an effective μ term is generated after S develops a VEV of order the electroweak/TeV scale [assuming the Yukawa coupling $h_s \sim O(1)$, as is well-motivated within semirealistic superstring models]. This framework is of particular interest because such extra $U(1)$ groups are often present in plausible extensions of the MSSM, and in fact are ubiquitous within many classes of four-dimensional superstring models. Additional nonanomalous $U(1)$ gauge groups are present in virtually all known 4D string models with semirealistic features, such as gauge structure which includes $SU(3)_c \times SU(2)_L \times U(1)_Y$ (or a viable GUT extension) and particle content which includes the MSSM fields.[4]

Within this class of models, the electroweak and $U(1)'$ symmetry breaking is driven by the soft SUSY breaking parameters, and hence the Z' mass is expected to be of order a few TeV or less. Such a Z' should be easily observable at either present or forthcoming colliders. The nonobservation to date of a Z' puts stringent limits on the Z' mass and mixing with the ordinary Z both from direct searches at the Tevatron [38] and indirect tests from precision electroweak measurements [39]. Although limits depend on the details of the Z' couplings, typically $M_{Z'} > 500$–800 GeV and the $Z - Z'$ mixing angle $\alpha_{Z-Z'} \lesssim O(10^{-3})$.[5] These models have been analyzed at tree level in [41–43], where it was found that there are corners of parameter space in which an acceptable $Z - Z'$ hierarchy can be achieved. Further studies of a different class of string-motivated $U(1)'$ models can be found in [44].

As the phase of the μ parameter filters into the amplitudes for many physical observables in the MSSM (and plays an important role in the Higgs sector at one-loop), it is worthwhile to analyze models which solve the μ problem in the presence of explicit *CP* violation. In this paper, we thus study the supersymmetric *CP* problem in $U(1)'$ models, focusing on the radiative corrections to the Higgs sector of the $U(1)'$ model of [41] in the case that the soft supersymmetry breaking parameters have general *CP*-violating phases (radiative corrections in the *CP*-conserving case have been studied in [47]). We begin by classifying the reparametrization invariant phase combinations and comment on the phe-

[2]However, there are unavoidable theoretical uncertainties involved in the determination of the hadronic EDMs and the atomic EDMs (see e.g. [14,15] for discussions). These uncertainties are particularly problematic for the mercury EDM, which yields the strongest constraints on the SUSY phases. For this reason, there are disagreements in the literature over how to include this bound. Here we take a conservative approach by including the Hg EDM constraint.

[3]The μ parameter can also be generated in models with no additional gauge groups, i.e. the next-to-minimal supersymmetric standard model (NMSSM). However, NMSSM models generically possess discrete vacua and the tensions of the walls separating them are too large to be cosmologically admissable [27].

[4]For example, many examples of such semirealistic models have been constructed within perturbative heterotic string theory (see e.g. [29] for an overview). An interesting class of constructions is the set of free fermionic models [30–32], in which a number of extra $U(1)$'s are always present at the string scale. Whether or not all of these $U(1)$s persist to the TeV scale depends on the details of the vacuum restabilization procedure. Although there are cases in which only the MSSM gauge structure remains at low energy [33], typically one or more extra $U(1)$s persists to the electroweak scale [34,35]. Additional $U(1)$s also are generic in supersymmetric braneworld models derived from type II string orientifolds [36] (due at least in part to the $U(N)$ gauge groups associated with a stacks of D branes). Phenomenological analyses also indicate that typically extra $U(1)$s are present in the low energy theory and broken at the electroweak/TeV scale [37].

[5]A potentially more stringent limit on the Z' mass arises from cosmology if the $U(1)'$ gauge symmetry forbids the standard implementation of the seesaw mechanism for neutrino masses. In such scenarios, the right-handed neutrinos may be light, and BBN constraints then require model-dependent limits that in some cases are as strong as $M_{Z'} \gtrsim 4$ TeV [40].

nomenological constraints on these phases from EDM bounds. We then turn to the Higgs sector, which conserves *CP* at tree level. As in the MSSM, phases enter the Higgs potential at one-loop, with the dominant contributions arising from the top squark mass-squared matrix. The VEV's of the electroweak Higgs doublets $H_{u,d}$ and singlet S are then determined by minimizing the loop-corrected Higgs potential. Within this framework, an effective μ parameter of the correct magnitude is generated which also has a phase governed by the phases of the soft SUSY breaking parameters. We study the pattern of Higgs masses and mixings including the EDM and Z' constraints, and discuss the phenomenological implications for Higgs searches.

II. THE SUSY *CP* PROBLEM IN $U(1)'$ MODELS

We study the class of $U(1)'$ models of [41], in which the gauge group is extended to

$$G=\mathrm{SU}(3)_c\times \mathrm{SU}(2)_L\times \mathrm{U}(1)_Y\times \mathrm{U}(1)', \tag{1}$$

with gauge couplings $g_3, g_2, g_Y, g_{Y'}$, respectively. The matter content includes the MSSM superfields and a SM singlet S, which are all generically assumed to be charged under the additional $U(1)'$ gauge symmetry. Explicitly, the particle content is $\hat{L}_i\sim(1,2,-1/2,Q_L)$, $\hat{E}_i^c\sim(1,1,1,Q_E)$, $\hat{Q}_i\sim(3,2,1/6,Q_Q)$, $\hat{U}_i^c\sim(\overline{3},1,-2/3,Q_U)$, $\hat{D}_i^c\sim(\overline{3},1,1/3,Q_D)$, $\hat{H}_d\sim(1,2,-1/2,Q_d)$, $\hat{H}_u\sim(1,2,1/2,Q_u)$, $\hat{S}\sim(1,1,0,Q_S)$, in which i is the family index.[6]

The superpotential includes a Yukawa coupling of the two electroweak Higgs doublets $H_{u,d}$ to the singlet S, as well as a top quark Yukawa coupling,

$$W=h_s\hat{S}\hat{H}_u\cdot\hat{H}_d+h_t\hat{U}_3^c\hat{Q}_3\cdot\hat{H}_u\,. \tag{2}$$

Gauge invariance of W under $U(1)'$ requires that $Q_u+Q_d+Q_S=0$ and $Q_{Q_3}+Q_{U_3}+Q_u=0$. This choice of charges not only forbids the "bare" μ parameter but also a Kähler potential coupling of the form $H_uH_d+\mathrm{H.c.}$ required for the Giudice-Masiero mechanism [49] (the Kähler potential is otherwise assumed to be of canonical form).[7]

Other than these constraints, we prefer to leave the $U(1)'$ charges unspecified because our aim is not to construct a specific model. In an explicit $U(1)'$ model, there will be additional constraints on the $U(1)'$ charges, most notably from anomaly cancellation. Indeed, the constraints on the charges from anomaly cancellation and gauge invariance of the full superpotential (Yukawa couplings for the quarks and leptons as well as the trilinear effective μ term) generically require the presence of exotic matter in the low energy spectrum. For example, one can construct $E(6)$-motivated anomaly-free $U(1)'$ models [42], which have additional vectorlike exotic quarks. In general, the exotic matter content and couplings (e.g. additional superpotential couplings and soft supersymmetry breaking terms) are highly model-dependent. Of course, such additional states and couplings can significantly alter the low energy phenomenology from that described in this minimal setup. In keeping with our model-independent approach, we will not address the presence of exotic matter explicitly in this paper, but we will comment throughout on how its presence may impact the general analysis in specific models.

The form of Eq. (2) is motivated by string models in which a given Higgs doublet only has $O(1)$ Yukawa couplings to a single (third) family. We will consider the small $\langle H_u\rangle/\langle H_d\rangle\equiv\tan\beta$ regime only[8] such that the Yukawa couplings of the b and τ can be safely neglected. The origin of the Yukawa couplings of the first and second generations of quarks and leptons is not addressed. As we are primarily interested in the third family, we shall suppress the family index in what follows.

The soft supersymmetry breaking parameters include gaugino masses $M_a(a=1,1',2,3)$, trilinear couplings A_s and A_t, and soft mass-squared parameters m_α^2:[9]

$$\begin{aligned}-\mathcal{L}_{soft}=&\Big(\sum_a M_a\lambda_a\lambda_a+A_sh_sSH_u\cdot H_d\\&+A_th_t\widetilde{U}^c\widetilde{Q}\cdot H_u+\mathrm{H.c.}\Big)\\&+m_u^2|H_u|^2+m_d^2|H_d|^2+m_s^2|S|^2+M_{\widetilde{Q}}^2|\widetilde{Q}|^2\\&+M_{\widetilde{U}}^2|\widetilde{U}|^2+M_{\widetilde{D}}^2|\widetilde{D}|^2+M_{\widetilde{E}}^2|\widetilde{E}|^2+M_{\widetilde{L}}^2|\widetilde{L}|^2.\end{aligned} \tag{3}$$

These soft SUSY breaking parameters are generically nonuniversal at low energies. We do not address the origin of these low energy parameters via RG evolution from high energy boundary conditions in this paper.

The gaugino masses M_a and soft trilinear couplings $A_{s,t}$ of Eq. (3) can be complex; if so, they can provide sources of *CP* violation (without loss of generality, the Yukawa couplings $h_{s,t}$ can be assumed to be real). However, not all of

[6]Note that if the $U(1)'$ charges are family nonuniversal they provide a tree-level source of FCNC. Phenomenological bounds thus dictate that the charges of the first and second families should be identical to avoid overproduction of FCNC without fine-tuning [48].

[7]We do not consider kinetic mixing in the analysis [50]. However, even if kinetic mixing is absent at tree level it will be generated through 1-loop RG running if $\mathrm{Tr}\,Q_YQ_{1'}\neq 0$ [50].

[8]Here low values of $\tan\beta$ such as $\tan\beta=1$ are allowed (this region is excluded in the MSSM). The reason is that the Higgs bosons are generically heavier in $U(1)'$ models (as in the NMSSM and other models with extended Higgs sectors), and even at tree level the lightest Higgs boson can easily escape LEP bounds.

[9]In explicit anomaly-free $U(1)'$ gauge models with exotic matter, there will be additional couplings in Eq. (3). In the general case, these couplings are complex, and hence the details of the supersymmetric *CP* problem will be altered depending on the model: they will affect the counting of the phases, etc. discussed below. The discussion below applies if the additional soft parameters and Yukawa couplings are real.

TABLE I. The $U(1)_{R,PQ}$ charge assignments for the MSSM fields and spurions.

Field	$\hat{Q}$	$\hat{U}^c$	$\hat{D}^c$	$\hat{H}_u$	$\hat{H}_d$	λ_a	M_a	μ	b	A_f	m^2_α
$U(1)_R$	1	1	1	0	0	1	-2	2	0	-2	0
$U(1)_{PQ}$	0	0	0	1	1	0	0	-2	-2	0	0

these phases are physical, just as the case in the MSSM. Let us first consider the MSSM. The reparametrization invariant combinations of phases in the MSSM are easily determined by forming invariants with respect to the global $U(1)_{PQ}$ and $U(1)_R$ symmetries present in the limit that the soft breaking parameters and the μ term are set to zero [51]; for reference, the $U(1)_{R,PQ}$ charge assignments are presented in Table I. A convenient basis of the resulting reparametrization invariant phases thus is $\theta_f=\phi_\mu+\phi_{A_f}-\phi_b$ and $\theta_a=\phi_\mu+\phi_{M_a}-\phi_b$, which enter the mass matrices of the sfermions and the gauginos/Higgsinos, respectively. An analysis of the MSSM tree level Higgs sector also suggests it is useful to exploit $U(1)_{PQ}$ to set $\phi_b=0$ (ϕ_b is then dropped from the invariants above), in which case the Higgs VEVs are real.

Performing the same exercise in the $U(1)'$ framework, one immediately notices that the $U(1)_{PQ}$ symmetry of the MSSM is embedded within the $U(1)'$ gauge symmetry. However, a nontrivial $U(1)_R$ symmetry remains; the $U(1)_R$ charges of the superfields and the associated spurion charges of the soft parameters are presented in Table II. The reparametrization invariant phase combinations are therefore $\theta_{ff'}=\phi_{A_f}-\phi_{A_{f'\neq f}}$, $\theta_{af}=\phi_{M_a}-\phi_{A_f}$, and $\theta_{ab}=\phi_{M_a}-\phi_{M_{b\neq a}}$, of which only two are linearly independent (e.g. $\theta_{ab}=\theta_{af}-\theta_{bf}$). We will see that (in analogy to the MSSM) the tree-level Higgs sector suggests it is convenient to measure all phases with respect to the phase of A_s. [In fact, one can go further and exploit the $U(1)_R$ symmetry to set $\phi_{A_s}=0$, although we prefer not to do that in this paper.] A basis of reparametrization invariant phase combinations can then be chosen as

$$\theta_{fs}=\phi_{A_f}-\phi_{A_s}$$

$$\theta_{as}=\phi_{M_a}-\phi_{A_s}. \tag{4}$$

To see this more explicitly and to lay the foundation for our analysis of the Higgs sector including one-loop radiative corrections, let us now review the tree-level Higgs potential analyzed in [41]. Gauge symmetry breaking is driven by the VEVs of the electroweak Higgs doublets H_u, H_d

$$H_u=\begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}, \quad H_d=\begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}, \tag{5}$$

and the singlet S. The tree level Higgs potential is a sum of F terms, D terms, and soft supersymmetry breaking terms:

$$V_{tree}=V_F+V_D+V_{soft}, \tag{6}$$

in which

$$V_F=|h_s|^2[|H_u\cdot H_d|^2+|S|^2(|H_u|^2+|H_d|^2)], \tag{7}$$

$$\begin{aligned} V_D=&\frac{G^2}{8}(|H_u|^2-|H_d|^2)^2 \\ &+\frac{g_2^2}{2}(|H_u|^2|H_d|^2-|H_u\cdot H_d|^2)+\frac{g_{Y'}^2}{2}(Q_u|H_u|^2 \\ &+Q_d|H_d|^2+Q_S|S|^2)^2, \end{aligned} \tag{8}$$

$$\begin{aligned} V_{soft}=&m_u^2|H_u|^2+m_d^2|H_d|^2+m_s^2|S|^2 \\ &+(A_s h_s S H_u\cdot H_d+\text{H.c.}), \end{aligned} \tag{9}$$

where $G^2=g_2^2+g_Y^2$ and $g_Y=\sqrt{3/5}g_1$, g_1 is the GUT normalized hypercharge coupling.

At the minimum of the potential, the Higgs fields are expanded[10] as follows:

$$\langle H_u\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}H_u^+ \\ v_u+\phi_u+i\varphi_u \end{pmatrix},$$

$$\langle H_d\rangle=\frac{1}{\sqrt{2}}\begin{pmatrix} v_d+\phi_d+i\varphi_d \\ \sqrt{2}H_d^- \end{pmatrix},$$

$$\langle S\rangle=\frac{1}{\sqrt{2}}e^{i\theta}(v_s+\phi_s+i\varphi_s), \tag{10}$$

in which $v^2\equiv v_u^2+v_d^2=(246\ \text{GeV})^2$. In the above, a phase shift $e^{i\theta}$ has been attached to $\langle S\rangle$. Since gauge invariance dictates that only the phase of the combination $SH_u\cdot H_d$ enters the potential, we can assume that the VEVs of $H_{u,d}$ are real and attach a phase only to S without loss of generality

TABLE II. The $U(1)_R$ charge assignments for the fields and spurions in the $U(1)'$ framework. Note that $\hat{S}$ (whose VEV induces an effective μ parameter) has nonzero R charge.

Field	$\hat{Q}$	$\hat{U}^c$	$\hat{D}^c$	$\hat{H}_u$	$\hat{H}_d$	$\hat{S}$	λ_a	M_a	A_f	m^2_α
$U(1)_R$	1	1	1	0	0	2	1	-2	-2	0

[10]As discussed in [41], gauge rotations can be used to set $\langle H_u^+\rangle=0$. However, $\langle H_d^-\rangle=0$ is not automatic and imposes constraints on the parameter space of the model: $\langle H_d^-\rangle=0$ implies that the physical charged Higgs boson is nontachyonic ($M^2_{H^\pm}>0$).

[this choice is also consistent with our assignment of $U(1)_R$ charges in Table II]. The effective μ parameter is generated by the singlet VEV $\langle S\rangle$,

$$\mu_{eff} \equiv \frac{h_s v_s}{\sqrt{2}} e^{i\theta}. \tag{11}$$

The only complex parameter which enters the Higgs potential at tree level is A_s. However, the global phases of the Higgs fields (more precisely, of the combination $SH_u \cdot H_d$) can always be chosen to absorb the phase ϕ_{A_s} of A_s by performing a $U(1)_R$ rotation on the fields, such that A_s and the VEV's can all be taken to be real without loss of generality [41]. To state this another way, the minimization conditions with respect to the *CP* odd directions $\varphi_{1,2,s}$ all lead to the condition

$$\sin(\theta + \phi_{A_s}) = 0, \tag{12}$$

such that $\theta = -\phi_{A_s}$ at tree level. With this condition, the Higgs sector is *CP* conserving. The Higgs mass eigenstates thus have definite *CP* quantum numbers, with three *CP* even Higgs bosons $H_{i=1,2,3}$ and one *CP* odd Higgs boson A^0, as well as a charged Higgs pair $H^{\pm}$. Expressions for their masses at tree level and a discussion of the associated Higgs phenomenology can be found in [41].

Although the Higgs sector conserves *CP* at tree level whether or not the soft SUSY breaking parameters are complex, this is generically not true for other sectors of the theory and care must be taken in the phenomenological analysis (e.g. for the EDM bounds) if there are nontrivial *CP*-violating phases in the soft terms even if the Higgs sector is only analyzed at tree level. Clearly, this is due to the fact that the phases which enter the mass matrices of the sfermion and the gaugino/Higgsino sectors involve the phase of the singlet VEV θ (i.e., the phase of the effective μ parameter μ_{eff}) as well as the phases of the A terms and the gaugino masses. For example, the reparametrization invariant phase combination which enters the chargino mass matrix within this class of $U(1)'$ models is

$$\theta_{\tilde{\chi}^{\pm}} = \theta + \phi_{M_2} = \phi_{M_2} - \phi_{A_s} + \dots = \theta_{2\ s} + \dots, \tag{13}$$

in which the terms represented by $(+\dots)$ are higher-loop contributions. As previously mentioned, this phase is strongly constrained by EDM experimental bounds (although the precise constraints can depend in detail on the other parameters of the model). More generally, the statement of (flavor-independent) SUSY *CP* violation within $U(1)'$ models is that if any of the phases θ_{fs} and θ_{as} defined in Eq. (4) are nonzero, they can lead to *CP*-violating effects which may be in conflict with experiment and must be checked. This is in direct analogy to the statement of flavor-independent SUSY *CP* violation in the MSSM. However, as μ is dynamically generated within $U(1)'$ models, its phase ϕ_μ is now a function of the phases of the other soft breaking parameters rather than an independent quantity.

Returning now to the question of *CP* violation in the Higgs sector, Eq. (12) suggests that it is natural to consider the combination of phases

$$\bar{\theta} \equiv \theta + \phi_{A_s} \tag{14}$$

as the parameter which governs *CP* violation in the Higgs sector. Note that $\bar{\theta}$ is a reparametrization invariant quantity, while θ is not ($\theta = \bar{\theta}$ in the basis in which $\phi_{A_s} = 0$). While $\bar{\theta} = 0$ at tree level, it acquires a nonzero value at one-loop if the sfermion and gaugino/Higgsino mass matrices have nontrivial phases. This calculation is outlined in the next section.

III. HIGGS SECTOR *CP* VIOLATION

Previously we discussed the SUSY *CP* problem within $U(1)'$ models, and reviewed the tree level Higgs sector (the patterns of gauge symmetry breaking which led to an acceptable $Z - Z'$ hierarchy were analyzed at tree level in [41]). In what follows, we will compute the dominant one-loop radiative corrections to the Higgs sector of this class of Z' models within a general framework including nontrivial *CP* violation (radiative corrections in the *CP*-conserving case were previously presented in [47]).

A. Radiative corrections to the Higgs potential

The effective potential approach provides an elegant way of determining the true vacuum state of a spontaneously broken gauge theory. The potential has the form

$$V = V_{tree} + \Delta V + \dots, \tag{15}$$

where V_{tree} is defined in Eq. (6), and the one-loop contribution ΔV has the Coleman-Weinberg form

$$\Delta V = \frac{1}{64\pi^2}\left\{ \mathrm{Str}\,\mathcal{M}^4(H_u, H_d, S) \times \left(\ln\frac{\mathcal{M}^2(H_u, H_d, S)}{Q^2} - \frac{3}{2} \right) \right\} \tag{16}$$

in the mass-independent renormalization scheme $\overline{\mathrm{DR}}$.[11] In the above, $\mathrm{Str} \equiv \Sigma_J (-1)^{2J+1}(2J+1)$ is the usual supertrace, Q is the renormalization scale, and $\mathcal{M}$ represents the Higgs

[11] See Martin's paper in [21] for a detailed discussion of the regularization and renormalization scheme dependence of the effective potential.

field-dependent mass matrices of the particles and sparticles of the theory.[12]

Within the MSSM, the dominant terms in the small $\tan\beta$ regime are due to top quark and scalar top quark loops; in the large $\tan\beta$ regime, loops involving the bottom quark and scalar bottom quark as well as chargino loops become important. Within the $U(1)'$ framework and neglecting highly model-dependent exotic matter couplings, the main contributions in the small $\tan\beta$ limit include the aforementioned top quark and top squark loops. There are also potentially significant Higgs/Higgsino self-loops arising from the presence of the effective μ term and its associated soft supersymmetry breaking term. However, such scalar self-loops clearly will not have appreciable contributions to the *CP*-violating mixings of the Higgs bosons, although they can have large contributions to the mixings of the *CP*-even Higgs bosons. Since our focus is the *CP*-violating mixings, we neglect these loops in this analysis. We also do not include loops involving the charginos and neutralinos, for which the contributions to the *CP*-violating mixings are typically suppressed by gauge couplings as well as EDM constraints (see [45,46] for the analysis in the MSSM case). We will comment more on such contributions later in the paper. Finally, we mention here that the exotic particles typically expected in $U(1)'$ models due to anomaly cancellation may generically have *CP*-violating couplings which can have an important impact on the *CP* violation in the Higgs sector. As these couplings are highly model-dependent, we do not include them in this analysis, but their presence would certainly need to be accounted for within any specific model framework.

For these reasons, here we will include only the dominant terms due to top quark and scalar top quark loops:

$$\Delta V=\frac{6}{64\pi^2}\left\{\sum_{k=1,2}(m^2_{\tilde{t}_k})^2\left[\ln\left(\frac{m^2_{\tilde{t}_k}}{Q^2}\right)-\frac{3}{2}\right]-2(m_t^2)^2\left[\ln\left(\frac{m_t^2}{Q^2}\right)-\frac{3}{2}\right]\right\} \quad (17)$$

in which the masses depend explicitly on the Higgs field components (note that ΔV naturally vanishes in the limit of exact SUSY). The top quark mass-squared is given by $m_t^2=h_t^2|H_u|^2$, and the top squark masses-squared are obtained by diagonalizing the mass-squared matrix

$$\tilde{M}^2=\begin{pmatrix} M^2_{LL} & M^2_{LR} \\ M^{2\,\dagger}_{LR} & M^2_{RR}\end{pmatrix} \quad (18)$$

via the unitary matrix S_t as $S_t^\dagger\tilde{M}^2S_t=\mathrm{diag}(m^2_{\tilde{t}_1},m^2_{\tilde{t}_2})$. The entries of $\tilde{M}^2$ are given by

$$M^2_{LL}=M^2_{\tilde{Q}}+h_t^2|H_u|^2-\frac{1}{4}\left(g_2^2-\frac{g_Y^2}{3}\right)(|H_u|^2-|H_d|^2)+g^2_{Y'}Q_Q(Q_u|H_u|^2+Q_d|H_d|^2+Q_s|S|^2) \quad (19)$$

$$M^2_{RR}=M^2_{\tilde{U}}+h_t^2|H_u|^2-\frac{1}{3}g_Y^2(|H_u|^2-|H_d|^2)+g^2_{Y'}Q_U(Q_u|H_u|^2+Q_d|H_d|^2+Q_s|S|^2) \quad (20)$$

$$M^2_{LR}=h_t(A_t^*H_u^{0*}-h_sSH_d^0), \quad (21)$$

in which we have emphasized the fact that the LR entry depends only on the neutral components of the Higgs fields. As the top squark LR mixing can be complex, the term $h_th_sA_tSH_u^0H_d^0+\mathrm{H.c.}$ present in $|M^2_{LR}|^2$ can provide a source of *CP* violation in the Higgs sector. From the discussion of the previous section, we can infer that this source is the phase $\theta_{ts}\equiv\phi_{A_t}-\phi_{A_s}$.

The vacuum state is characterized by the vanishing of all tadpoles and positivity of the resulting Higgs boson masses. Recalling the expressions for the Higgs fields in Eq. (10), the vanishing of tadpoles for V along the *CP* even directions $\phi_{u,d,s}$ enables the soft masses $m^2_{u,d,s}$ to be expressed in terms of the other parameters of the potential:

$$m_u^2=M_0^2\cos^2\beta-\lambda_uv_u^2-\frac{1}{2}(\lambda_{ud}v_d^2+\lambda_{us}v_s^2)-\frac{1}{v_u}\left(\frac{\partial\Delta V}{\partial\phi_u}\right)_0 \quad (22)$$

$$m_d^2=M_0^2\sin^2\beta-\lambda_dv_d^2-\frac{1}{2}(\lambda_{ud}v_u^2+\lambda_{ds}v_s^2)-\frac{1}{v_d}\left(\frac{\partial\Delta V}{\partial\phi_d}\right)_0 \quad (23)$$

$$m_s^2=M_0^2\cot^2\alpha-\lambda_sv_s^2-\frac{1}{2}(\lambda_{us}v_u^2+\lambda_{ds}v_d^2)-\frac{1}{v_s}\left(\frac{\partial\Delta V}{\partial\phi_s}\right)_0, \quad (24)$$

in which the subscript 0 indicates that the derivatives of ΔV are to be evaluated at $\phi_i=0$ and $\varphi_i=0$. Here the various λ coefficients represent the quartic couplings in the potential

[12]While the complete effective potential is scale invariant, it is scale dependent when truncated to any finite loop order in perturbation theory due to the renormalization of the parameters and the Higgs wave functions. In the MSSM, most of the scale-dependent terms can be collected in the pseudoscalar mass, which itself can be regarded as a free parameter of the theory. The remaining Q^2-dependence arises from the D term contributions generated by wave-function renormalization, such that in the limit in which $g_2=g_Y=0$ all of the scale dependence can be absorbed into the pseudoscalar mass. For the $U(1)'$ models, the scale dependence can be absorbed into the pseudoscalar mass only if the D term contributions vanish *and* the superpotential parameter $h_s=0$, because the potential also includes quartic Higgs couplings which arise from F terms. These properties are manifest in the Higgs mass-squared matrix presented below.

$$\lambda_{u,d}=\frac{1}{8}G^2+\frac{1}{2}Q_{u,d}^2 g_{Y'}^2, \quad \lambda_s=\frac{1}{2}Q_s^2 g_{Y'}^2,$$

$$\lambda_{ud}=-\frac{1}{4}G^2+Q_uQ_d g_{Y'}^2+h_s^2,$$

$$\lambda_{us,ds}=Q_sQ_{u,d}g_{Y'}^2+h_s^2. \tag{25}$$

The Higgs soft masses (22) are written in terms of two angle parameters: (i) $\tan\beta$, which measures the hierarchy of the Higgs doublet VEVs, and (ii) $\cot\alpha\equiv(v\sin\beta\cos\beta)/v_s$, which is an indication of the splitting between the $U(1)'$ and electroweak breaking scales. For convenience, we have also introduced the mass parameter

$$M_0^2=\frac{h_s|A_s|v_s\cos\bar{\theta}}{\sqrt{2}\sin\beta\cos\beta}, \tag{26}$$

which corresponds to the mass parameter of the *CP* odd Higgs boson of the MSSM after using the definition of the effective μ parameter in Eq. (11).

While the vanishing of the tadpoles along the *CP* odd directions $\varphi_{u,d,s}$ led to Eq. (12) at tree level, once the loop corrections are included they lead to the following conditions:

$$M_0^2\sin\beta\cos\beta\tan\bar{\theta}=\frac{1}{v_d}\left(\frac{\partial\Delta V}{\partial\varphi_u}\right)_0 \tag{27}$$

$$M_0^2\sin\beta\cos\beta\tan\bar{\theta}=\frac{1}{v_u}\left(\frac{\partial\Delta V}{\partial\varphi_d}\right)_0 \tag{28}$$

$$M_0^2\cot\alpha\tan\bar{\theta}=\frac{1}{v_s}\left(\frac{\partial\Delta V}{\partial\varphi_s}\right)_0, \tag{29}$$

demonstrating explicitly that the phase $\bar{\theta}=\theta+\phi_{A_s}$ associated with the phase θ of the singlet VEV is a radiatively induced quantity. Indeed, the derivatives of ΔV with respect to $\varphi_{u,d,s}$ are nonvanishing provided that there is a nontrivial phase difference between A_t and A_s (i.e., if $\theta_{ts}\neq 0$). In fact, Eqs. (27)–(29) all lead to the same relation for $\bar{\theta}$,

$$\sin\bar{\theta}=-\beta_{h_t}\frac{|A_t|}{|A_s|}\sin\theta_{ts}\mathcal{F}(Q^2,m_{\tilde{t}_1}^2,m_{\tilde{t}_2}^2) \tag{30}$$

in which $\beta_{h_t}=3h_t^2/(32\pi^2)$ is the beta function for the top quark Yukawa coupling, and the loop function

$$\mathcal{F}(Q^2,m_{\tilde{t}_1}^2,m_{\tilde{t}_2}^2)=-2+\ln\left(\frac{m_{\tilde{t}_1}^2m_{\tilde{t}_2}^2}{Q^4}\right)+\frac{m_{\tilde{t}_1}^2+m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2-m_{\tilde{t}_2}^2}\ln\left(\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2}\right) \tag{31}$$

depends explicitly on the renormalization scale. In the above, $\theta_{ts}=\phi_{A_t}+\theta=\phi_{A_t}-\phi_{A_s}$ up to one loop accuracy determined by Eq. (30).[13]

B. The Higgs mass calculation

We now turn to the Higgs mass calculation at one-loop in the presence of *CP* violation in the top squark LR mixing. The mass-squared matrix of the Higgs scalars is

$$\mathcal{M}_{ij}^2=\left(\frac{\partial^2}{\partial\Phi_i\partial\Phi_j}V\right)_0, \tag{33}$$

subject to the minimization constraints Eqs. (22)–(24) and (30). In the above, $\Phi_i=(\phi_i,\varphi_i)$. Clearly, two linearly independent combinations of the pseudoscalar components $\varphi_{u,d,s}$ are the Goldstone bosons G_Z and $G_{Z'}$, which are eaten by the Z and Z' gauge bosons when they acquire their masses. These two modes are given by

$$G_Z=-\sin\beta\varphi_u+\cos\beta\varphi_d,$$

$$G_{Z'}=\cos\beta\cos\alpha\varphi_u+\sin\beta\cos\alpha\varphi_d-\sin\alpha\,\varphi_s, \tag{34}$$

and hence the orthogonal combination

$$A=\cos\beta\sin\alpha\varphi_u+\sin\beta\sin\alpha\varphi_d+\cos\alpha\varphi_s \tag{35}$$

is the physical pseudoscalar Higgs boson in the *CP*-conserving limit. In the decoupling limit, $v_s\gg v$, $\sin\alpha\to 1$ and $\cos\alpha\to 0$, in which case G_Z and A reduce to their MSSM expressions. In the basis of scalars $\mathcal{B}=\{\phi_u,\phi_d,\phi_s,A\}$, the Higgs mass-squared matrix $\mathcal{M}^2$ takes the form

[13] Chargino and neutralino loops can also contribute to $\bar{\theta}$. Their contributions are of the form [45,46]

$$\sin\bar{\theta}\sim\frac{g_a^2}{16\pi^2}\frac{|\mu_{eff}||M_a|}{m_{SUSY}^2}\sin\theta_{as}f, \tag{32}$$

in which f denotes an appropriate loop function [for the chargino case it would be $\mathcal{F}(Q^2,m_{\chi_1^\pm}^2,m_{\chi_2^\pm}^2)$ [45]]. Such contributions are in general subleading to Eq. (30) not only from the gauge coupling suppression, but also because the θ_{as} are subject to EDM constraints (see Sec. IV B). Note that these contributions arise from gaugino-Higgsino mixing; the purely h_s-dependent terms ($\sim\mu^4$) do not contribute to *CP* violation.

$$\begin{pmatrix} M_{uu}^2+M_A^2\cos^2\beta & M_{ud}^2-M_A^2\sin\beta\cos\beta & M_{us}^2-M_A^2\cot\alpha\cos\beta & M_{uA}^2\sin\theta_{ts} \\ M_{ud}^2-M_A^2\sin\beta\cos\beta & M_{dd}^2+M_A^2\sin^2\beta & M_{ds}^2-M_A^2\cot\alpha\sin\beta & M_{dA}^2\sin\theta_{ts} \\ M_{us}^2-M_A^2\cot\alpha\cos\beta & M_{ds}^2-M_A^2\cot\alpha\sin\beta & M_{ss}^2+M_A^2\cot^2\alpha & M_{sA}^2\sin\theta_{ts} \\ M_{uA}^2\sin\theta_{ts} & M_{dA}^2\sin\theta_{ts} & M_{sA}^2\sin\theta_{ts} & M_P^2 \end{pmatrix}, \tag{36}$$

in which our notation explicitly demonstrates that all of the entries $\mathcal{M}_{iA}^2$ $(i=u,d,s)$ identically vanish in the *CP*-conserving limit $\theta_{ts}\to\phi_0$. In the above,

$$M_A^2=M_0^2\left(1+\beta_{h_t}\frac{|A_t|}{|A_s|}\frac{\cos\theta_{ts}}{\cos\bar{\theta}}\mathcal{F}\right), \tag{37}$$

which depends explicitly on the renormalization scale, and

$$M_P^2=\frac{M_A^2}{\sin^2\alpha}+4\beta_{h_t}\frac{m_t^2|\mu_{eff}|^2|A_t|^2}{(m_{\tilde{t}_1}^2-m_{\tilde{t}_2}^2)^2}\frac{\sin^2\theta_{ts}}{\sin^2\alpha\sin^2\beta}\mathcal{G} \tag{38}$$

is the one-loop pseudoscalar mass in the *CP*-conserving limit. The loop function $\mathcal{G}$ is independent of the renormalization scale and has the functional form

$$\mathcal{G}(m_{\tilde{t}_1}^2,m_{\tilde{t}_2}^2)=2-\frac{m_{\tilde{t}_1}^2+m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2-m_{\tilde{t}_2}^2}\ln\left(\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2}\right). \tag{39}$$

We now turn to the mass parameters M_{ij}^2 $(i,j=u,d,s)$ which appear in $\mathcal{M}^2$. These entries may be represented as

$$M_{ij}^2=v_iv_j\left\{\overline{\lambda}_{ij}+\frac{3}{(4\pi)^2}\left[\frac{(\rho_i\widetilde{m}_j^2+\widetilde{m}_i^2\rho_j)}{m_{\tilde{t}_1}^2+m_{\tilde{t}_2}^2}(2-\mathcal{G})+\left(\rho_i\rho_j+\zeta_i\zeta_j+\delta_{id}\delta_{js}\frac{h_t^2h_s^2}{4}\right)\mathcal{F}\right.\right.$$
$$\left.\left.+\left(\rho_i\rho_j+\frac{\widetilde{m}_i^2\widetilde{m}_j^2}{(m_{\tilde{t}_1}^2-m_{\tilde{t}_2}^2)^2}\right)\mathcal{G}-\delta_{iu}\delta_{ju}h_t^4\ln\left\{\frac{m_t^4}{Q^4}\right\}\right]\right\}, \tag{40}$$

in which $\overline{\lambda}_{ij}=\lambda_{ij}$ for $i\neq j$, $\overline{\lambda}_{ij}=2\lambda_i$ for $i=j$. For notational purposes we have also introduced the dimensionless quantities

$$\rho_u=h_t^2-\lambda_u,\quad \rho_d=(h_s^2-\lambda_{ud})/2,\quad \rho_s=(h_s^2-\lambda_{us})/2 \tag{41}$$

as well as the dimensionful ones,

$$\widetilde{m}_u^2=\zeta_u\delta+h_t^2|A_t|(|A_t|-|\mu_{eff}|\cot\beta\cos\theta_{ts}) \tag{42}$$

$$\widetilde{m}_d^2=\zeta_d\delta+h_t^2|\mu_{eff}|(|\mu_{eff}|-|A_t|\tan\beta\cos\theta_{ts}) \tag{43}$$

$$\widetilde{m}_s^2=\zeta_s\delta+\frac{v_d^2}{v_s^2}h_t^2|\mu_{eff}|(|\mu_{eff}|-|A_t|\tan\beta\cos\theta_{ts}), \tag{44}$$

with $\delta=M_{\tilde{Q}}^2-M_{\tilde{U}}^2+\zeta_uv_u^2+\zeta_dv_d^2+\zeta_sv_s^2$. The new dimensionless couplings appearing here are pure D term contributions

$$\zeta_u=-\frac{1}{8}\left(g_2^2-\frac{5}{3}g_Y^2\right)+\frac{1}{2}(Q_Q-Q_U)Q_ug_{Y'}^2, \tag{45}$$

$$\zeta_d=\frac{1}{8}\left(g_2^2-\frac{5}{3}g_Y^2\right)+\frac{1}{2}(Q_Q-Q_U)Q_dg_{Y'}^2, \tag{46}$$

$$\zeta_s=-(\zeta_u+\zeta_d). \tag{47}$$

Finally, the scalar-pseudoscalar mixing entries M_{iA}^2 $(i=u,d,s)$, which exist only if there are sources of *CP* violation in the Lagrangian (as has been made explicit in $\mathcal{M}^2$ by factoring out $\sin\theta_{ts}$), are given by

$$M_{iA}^2 = 2\beta_{h_t}\frac{v v_i}{\sin\alpha}\frac{|\mu_{eff}||A_t|}{m_{\tilde t_1}^2 - m_{\tilde t_2}^2}\left[2\rho_i\frac{m_{\tilde t_1}^2 - m_{\tilde t_2}^2}{m_{\tilde t_1}^2 + m_{\tilde t_2}^2} + \left(\frac{\widetilde m_i^2}{m_{\tilde t_1}^2 - m_{\tilde t_2}^2} - \rho_i\frac{m_{\tilde t_1}^2 - m_{\tilde t_2}^2}{m_{\tilde t_1}^2 + m_{\tilde t_2}^2}\right)\mathcal{G}\right], \tag{48}$$

and are scale independent. These results agree with the tree level computations of [41]. After identifying Eq. (11) with the $|\mu_{eff}|$ parameter of the MSSM, the doublet sector of the mass-squared matrix agrees with that of the MSSM [22]. Finally, the results also agree with those of [47] in the *CP*-conserving limit ($\sin\theta_{ts}=0$).

As previously stated, there are three *CP* even and one *CP* odd Higgs boson in the *CP*-conserving limit. The mass of the *CP* odd Higgs boson A is given in Eq. (38), while the masses of the *CP* even scalars arise from the diagonalization of the upper 3×3 subblock of Eq. (36). The masses and mixings then differ from their tree level values by the inclusion of radiative effects. In this limit, the only source of *CP* violation is the CKM matrix and one easily evades constraints from the absence of permanent EDMs for leptons and hadrons. The lightest Higgs boson has a larger mass than M_Z even at tree level, and the radiative effects modify it sizeably [47]. Once the radiative corrections are included a direct comparison with experimental results is possible. In principle, one can constrain certain portions of the parameter space using the post-LEP indications for a light scalar with mass $\gtrsim 114$ GeV.

In the presence of *CP* violation, there are four scalar bosons with no definite *CP* quantum number. This results from the mixing between the *CP* even scalars $\phi_{u,d,s}$ with the *CP* odd scalar A via the entries $M_{iA}^2\sin\theta_{ts}$ in Eq. (36). The main impact of the *CP* breaking Higgs mixings on the collider phenomenology comes via the generation of novel couplings for Higgs bosons which eventually modify the event rates and asymmetries. In particular, a given Higgs boson can couple to both scalar and pseudoscalar fermion densities depending on the strength of *CP* violation [22]. Moreover, the coupling of the lightest Higgs boson to Z bosons can be significantly suppressed, avoiding the existing bounds from the LEP data [24,25]. The *CP*-violating entries of $\mathcal{M}^2$ grow with $|\mu_{eff}A_t|$ as in the MSSM. The mass-squared matrix is diagonalized by a 4×4 orthonormal matrix $\mathcal{R}$

$$\mathcal{R}\cdot M_h^2\cdot\mathcal{R}^T = \mathrm{diag}\cdot(M_{H_1}^2, M_{H_2}^2, M_{H_3}^2, M_{H_4}^2). \tag{49}$$

To avoid discontinuities in the eigenvalues it is convenient to adopt an ordering: $M_{H_1}<M_{H_2}<M_{H_3}<M_{H_4}$. The mass eigenstates H_i can then be expressed as

$$H_i = \mathcal{R}_{iu}\phi_u + \mathcal{R}_{id}\phi_d + \mathcal{R}_{is}\phi_s + \mathcal{R}_{iA}A \tag{50}$$

in which e.g. $|\mathcal{R}_{iA}|^2$ is a measure of the *CP* odd composition of H_i. The elements of $\mathcal{R}$ determine the couplings of Higgs bosons to the MSSM fermions, scalars, and gauge bosons.

C. Comparison with MSSM

Before turning to the numerical analysis, it is instructive to compare the origin of Higgs sector *CP* violation in the $U(1)'$ models to that within the MSSM. Let us first consider the case of the MSSM, in which the Higgs sector consists of the two electroweak Higgs doublets $H_{u,d}$. It is useful to start with the most general renormalizable Higgs potential for a two Higgs doublet model (2HDM), which must be built out of the gauge invariant combinations $|H_u|^2$, $|H_d|^2$, and $H_u\cdot H_d$ as follows:

$$\begin{aligned}V_{ren}^{2HDM} &= m_u^2|H_u|^2 + m_d^2|H_d|^2 + (m_3^2 H_u\cdot H_d + \text{H.c.}) + \lambda_1|H_u|^4 \\ &\quad + \lambda_2|H_d|^4 + \lambda_3|H_u|^2|H_d|^2 + \lambda_4|H_u\cdot H_d|^2 \\ &\quad + [\lambda_5(H_u\cdot H_d)^2 - (\lambda_6|H_d|^2 + \lambda_7|H_u|^2)H_u\cdot H_d \\ &\quad + \text{H.c.}],\end{aligned} \tag{51}$$

in which m_3^2, $\lambda_{5,6,7}$ can be complex. In a general 2HDM, the Higgs sector exhibits *CP* violation if any two of these couplings have nontrivial relative phases. Spontaneous *CP* violation can also occur for certain ranges of the parameters [52]. However, at tree level the MSSM is a special 2HDM, with $m_3^2 = B\mu \equiv b$, $m_{u,d}^2 = m_{H_{u,d}}^2$, and

$$\begin{aligned}&\lambda_1=\lambda_2=G^2/4;\lambda_3=(g_2^2-g_Y^2)/4;\\ &\lambda_4=-g_2^2/2;\lambda_5=\lambda_6=\lambda_7=0.\end{aligned} \tag{52}$$

As previously discussed, there is only one complex coupling $B\mu$ in the MSSM Higgs potential at tree level, and hence its phase can always be eliminated by a suitable PQ rotation of the Higgs fields. Although the Higgs sector is *CP*-conserving at tree level, *CP* violation occurs at the loop level if θ_f and/or θ_a are nonzero, with the dominant contribution involving θ_t. If $\theta_t\neq0$, a relative phase θ between the VEVs of H_u and H_d is generated [22].

Essentially, while the $U(1)_{PQ}$ symmetry of the MSSM forbids nonzero values of $\lambda_{5,6,7}$ at tree level, these couplings are generated by radiative corrections because $U(1)_{PQ}$ is softly broken by the $B\mu$ term. For example, the effective λ_5 coupling which is generated at one-loop is approximately

$$\lambda_5 \sim \frac{h_t^2}{16\pi^2 m_{SUSY}^4}(\mu A_t)^2; \tag{53}$$

see [22] for the explicit expressions.[14]

Within the $U(1)'$ models, the tree level Higgs potential does not allow for explicit or spontaneous *CP* violation. However, it is possible to make a stronger statement: unlike the MSSM, the Higgs potential in this class of $U(1)'$ models does not allow for *CP* violation at the renormalizable level at any order in perturbation theory. To see this more clearly, consider the most general renormalizable Higgs potential for H_u, H_d, and S. The potential can be expressed as a function of the gauge-invariant quantities $|H_u|^2$, $|H_d|^2$, $|H_d\cdot H_u|^2$, and $SH_u\cdot H_d$:

$$\begin{aligned}V_{ren}=&m_u^2|H_u|^2+m_d^2|H_d|^2+m_S^2|S|^2\\&+(m_{12}SH_u\cdot H_d+\text{H.c.})+\lambda_u|H_u|^4+\lambda_d|H_d|^4+\lambda_s|S|^4\\&+\lambda_{ud}|H_u|^2|H_d|^2+\lambda_{us}|H_u|^2|S|^2\\&+\lambda_{ds}|H_d|^2|S|^2+\tilde{\lambda}_{ud}|H_u\cdot H_d|^2.\end{aligned}\quad(54)$$

At tree level, the dimensionful parameters $m_{u,d}^2=m_{H_{u,d}}^2$ and $m_{12}=h_sA_s$, and the dimensionless couplings have all been listed before except $\tilde{\lambda}_{ud}=\frac{1}{2}g_2^2-h_s^2$. Therefore, even in the most general renormalizable Higgs potential there is only one coupling which can be complex (m_{12}); this is because the gauge-invariant operator $SH_u\cdot H_d$ is already dimension 3. Hence, the global phases of the Higgs fields (more precisely of the combination SH_uH_d) can always be chosen such that the phase of m_{12} is absorbed. Note that this statement, while true for the tree-level potential of Eq. (6), does not depend in any way on perturbation theory.[15]

As the Higgs potential conserves *CP* to all orders at the renormalizable level, *CP* violation can enter the theory only through loop-induced nonrenormalizable operators. The form of Eq. (17) demonstrates that the one-loop contributions to the Higgs potential include an infinite series of terms involving powers of the Higgs fields. While these terms include contributions to the potential at the renormalizable level, they also include a tower of nonrenormalizable terms, such as

$$V_{nr}=\dots+\left(\frac{\lambda}{m_{SUSY}^2}(SH_u\cdot H_d)^2+\text{H.c.}\right)+\dots,\quad(55)$$

in which m_{SUSY} denotes a typical sfermion mass. By $U(1)_R$ invariance, the coupling λ of the $(SH_u\cdot H_d)^2$ term is proportional to $\lambda\sim A_t^2/(16\pi^2m_{SUSY}^2)$. Such a term is generated by the one-loop diagram formed from the Lagrangian interactions $h_sh_t^*SH_d^0\tilde{u}_L^*\tilde{u}_L^{c*}+\text{H.c.}$ (from F terms) and the soft SUSY breaking interaction $h_tA_t\tilde{u}_L\tilde{u}_L^c+\text{H.c.}$ For $\langle S\rangle\gg\langle H_{u,d}\rangle$, Eq. (55) effectively leads to the coupling[16]

$$\frac{\lambda_5^{eff}}{m_{SUSY}^2}(H_u\cdot H_d)^2,\quad(56)$$

with

$$\lambda_5^{eff}\sim\frac{(A_t\langle S\rangle)^2}{16\pi^2m_{SUSY}^2}.\quad(57)$$

In general, one can expand the one-loop potential in powers of the phase-sensitive gauge-invariant operator $SH_u\cdot H_d$:

$$\begin{aligned}\delta V=&\dots-\beta_{h_t}h_s\mathcal{F}(Q^2,m_{\tilde{t}_1}^2,m_{\tilde{t}_2}^2)A_tSH_u\cdot H_d\\&+\beta_{h_t}h_t^2h_s^2\mathcal{G}(m_{\tilde{t}_1}^2,m_{\tilde{t}_2}^2)\frac{A_t^2}{(m_{\tilde{t}_1}^2-m_{\tilde{t}_2}^2)^2}(SH_u\cdot H_d)^2\\&+\text{H.c.}+\dots,\end{aligned}\quad(58)$$

in which we have presented only the phase-sensitive corrections up to quadratic order (this expansion can of course be continued to higher orders with no difficulty at all). The first term renormalizes the $h_sA_sSH_u\cdot H_d$ operator in the tree level potential, while the second term is a new higher-dimensional operator. Both terms violate *CP* through the phase of $A_t\langle S\rangle$ (recall this phase is irremovable if A_t and A_s have a nontrivial relative phase θ_{ts}). The effective theory at scales below $\langle S\rangle$ is equivalent to the MSSM (with μ and $B\mu$ parameters related to the other soft parameters of the model). One concludes from Eq. (58) that the size of the *CP* violation in the Higgs sector depends on the extent to which the $U(1)'$ breaking scale is split from the electroweak scale. Below the scale $\langle S\rangle$, the coefficients of the *CP*-violating effective operators in Eq. (51) grow with $|A_t|v_s$ (or equivalently $|A_t||\mu_{eff}|$), in agreement with the *CP*-violating $M_{(u,d,s)A}^2$ entries of the Higgs mass-squared matrix.

IV. PHENOMENOLOGICAL IMPLICATIONS

In this section we discuss the existing constraints on $U(1)'$ models as well as their phenomenological implications with explicit *CP* violation.

A. Constraints from Z-Z' mixing

In the previous section, we computed the radiatively corrected Higgs boson mass-squared matrix (36). If the eigenvalues of the Higgs boson mass-squared are all positive definite, the parameter space under concern corresponds to a minimum of the potential. The parameter space is of course

[14]Note that spontaneous *CP* violation (SCPV) requires $m_3^2<\lambda_5v_uv_d$. As λ_5 is loop suppressed in the MSSM, SCPV would require a very small m_3^2, leading to an unacceptably light pseudoscalar Higgs mass [52].

[15]Note that the structure of the potential is very different in the case of the NMSSM, in which the S is a total gauge singlet. As gauge invariance then does not restrict the possible S couplings, the Higgs sector generically violates *CP* at tree level [53].

[16]Note that SCPV is also not viable in this potential, for the same reason as in the MSSM.

also constrained by the fact that direct collider searches have yielded lower bounds on the sparticle and Higgs masses. Within $U(1)'$ models, further constraints arise from the nonobservation to date of a Z', both from direct searches [38] and indirect precision tests from Z pole, LEP II and neutral weak current data [28,39]. The strongest constraints arise from the mixing mass term between the Z and the Z' induced by electroweak breaking,

$$M_{Z-Z'}=\begin{pmatrix} M_Z^2 & \Delta^2 \\ \Delta^2 & M_{Z'}^2 \end{pmatrix}, \tag{59}$$

in which

$$M_Z^2=G^2v^2/4 \tag{60}$$

$$M_{Z'}^2=g_{Y'}^2(Q_u^2v_u^2+Q_d^2v_d^2+Q_s^2v_s^2) \tag{61}$$

$$\Delta^2=\frac{1}{2}g_{Y'}G(Q_uv_u^2-Q_dv_d^2). \tag{62}$$

Current data require $\Delta^2 \ll M_{Z'}^2, M_Z^2$, because the $Z-Z'$ mixing angle

$$\alpha_{Z-Z'}=\frac{1}{2}\arctan\left(\frac{2\Delta^2}{M_{Z'}^2-M_Z^2}\right) \tag{63}$$

must not exceed a few$\times 10^{-3}$ in typical models.

Let us review the implications of this constraint, which was studied in [28,41]. One can see from Eq. (63) that unless $M_{Z'} \gg M_Z$, the $Z-Z'$ mixing angle is naturally of $O(1)$. Therefore, a small $\alpha_{Z-Z'}$ requires a cancellation in the mixing term Δ^2 for a given value of $\tan\beta$. For models in which $M_{Z'} \sim O(M_Z)$, this cancellation must be nearly exact; this can be slightly alleviated if the Z' mass is near its natural upper limit of a few TeV. Hence, $\tan^2\beta$ must be tuned around Q_d/Q_u with a precision determined by the size of $\alpha_{Z-Z'}$. In our analysis, we will eliminate $\tan\beta$ from Eq. (63) for a given value of $\alpha_{Z-Z'}$:

$$\tan^2\beta=\frac{\eta Q_d-\alpha_{Z-Z'}\left[-1+\eta^2\left(Q_d^2+Q_s^2\frac{v_s^2}{v^2}\right)\right]}{\eta Q_u+\alpha_{Z-Z'}\left[-1+\eta^2\left(Q_u^2+Q_s^2\frac{v_s^2}{v^2}\right)\right]}, \tag{64}$$

in which $\eta=2g_{Y'}/G$, and we used $\tan(2\alpha_{Z-Z'}) \approx 2\alpha_{Z-Z'}$. Having fixed $\tan\beta$ in this way, a multitude of parameters remain which can be varied continuously as long as all collider constraints are satisfied. In [41], two phenomenologically viable scenarios were identified:

(i) *Light Z' Scenario*. Clearly, the $U(1)'$ symmetry can be broken along with the SM gauge symmetries at the electroweak scale.[17] In this case $v_s \sim v$, $\tan\beta \sim \sqrt{|Q_d|/|Q_u|}$, and $M_{Z'}$ is of order M_Z (the precise factor depends on the size of $g_{Y'}|Q_s|$). However, the collider constraints on such a light Z' are severe within typical models, and hence it can be accommodated in the spectrum only if it is sufficiently leptophobic. Note that within this framework, leptophobic $U(1)'$ couplings lead to a generic difficulty related to lepton mass generation: as $Q_{H_d} \neq 0$, if the electron mass is induced via the Yukawa coupling $h_e\hat{L}_1\hat{H}_d\hat{E}_1^c$, the leptons necessarily have nonvanishing $U(1)'$ charges. The electron mass (and perhaps all light fermion masses) then must be generated via nonrenormalizable interactions which guarantee the neutrality of $\hat{L}_1$ and $\hat{E}_1^c$ under the $U(1)'$. In practice, this would need to be investigated within specific models.[18]

(ii) *Heavy Z' Scenario.* In this scenario, the $U(1)'$ breaking is radiative (driven by the running of m_S^2 to negative values in the infrared) and occurs at a hierarchically larger scale than the electroweak scale. However, gauge invariance does not allow for the $U(1)'$ and electroweak breakings to decouple completely (as $Q_{H_{u,d}} \neq 0$). The electroweak scale is then achieved by a cancellation among the soft masses, which are typically of $O(M_{Z'})$, with a fine-tuning $O(M_{Z'}/M_Z)$. As discussed in [41], excessive fine tuning is avoided if $M_{Z'}$ in units of the heavy scale is roughly bounded by the ratio of the charges, min$[|Q_s/Q_d|,|Q_s/Q_d|]$. There are several advantages of the heavy Z' scenario. First, the $Z-Z'$ mixing can be kept small enough with less fine-tuning of the Δ^2 in Eq. (59); in particular, $Q_u=Q_d$ is no longer a requirement. In addition, the collider constraints are less severe for Z' bosons with TeV-scale masses in typical models; for example, leptophobic couplings are not generically a phenomenological necessity.

B. Constraints from dipole moments

Let us now turn to dipole moment constraints. Recall in SUSY theories dipole moments of the fundamental fermions

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[17]As shown in [41], at tree level a light Z' boson with a vanishing $Z-Z'$ mixing (for $Q_u=Q_d$) naturally arises when $|A_s|$ is the dominant soft mass in the Higgs potential. Such trilinear coupling induced minima can also accommodate a heavy Z' boson. This can happen in models in which there are several additional singlets in a secluded sector coupled to the Higgs fields $H_{u,d}$ and S via the gauge or gravitational interactions [44]. Furthermore, these large trilinear coupling scenarios (with light Z' bosons) also have interesting implications for baryogenesis, due to the first order phase transition at tree level. If the phase transition remains first order after radiative corrections are included, then $\bar{\theta}$ may be sufficient to generate the baryon asymmetry. The electroweak phase transition in Z' models with a secluded sector is strongly first order (with a heavy enough Z' without any fine-tuning), and electroweak baryogenesis in such models can be viable in a greater region of parameter space than in the minimal model [54].

[18]However, the kinetic mixing between the hypercharge and Z' gauge bosons can be used to decouple leptons from Z' though all leptons, with nonzero $U(1)'$ charges, acquire their masses from their Yukawa couplings [50].

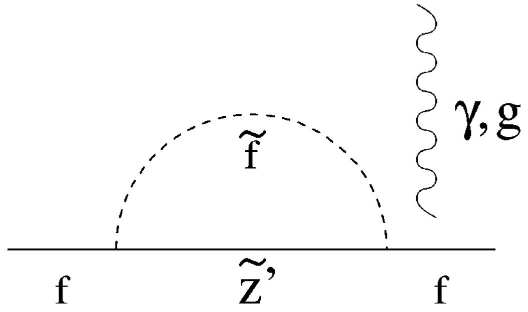

FIG. 1. The $\tilde{Z}'$-sfermion diagram which contributes to the (chromo-)electric and (chromo-)magnetic dipole moments of the fermion *f*. The photon (γ) or gluon (g) are to be attached in all possible ways.

are generated by gaugino/Higgsino exchanges accompanied by sfermions of the appropriate flavor. The dipole moment under concern may (e.g. the electric and chromoelectric dipole moments of the quarks) or may not (e.g. the anomalous magnetic moment of muon) require explicit sources of *CP* violation.

In the MSSM, dipole moments can provide important constraints on the parameter space. For example, the anomalous magnetic moment of the muon is in principle an important observable either for discovering SUSY indirectly or constraining SUSY parameter space; however, at present the theoretical uncertainties present in certain nonperturbative SM contributions lead to difficulties in carrying out this procedure using recent data (see e.g. [55] for a review of the basic physics and [56] for the most recent experimental results). At present, the most stringent constraints arise from electric dipole moments (EDMs). As is well known, the experimental upper bounds on the EDMs of the electron, neutron, and certain atoms impose particularly severe constraints on the parameter space of general SUSY models. In contrast to the SM, in which EDMs are generated only at three-loop order (as the only source of *CP* violation is in the CKM matrix), the sources of explicit *CP* violation in SUSY theories include phases in flavor-conserving couplings which, if present, lead to nonvanishing one-loop contributions to the EDMs which can exceed the experimental bounds. As these phases generically filter into the Higgs sector, it is important to include the parameter space constraints provided by the EDM bounds.

Let us consider the dipole moments which arise within this class of $U(1)'$ models. Here we will neglect any possible (but highly model-dependent) contributions involving exotic matter, although this issue should be examined within specific models on a case-by-case basis. After replacing the μ parameter of the MSSM by μ_{eff} in Eq. (11), all one-loop dipole moments are found to be identical to their MSSM counterparts except for an additional contribution generated by the $\tilde{Z}'-\tilde{f}$ diagram in Fig. 1. Here $\tilde{Z}'$ is the $U(1)'$ gaugino with mass $M_{1'}$. This diagram generates the operator $D_f \tilde{f}_L \sigma^{\mu\nu} F_{\mu\nu} f_R$, in which

$$D_f(\tilde{Z}') \sim \frac{g_{Y'}^2 Q_f^2}{16\pi^2} \frac{m_f |M_{1'}|}{M_{\tilde{f}}^4} [|A_f| e^{i(\theta_{1's} - \theta_{fs})} - R_f |\mu_{eff}| e^{i(\theta_{1's} + \bar{\theta})}], \quad (65)$$

in units of the electromagnetic or strong coupling. In the above $R_f = (\tan\beta)^{-2I_f^3}$, $M_{\tilde{f}}$ characterizes the typical sfermion mass, and recall that the reparametrization invariant phases are defined in Eq. (4). Clearly, the (chromo-)electric and (chromo-)magnetic dipole moments are generated, respectively, by Im$[D_f]$ and Re$[D_f]$. The expression above is an approximate estimate (valid in the limit that $M_{\tilde{f}} \gg M_{1'}$) of the exact amplitude; a more precise treatment would take into account the mixing of all six neutral fermions. The amplitude (65) is similar to the *B*-ino exchange contribution in the MSSM.

Within the aforementioned light Z' scenario, for phenomenologically viable models $D_e(\tilde{Z}')$ vanishes because the lepton couplings to the Z' are necessarily leptophobic. Therefore, for instance, the electron EDM is completely decoupled from the presence of an electroweak scale $U(1)'$ symmetry. This conclusion extends to other leptons for family universal Z' models. This may also be relevant for the hadronic dipole moments depending on whether or not the Z' boson is hadrophobic (assuming it is detected in present and/or forthcoming colliders). As $|\mu_{eff}| \ll M_{\tilde{f}}$ within the light Z' scenario, the dipole moments of both the up-type and down-type fermions are largely controlled by the corresponding A_f parameters. In contrast, the $U(1)'$ charges are not necessarily suppressed for any fermion flavor in the heavy Z' scenario and thus the $D_f(\tilde{Z}')$ contribution to dipole moments can compete with the MSSM amplitudes. In this scenario, $|\mu_{eff}| \sim M_{Z'} \gg M_Z$, and hence both terms in $D_f(\tilde{Z}')$ are important. The dipole moments become sensitive to $\theta_{1's} + \bar{\theta}$ if the A_f parameters are sufficiently small compared to $|\mu_{eff}|$.

As the dipole moments generically scale as $m_f/M_{\tilde{f}}^2$, [which is clear from the form of Eq. (65)], when $M_{\tilde{f}} \sim O(M_Z)$ the EDMs typically exceed the existing bounds by 2 to 3 orders of magnitude if the phases are $O(1)$. As discussed briefly in the Introduction, one possibility for satisfying the experimental bounds while retaining $O(1)$ phases is to raise $M_{\tilde{f}}$ to multi-TeV values, which in effect requires the sfermions of first and second generations to be ultraheavy [2,4]. Another way of suppressing the EDMs is to invoke accidental cancellations between different contributions, i.e., to find regions of parameter space in which the SUSY amplitudes interfere destructively. In the MSSM with low values of $\tan\beta$, this has been shown to occur with almost no constraint on any of the invariant phases except $|\theta_{\tilde{\chi}^\pm}| = |\phi_\mu + \phi_{M_2}| \lesssim \pi/10$ [4–8]. This strong constraint follows from the fact that $g_Y \ll g_2$, and thus the $SU(2)$ gauginos dominate the EDMs. Within the $U(1)'$ framework, the EDM constraints can have varying implications depending on the size of Eq. (65).

(i) If $g_{Y'} \sim O(g_Y)$ or (more generally) $g_{Y'} \ll g_2$, the EDM constraints on the parameter space are similar to that of the

MSSM except for a slight folding of the cancellation domain due to the inclusion of Eq. (65). Once again, the most strongly constrained phase is $\theta_{\tilde{\chi}^{\pm}}$, with $|\theta_{\tilde{\chi}^{\pm}}|\lesssim\pi/10$ in the low $\tan\beta$ regime. As $\theta_{\tilde{\chi}^{\pm}}=\theta_{2s}+\bar{\theta}$ and $\bar{\theta}$ is a loop-suppressed angle (30), the EDMs provide a constraint on θ_{2s}: $|\theta_{2s}|\lesssim\pi/10$. Consequently, the dynamical solution to the μ problem present in this class of $U(1)'$ models also solves the SUSY *CP* hierarchy problem in specific models of the soft parameters in which (at least) the $SU(2)$ gaugino mass has the same phase as the A_s parameter (then $\theta_{2s}=0$ by definition).[19]

(ii) If $g_{Y'}\gtrsim g_2$, the dipole moment amplitude $D_f(\tilde{Z}')$ becomes comparable to or larger than the $SU(2)$ gaugino contribution, and the cancellation domain found in the MSSM will be significantly folded. In this case, the EDMs will constrain a combination of the phases in Eq. (65) and $\theta_{\tilde{\chi}^{\pm}}$. Such a scenario, however, can have tension with the standard picture of gauge coupling unification at a high fundamental scale (although in principle it could be considered as a possibility in generic low scale realizations).

Until this point, we have only discussed one-loop EDMs. It was pointed out a while ago [11] that in certain regions of MSSM parameter space certain two-loop contributions which exclusively depend on the third generation sfermions can be non-negligible. These contributions, which are particularly relevant if the one-loop EDMs are suppressed solely by ultraheavy first and second generation sfermion masses, involve the same phases which predominantly filter into the Higgs potential at one-loop (i.e. the phases present in the top squark mass-squared matrix). However, these two-loop EDMs become sizeable only at large $\tan\beta$. In this paper, we have restricted our attention to small $\tan\beta$ values, which is a well-motivated parameter regime (e.g., $\tan\beta=1$ is allowed within this framework, in contrast to the MSSM), and further neglected the (subleading) gaugino/Higgsino loops in the analysis of the Higgs sector. Hence, neither the one-loop nor the two-loop contributions will provide significant parameter space constraints in our numerical analysis.

C. Numerical estimates for Higgs sector *CP* violation

In this section, we present sample numerical calculations of the Higgs boson masses and mixings derived in Sec. III B, taking into account the phenomenological constraints on the parameter space discussed in Secs. IV A and IV B.

In the absence of *CP* violation, the scalar-pseudoscalar mixing terms of the Higgs mass-squared matrix (36) vanish ($\sin\bar{\theta}=0$), and hence Eq. (36) takes on a block diagonal form. The structure of Eq. (36) demonstrates that in this limit there is one *CP* even scalar with mass $\propto v_s$ and a *CP* odd scalar with mass proportional to $\sqrt{|A_s|v_s}$. In addition, there is a light *CP* even scalar of mass $\sim M_Z$ and a heavier *CP* even scalar with its mass controlled by a combination of v and M_A. However, in the presence of explicit *CP* violation, the Higgs bosons cease to have definite *CP* parities. Including only the dominant loops involving the top squarks, the strength of *CP* violation in the Higgs sector is parametrized by the reparametrization invariant phase θ_{ts}, which induces a nonvanishing $\bar{\theta}$ through the relation (30). The induced phase $\bar{\theta}$ is a loop-induced and scale-dependent quantity which is particularly enhanced in parameter regions with a low M_A.

As discussed in Sec. IV B, while the one-loop EDM constraints strongly constrain the phase θ_{2s}, this phase is not the dominant source of *CP* violation in the Higgs sector for small values of $\tan\beta$ and hence this constraint does not restrict the parameter space for our analysis. The dominant corrections to the Higgs potential arise from top quark and top squark loops, and the dipole moments of the fermions in the first two generations feel such effects only at the two-loop level. In fact, in the low $\tan\beta$ limit (which is the domain in which our analysis of the Higgs potential is valid), such effects are completely negligible [11]. Therefore, the EDM constraints do not have a direct impact on our analysis of *CP* violation in the Higgs sector (we simply assume that the dipole moment constraints have been saturated either via cancellations or by choosing the first and second generation sfermion masses heavy enough; we could also simply assume that all phases except θ_{ts} are small). Of course, in a more general analysis which includes the subdominant contributions from the charginos and neutralinos to *CP* violation in the Higgs sector, the EDM constraints would more significantly constrain the parameter space.

We now turn to the analysis of the parameter space, including the nontrivial constraints arising from $Z-Z'$ mixing. The fundamental parameters relevant for the Higgs sector include $\{v_s, A_s, A_t, M_{\tilde{Q}}, M_{\tilde{U}}, h_s, Q_u, Q_d, g_{Y'}, \theta_{ts}\}$. We fix a subset of these parameters as follows: (i) $\alpha_{Z-Z'}=10^{-3}$, which is well below the present bounds; (ii) $g_{Y'}^2=(5/3)G^2\sin^2\theta_W$, as inspired from one-step GUT breaking; (iii) $h_s=1/\sqrt{2}$, as motivated by the RGE analysis of [41]; (iv) $Q_u=Q_d=-1$, such that $\tan\beta$ remains close to unity [as can be seen from Eq. (64)]; and finally (v) $M_{\tilde{Q}}=M_{\tilde{U}}$. The remaining parameters can be fixed on a case by case basis depending on the range of values assumed for $M_{Z'}$. A few notational comments are also in order. Although Eq. (36) suggests that M_A can be chosen to be a fundamental parameter and this is what is traditionally done in the MSSM, we prefer to work instead with A_s for consistency with previous discussions in this paper as well as the tree level analysis of [41]. In addition, in our numerical results we fix the renormalization scale to be $Q=(2m_t+M_{Z'})/2$. This differs once again from the MSSM, where the renormalization scale is chosen to be $Q=m_t$ in order to minimize the next-to-leading order corrections. Such higher order corrections are beyond the scope of this paper; our choice for Q can be regarded as

[19]In this paper, we have not addressed the origin of the phases of the soft parameters in Eq. (3), and hence we cannot make any claims about how one solves the SUSY *CP* hierarchy problem within this framework. However, it is worthwhile to note that models of the soft parameters in which the gaugino masses and *A* terms have the same phases are quite common within various classes of four-dimensional string models (at least at tree level) under plausible assumptions [57].

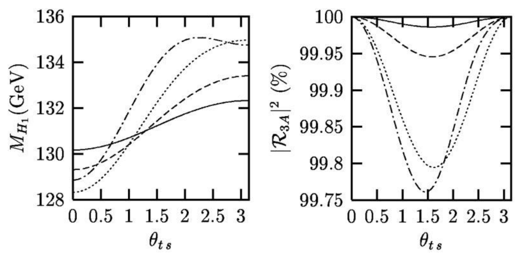

FIG. 2. The θ_{ts} dependence of the lightest Higgs mass and the *CP*-odd composition of H_3 in the light Z' scenario. The solid, dashed, dotted, and dot-dashed curves correspond to, respectively, $|A_t|/v_s = 1/2$, 1, 2 and 4 with $v_s = v/\sqrt{2}$.

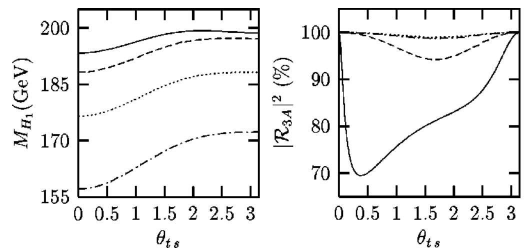

FIG. 3. The θ_{ts} dependence of the lightest Higgs mass (left panel) and the *CP*-odd composition of H_3 in the heavy Z' scenario. Here solid, dashed, dotted, and dot-dashed curves correspond, respectively, to $|A_s|/v_s = 1/5$, 1/2, 3/4 and 1 with $v_s = 1$ TeV.

some nominal value in between the electroweak and $U(1)'$ breaking scales.

We begin with an analysis of the light Z' scenario. For purposes of definiteness, we set $M_{\tilde{Q}} = 2v_s$, $v_s = v/\sqrt{2} \simeq m_t$, and $|A_s| = v_s$, in which case $M_{Z'} \simeq 2M_Z$ and $|\mu_{eff}| \simeq M_Z$. The SUSY phase θ_{ts} influences both the Higgs masses and their mixings, as shown in Fig. 2. In the left panel, the variation of the lightest Higgs mass with θ_{ts} is displayed for several values of $|A_t|/v_s$. For $|A_t|/v_s = 1/2$,1 and 2, M_{H_1} grows gradually with θ_{ts}, peaking at $\theta_{ts} = \pi$. This behavior is easy to understand: as the magnitude of the top squark LR mixing depends strongly on θ_{ts}, the variation of M_{H_1} with respect to θ_{ts} simply displays the well-known fact that the lightest Higgs mass depends strongly on the value of the top squark mixing,

$$\frac{|M^2_{LR}|_{\theta_{ts}=\pi}}{|M^2_{LR}|_{\theta_{ts}=0}} = \frac{|A_t| + |\mu_{eff}|\cot\beta}{|A_t| - |\mu_{eff}|\cot\beta}, \tag{66}$$

which becomes large when $|A_t|$ and $|\mu_{eff}|$ are of comparable size. The ratio (66) gets saturated with further increase of $|A_t|$; however, in this case $|A_t||\mu_{eff}|$ also becomes large, which affects both the M^2_P and M^2_{iA} entries of the Higgs mass-squared matrix. While the former shifts the peak value of M_{H_1} towards the point of maximal *CP* violation (see the dot-dashed curve in the figure), the latter enhances the scalar-pseudoscalar mixings. The generic strength of the scalar-pseudoscalar mixings can be determined e.g. by working out the *CP*-odd composition of H_3 (the would-be pseudoscalar Higgs boson). The result is shown in the right panel of Fig. 2. Clearly, the $M^2_{iA}\sin\theta_{ts}$ elements of the Higgs mass-squared matrix are not large enough to enhance such mixings ($|\mathcal{R}_{3A}|^2$ falls at most to 99.75% for $|A_t| = 4v_s$).

The functional dependence of the heavier Higgs boson masses on θ_{ts} is opposite that of M_{H_1} in that the masses tend to decrease as θ_{ts} ranges from 0 to π; e.g. when $|A_t| = 4v_s$, $(M_{H_4}, M_{H_3}, M_{H_2})$ fall from (245,224,191) to (234,206,182) GeV. In accord with the analytical expression (30), $\overline{\theta}$ grows with $|A_t|$ until it arrives at the peak value of $\sim$30% for $|A_t| = 4v_s$ for maximal *CP* violation. For low $M_{Z'}$ minima, the scalar-pseudoscalar mixings (which govern the novel *CP* violating effects in the Higgs couplings to fermions, gauge bosons and other Higgs bosons) are typically small due to the low value of $|\mu_{eff}|$.

We now discuss the heavy Z' scenario, setting $v_s = 1$ TeV, $M_{\tilde{Q}} = 750$ GeV, and $|A_t| = 2M_{\tilde{Q}}$. Figure 3 depicts the variations of the lightest Higgs mass (left panel) and the *CP*-odd composition of the would-be Higgs scalar as a function of θ_{ts} and $|A_s|$. In both figures, the solid, dashed, dotted, and dot-dashed curves correspond, respectively, to $|A_s|/v_s = 1/5$, 1/2, 3/4 and 1. In contrast to the light Z' scenario as shown in Fig. 2, here we illustrate the dependence on $|A_s|$ (or equivalently M_A), as this parameter remains largely free in the heavy Z' limit [41]. As the left panel of the figure shows, the mass of the lightest Higgs boson is typically larger than that in the light Z' scenario. The lightest Higgs mass is also a steep function of $|A_s|$, which becomes increasingly smaller as M_A increases due to decoupling. Note also that the dependence of M_{H_1} on θ_{ts} in this scenario is similar to the case within the light Z' scenario; once again, this is because the radiative corrections to the lightest Higgs mass strongly depend on the value of the top squark mixing parameter.

However, in contrast to the light Z' scenario, the scalar-pseudoscalar mixings in the heavy $M_{Z'}$ limit are sizeable, as shown in the right panel of Fig. 3. This feature is expected because the strength of the Higgs sector *CP* violation is governed by the size of the singlet VEV, i.e. the effective μ parameter, and in this scenario $|\mu_{eff}| \sim M_{Z'}$. In general, the *CP*-violating mixings grow larger as A_s decreases, because in this case $M^2_{iA}\sin\theta_{ts}$ can be comparable to M_A, which facilitates scalar-pseudoscalar transitions. For $|A_s| = v_s/5$, the *CP*-odd composition of the would-be pseudoscalar Higgs boson falls down to 70% around $\theta_{ts} \sim \pi/6$. However, as $|A_s|$ increases (while keeping $|\mu_{eff}|$ and $|A_t|$ fixed), the diagonal elements of Eq. (36) also increase, with the result that the *CP*-violating effects become weaker. The large variations in $|\mathcal{R}_{3A}|^2$ depicted here are due to the mixings between H_3 and H_2. For $|A_s|/v_s = 1/5$,1/2,3/4 and 1 the two masses are strongly degenerate, with (M_{H_2}, M_{H_3}) starting at (476,477), (722,726), (876,881), (1013,1007) and decreasing to (417,418), (685,688), (846,851), (987,981) GeV as θ_{ts} varies from 0 to π. Note that the scalar-pseudoscalar conversions are more efficient when the two masses are highly degenerate.

For the values of $|A_s|/v_s$ exhibited above, the Higgs sector is within the decoupling regime ($M_A > 2M_Z$),[20] in which the lightest Higgs boson resembles the SM Higgs boson, the heaviest Higgs boson is singlet-dominated with a mass of order $M_{Z'}$, and the two intermediate mass Higgs bosons (the *CP* odd scalar and the second heaviest *CP* even scalar in the absence of *CP* violation) are strongly degenerate. The lightest Higgs boson is essentially *CP* even ($|R_{1A}|^2 \leqslant 0.1\%$ for $|A_s|/v_s \geqslant 1/15$) and hence is decoupled from *CP*-violating effects, although its mass depends strongly on θ_{ts}. However, there are phenomenologically interesting corners of parameter space with sufficiently small values of $|A_s|/v_s$ in which the lightest Higgs boson can have a significant mixing with the would-be pseudoscalar. As the lightest Higgs mass is a steep function of $|A_s|/v_s$, for a value of M_{H_1} consistent with LEP bounds the *CP*-odd composition of H_1 cannot be larger than 20%. It is important to keep in mind though that the couplings of the lightest Higgs boson to gauge bosons and fermions are modified when the lightest Higgs boson has a significant mixing with the would-be pseudoscalar (the modifications grow with the *CP*-odd composition of the lightest Higgs boson), such that the existing LEP bounds may not be applicable (see e.g. [24,25] for discussions within the MSSM).

Our results demonstrate that the *CP*-violating effects in the Higgs sector, or more precisely, the mixing between the would-be scalars and pseudoscalars in the *CP* conserving limit, are generically highly suppressed in the light Z' models but can be sizeable in the heavy Z' scenario, even though the masses can vary strongly with θ_{ts} (which is of course a *CP*-conserving effect). This behavior is exactly in accordance with the general discussion of Sec. III C, in which we demonstrated that the *CP*-violating terms in the Higgs potential necessarily originate from nonrenormalizable terms present at one-loop (such terms are encoded within the full Coleman-Weinberg potential). The strength of such terms in e.g. the doublet sector then scale according to the ratio of the singlet VEV $v_s \simeq |\mu_{eff}|$ to the scale of a typical soft mass. Hence, within the light Z' scenario (in which the effective μ parameter is small) *CP*-violating effects are suppressed, while the large $|\mu_{eff}|$ present in the heavy Z' scenario can allow for spectacular effects of *CP* violation.

We close this section with a brief discussion of the implications for collider searches. In general, at least a subset of the Higgs masses within this class of $U(1)'$ models can be observable at forthcoming colliders. Within light Z' models, all of the Higgs bosons remain light after including radiative corrections, but such models generically have very small *CP*-violating Higgs couplings. In contrast, large Z' models can have large *CP*-violating Higgs couplings. As the viable regions of space typically correspond to the decoupling limit in which all of the Higgs bosons except the lightest Higgs boson are heavy, detecting the *CP*-violating effects within this scenario is similar to that within the MSSM for a large μ parameter. Such effects have been studied in [22,24,25], where it is known that Higgs sector *CP* violation can introduce sizeable modifications in the couplings of the Higgs bosons to fermions and vector bosons, and strongly affect the bounds inferred from the *CP*-conserving theory. Furthermore, the *CP* purity of the Higgs bosons (assuming that the collider searches establish their existence) can be tested by measuring *CP* violation in its decays into heavy quarks or vector bosons [58].

[20]See [22,23] for a more precise definition of the decoupling regime in the *CP*-violating MSSM.

V. SUMMARY

In this paper, we have discussed the nature and implications of explicit *CP* violating phases present in the soft breaking Lagrangian within supersymmetric models with an additional $U(1)$ gauge symmetry and an additional SM gauge singlet *S*. This class of models is worthy of further study not only because such gauge extensions are ubiquitous within four-dimensional string models (and other plausible extensions of the MSSM), but also they provide an elegant framework in which to study the μ problem of the MSSM. The solution, which is to forbid the bare μ term by $U(1)'$ gauge invariance and generate an effective μ parameter through the VEV of the singlet *S*, is similar to that found within the NMSSM (but without its generic cosmological and *CP* problems). Our results can be summarized as follows:

(i) All reparametrization invariant phases can be expressed as linear combinations of $\theta_{fs} \equiv \phi_{A_f} - \phi_{A_s}$ and $\theta_{as} \equiv \phi_{M_a} - \phi_{A_s}$, and hence a "natural" basis can be obtained by using $U(1)_R$ to set $\phi_{A_s} = 0$.

(ii) The Higgs sector is manifestly *CP* conserving at tree level (and at the renormalizable level to all orders in perturbation theory). However, the *CP*-violating phases present in the top squark mass-squared matrix filter into the Higgs sector at the nonrenormalizable level at one-loop. The *CP*-violating effects are particularly enhanced when $U(1)'$ symmetry is broken near the sparticle thresholds.

(iii) The spontaneous breakdown of the $U(1)'$ symmetry near the weak scale stabilizes not only the modulus of μ but also its phase. The phase of μ itself is of course not a basis-independent quantity; however, in the "natural" basis defined above, this phase ($\bar{\theta}$ in this basis) arises only at the loop level and is typically 1–10 %, depending on the size of M_A (the pseudoscalar Higgs boson mass in the *CP* conserving limit).

(iv) The absence of permanent EDMs for leptons and hadrons (even assuming either cancellations and/or heavy first and second generation sfermions) strongly bounds the reparametrization invariant phase present in the chargino mass matrix $(\phi_\mu + \phi_{M_2}) = (\bar{\theta} - \phi_{A_s} + \phi_{M_2})$, while the other SUSY phases remain largely unconstrained. In specific models in which the phase difference between [at least the $SU(2)$] gaugino mass parameters and A_s is vanishingly small, this "SUSY *CP* hierarchy problem" is resolved because the radiative phase $\overline{\theta}$ is sufficiently small to be easily allowed by EDM bounds.

(v) The *CP*-violating effects in the Higgs sector are quite distinct for the two phenomenologically viable scenarios with acceptably small $Z-Z'$ mixing, because these effects are proportional to the size of the effective μ term. In scenarios with a light Z', *CP*-violating effects are suppressed, while heavy Z' models can exhibit significant *CP*-violating scalar-pseudoscalar mixings, with phenomenological implications similar to that of the MSSM with large μ parameter.

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