

# Supersymmetric effects on the forward-backward asymmetries of $B \rightarrow K \tau^+ \tau^-$

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Leptonic and semileptonic rare decays of  $B$  mesons provide significant (both theoretically and experimentally) signatures of any new physics beyond the standard model (SM). More specifically the decay  $B \rightarrow K \ell^- \ell^+$  has been theoretically observed to be very sensitive to new physics, as the forward-backward (FB) asymmetry in this decay mode vanishes in the SM. Supersymmetry, however, predicts a nonvanishing value of this asymmetry. In this work we will study the polarized lepton pair FB asymmetry, i.e. the FB asymmetry of the lepton when one (or both) final state lepton(s) are polarized. We will study these asymmetries both within the SM and for supersymmetric corrections to the SM.

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## I. INTRODUCTION

Lately there has been enormous progress in the study of flavor physics, where the  $B$  system has provided us with one of the most ideal environments for this type of study. Of the decay modes considered, theoretically and experimentally, the study of the “rare decays” of  $B$  mesons is of particular interest. Here the name *rare* has been given to those decay modes which arise from flavor changing neutral currents (FCNC). FCNC processes are absent at the tree level in the standard model (SM) but can occur through loop diagrams, their strength being proportional to the Fermi constant. FCNC processes involving, for example  $b \rightarrow s$  and  $b \rightarrow d$  transitions are therefore more sensitive to the details of the SM interactions and thus are suited to the study of possible new physics beyond the SM. Theoretically inclusive FCNC processes such as  $B \rightarrow X_{s(d)} \ell^+ \ell^-$  are relatively cleaner than their exclusive counterparts, since they are relatively independent of the quark structure of the hadrons involved. They are however difficult to measure experimentally (for details, refer to Chap. 7 in Ref. [1]) and one may expect a substantial amount of experimental information regarding various exclusive  $B$  decay processes from  $B$  factories. Amongst the important FCNC exclusive  $B$ -decay processes are  $B \rightarrow$  charmless meson  $+\gamma$  and  $B \rightarrow$  charmless meson  $+\text{lepton pair}$ . This second process potentially provides a very rich set of experimental observables involving various momenta and spin polarization correlations. It is thus important to theoretically calculate all possible measurable parameters of these processes.

In recent times there have been many calculations of such

processes such as  $B \rightarrow K(K^*) \ell^+ \ell^-$  [2–4],  $B \rightarrow \pi(\rho) \ell^+ \ell^-$  [5],  $B_{s,d} \rightarrow \ell^+ \ell^-$  [6] and  $B_{s,d} \rightarrow \ell^+ \ell^- \gamma$  [7]. Amongst these, the ones involving a quark level  $b \rightarrow s$  transition are expected to have relatively large branching ratios. For the  $B \rightarrow K^* \ell^+ \ell^-$  transition possible experimentally accessible parameters such as forward-backward asymmetry, lepton polarization asymmetry, etc., have been studied [2,4,8,9]. In particular, polarization correlations between the two leptons, which were suggested recently by Bensalem *et al.* [10], have also been studied in the context of this process [11]. In this paper we carry out an analysis of the polarized forward-backward (FB) asymmetries in the process  $B \rightarrow K \ell^+ \ell^-$ .

The theoretical basis for the study of FCNC  $B$ -decay processes is now a well established formalism based on the operator product expansion and use of the renormalization group [12]. The formalism ultimately produces an effective Hamiltonian for every process involving low dimensional hadronic operators, in the form of currents, with numerical multiplying coefficients called the Wilson coefficients. The Wilson coefficients encrypt short distance properties of the weak Hamiltonian and are sensitive to physics beyond the SM at high energy scales, in particular, to supersymmetric extensions of the SM. A great deal of theoretical work has gone, in recent times, to evaluating their values both within the SM and in the context of the minimal extension of the standard model (MSSM). Evaluation of the matrix elements of the hadronic currents on the other hand involves the relatively long distance quark structure of the hadrons. The most ideal way to evaluate them would be through lattice gauge theory calculations, which do not exist for all  $B$ -meson processes at the moment. Alternatively, one relies on evaluations based on light cone sum rules and these also have been compiled for a variety of processes. We shall make extensive use of these in our calculations.

This paper is organized as follows. In Sec. II we will discuss the effective Hamiltonian for the process under consideration. In Sec. III we will introduce our notation for the

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polarized FB asymmetry. Finally, in Sec. IV we will present results of our numerical analysis and the conclusions.

## II. EFFECTIVE HAMILTONIAN

In this paper we are interested in the process  $B \rightarrow K \ell^- \ell^+$ , which has the basic quark level transition  $b \rightarrow s$ . The effective Hamiltonian for such a transition has been summarized in the literature [12]. The effective Hamiltonian is arrived at by integrating out the heavy degrees of freedom from the full theory. In the SM the heavy degrees of freedom are  $W^\pm, Z$  and the top quark; in the MSSM all the new SUSY particles shall also be counted. From such considerations we arrive at the effective Hamiltonian [13,29–31],

$$\mathcal{H}_{eff} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[ \sum_{i=1}^{10} C_i(\mu) O_i(\mu) + \sum_{i=1}^{10} C_{Q_i}(\mu) Q_i(\mu) \right] \quad (2.1)$$

where the  $O_i$  are the current-current ( $i=1,2$ ), penguin ( $i=3, \dots, 6$ ), magnetic penguin ( $i=7,8$ ) and semileptonic ( $i=9,10$ ) operators, whereas the  $C_i(\mu)$  are the corresponding Wilson coefficients renormalized at scale  $\mu$ . The value of these coefficients has been given in Refs. [14,15]. The additional operators  $Q_i$  ( $i=1, \dots, 10$ ) and their corresponding Wilson coefficients are due to the neutral Higgs boson (NHB) exchange diagrams and are given in Refs. [6,13,30].

Different FCNC decays involve different combinations of  $C_i$ 's and  $C_{Q_i}$ 's and thus provide us with independent information on these coefficients. At the quark level the transition  $b \rightarrow s \gamma$  is sensitive only to the magnitude of  $C_7$ , whereas the semileptonic transition  $b \rightarrow s \ell^- \ell^+$  is sensitive to  $C_9, C_{10}, C_{Q_1}$  and  $C_{Q_2}$  as well.<sup>1</sup>

From the effective Hamiltonian given in Eq. (2.1) the decay amplitude for  $B \rightarrow K \ell^- \ell^+$  is calculated to be

$$\begin{aligned} \mathcal{M} = \frac{\alpha G_F}{\sqrt{2} \pi} V_{tb} V_{ts}^* \left\{ -2C_7^{eff} \frac{m_b}{q^2} (\bar{s} i \sigma_{\mu\nu} q^\nu P_R b) (\bar{\ell} \gamma^\mu \ell) \right. \\ + C_9^{eff} (\bar{s} \gamma_\mu P_L b) (\bar{\ell} \gamma^\mu \ell) + C_{10} (\bar{s} \gamma_\mu P_L b) (\bar{\ell} \gamma^\mu \gamma_5 \ell) \\ \left. + C_{Q_1} (\bar{s} P_R b) (\bar{\ell} \ell) + C_{Q_2} (\bar{s} P_R b) (\bar{\ell} \gamma_5 \ell) \right\} \quad (2.2) \end{aligned}$$

where  $q$  is the momentum transferred to the lepton pair, given as  $q = p_- + p_+$ , where  $p_-$  and  $p_+$  are the momenta of  $\ell^-$  and  $\ell^+$  respectively. The  $V_{tb} V_{ts}^*$  are the CKM factors and  $P_{L,R} = (1 \mp \gamma_5)/2$ . In writing the above matrix element (and in future analysis) we will neglect the mass of the strange quark, whereas lepton masses shall be retained.

The free quark decay amplitude given in Eq. (2.2) contains certain long distance effects which are absorbed in the

<sup>1</sup>It has also been shown in many works that  $b \rightarrow s \ell^- \ell^+$  is sensitive even to the signs of these Wilson coefficients and hence this decay channel will provide information not only on the magnitude but also the sign of these coefficients.

redefinition of the  $C_9$  Wilson coefficient (where we use the prescription given in Ref. [16]),

$$C_9^{eff}(\hat{s}) = C_9 + Y(\hat{s}). \quad (2.3)$$

The  $Y(\hat{s})$  part has a perturbative as well as a nonperturbative part. The origin of the nonperturbative part is from the resonance corrections to the perturbative quark loops [which gives the perturbative contribution to  $Y(\hat{s})$ ]. We will also use the usual Breit-Wigner prescription to take care of the resonant contribution [9,16]. This prescription implies adding resonant terms to  $C_9^{eff}$ ,

$$C_9^{res} \propto \kappa \sum_{V=\psi} \frac{\hat{m}_V Br(V \rightarrow \ell^- \ell^+) \hat{\Gamma}_{total}^V}{\hat{s} - \hat{m}_V^2 + i \hat{m}_V \hat{\Gamma}_{total}^V} \quad (2.4)$$

where all the symbols above have been explained in the work of Krüger and Sehgal [9]. For the phenomenological factor,  $\kappa$ , we will choose a value 2.3.

Using the definitions of the form factors given in Appendix A we can write the matrix element given in Eq. (2.2) as

$$\begin{aligned} \mathcal{M} = \frac{\alpha G_F}{2\sqrt{2} \pi} V_{tb} V_{ts}^* \{ A(p_K)_\mu (\bar{\ell} \gamma^\mu \ell) + B(p_K)_\mu (\bar{\ell} \gamma^\mu \gamma_5 \ell) \\ + C(\bar{\ell} \ell) + D(\bar{\ell} \gamma_5 \ell) \} \quad (2.5) \end{aligned}$$

where the coefficients  $A, B, C$  and  $D$  in Eq. (2.11) are given as<sup>2</sup>

$$A = 4f_T \frac{\hat{m}_b C_7^{eff}}{1 + \hat{m}_K} + 2f_+ C_9^{eff} \quad (2.6)$$

$$B = 2f_+ C_{10} \quad (2.7)$$

$$C = f_0 \frac{1 - \hat{m}_K^2}{\hat{m}_b} C_{Q_1} \quad (2.8)$$

$$\begin{aligned} D = f_0 \frac{1 - \hat{m}_K^2}{\hat{m}_b} C_{Q_2} + 2\hat{m}_\ell C_{10} f_+ \\ - 2\hat{m}_\ell f_+ \frac{1 - \hat{m}_K^2}{\hat{s}} C_{10} + 2\hat{m}_\ell f_0 \frac{1 - \hat{m}_K^2}{\hat{s}} C_{10}. \quad (2.9) \end{aligned}$$

With the above expression of the matrix element, Eq. (2.5), we can obtain the expression for the differential decay rate as

<sup>2</sup>In writing Eq. (2.11) we have used the equations of motion:  $p_\mu \bar{\ell} \gamma^\mu \ell = 0$ ;  $p_\mu \bar{\ell} \gamma^\mu \gamma_5 \ell = 2m_\ell (\bar{\ell} \ell)$ .

$$\frac{d\Gamma}{d\hat{s}} = \frac{\alpha^2 G_F^2 m_B^5}{2^{12} \pi^5} |V_{tb} V_{ts}^*|^2 \lambda^{1/2} \sqrt{1 - \frac{4\hat{m}_\ell^2}{\hat{s}}} \Delta_{up} \quad (2.10)$$

where

$$\begin{aligned} \Delta = & \frac{2}{3} \lambda \left( 1 + \frac{2\hat{m}_\ell^2}{\hat{s}} \right) |A|^2 + \frac{2}{3} \left[ \lambda \left( 1 + \frac{2\hat{m}_\ell^2}{\hat{s}} \right) + 24\hat{m}_K^2 \hat{m}_\ell^2 \right] |B|^2 \\ & + 4(\hat{s} - 4\hat{m}_\ell^2) |C|^2 + 4\hat{s} |D|^2 \\ & + 8\hat{m}_\ell(1 - \hat{m}_K^2 - \hat{s}) \text{Re}(B^* D) \end{aligned} \quad (2.11)$$

with  $\lambda = 1 + \hat{m}_K^4 + \hat{s}^2 - 2\hat{m}_K^2 \hat{s} - 2\hat{m}_\ell^2 - 2\hat{s}$ . In the next section we will use this expression for the differential decay rate to introduce and define the polarized FB asymmetries, followed by our analytical expressions for these asymmetries.

### III. POLARIZED FB ASYMMETRIES

Firstly we will define the polarization vectors of  $\ell^-$  and  $\ell^+$ , where in this definition we will use the convention followed in many earlier works [2,3,9,11]. In order to evaluate the polarized FB asymmetries we introduce a spin projection operator defined by  $N = (1 + \gamma_5 \mathcal{S}_x)/2$  for  $\ell^-$  and  $M = (1 + \gamma_5 \mathcal{W}_x)/2$  for  $\ell^+$ , where  $x = L, N$ , or  $T$  (corresponding to the longitudinal, normal and transverse polarization asymmetries respectively). Next we define the orthogonal unit vectors  $S_x$  for  $\ell^-$  and  $W_x$  for  $\ell^+$  in the rest frames of  $\ell^-$  and  $\ell^+$  respectively as

$$\begin{aligned} S_L^\mu &\equiv (0, \mathbf{e}_L) = \left( 0, \frac{\mathbf{p}_-}{|\mathbf{p}_-|} \right) \\ S_N^\mu &\equiv (0, \mathbf{e}_N) = \left( 0, \frac{\mathbf{p}_K \times \mathbf{p}_-}{|\mathbf{p}_K \times \mathbf{p}_-|} \right) \\ S_T^\mu &\equiv (0, \mathbf{e}_T) = (0, \mathbf{e}_N \times \mathbf{e}_L) \\ W_L^\mu &\equiv (0, \mathbf{w}_L) = \left( 0, \frac{\mathbf{p}_+}{|\mathbf{p}_+|} \right) \\ W_N^\mu &\equiv (0, \mathbf{w}_N) = \left( 0, \frac{\mathbf{p}_K \times \mathbf{p}_+}{|\mathbf{p}_K \times \mathbf{p}_+|} \right) \\ W_T^\mu &\equiv (0, \mathbf{w}_T) = (0, \mathbf{w}_N \times \mathbf{w}_L) \end{aligned} \quad (3.1)$$

where  $\mathbf{p}_-, \mathbf{p}_+$  and  $\mathbf{p}_K$  are the three momenta of  $\ell^-, \ell^+$  and the K-meson in the dilepton center-of-mass (CM) frame. From the rest frames of the leptons we boost the four vectors  $S_x$  and  $W_x$  to the dilepton CM frame. Only the longitudinal vectors  $S_L$  and  $W_L$  will be boosted by the Lorentz transformation, to a value of

$$\begin{aligned} S_L^\mu &= \left( \frac{|\mathbf{p}_-|}{m_\ell}, \frac{E_\ell \mathbf{p}_-}{m_\ell |\mathbf{p}_-|} \right) \\ W_L^\mu &= \left( \frac{|\mathbf{p}_+|}{m_\ell}, -\frac{E_\ell \mathbf{p}_-}{m_\ell |\mathbf{p}_-|} \right) \end{aligned} \quad (3.2)$$

where  $E_\ell$  is the energy of either of the leptons (both having the same energy in this frame) in the dileptonic CM frame.

The definition of the differential forward-backward (FB) asymmetry is given in Refs. [8,17],

$$\bar{A}(\hat{s}) = \int_0^1 \frac{d^2\Gamma}{d\hat{s}dz} dz - \int_{-1}^0 \frac{d^2\Gamma}{d\hat{s}dz} dz. \quad (3.3)$$

Consider the case where we shall not sum over the spins of the outgoing leptons. In general the FB asymmetry will be a function of the spins of the final state leptons, and as such can be defined as

$$\bar{A}(s^-, s^+, \hat{s}) = \int_0^1 \frac{d^2\Gamma(s^-, s^+)}{d\hat{s}dz} dz - \int_{-1}^0 \frac{d^2\Gamma(s^-, s^+)}{d\hat{s}dz} dz. \quad (3.4)$$

From an experimental viewpoint the normalized FB asymmetry is more useful. Therefore we shall normalize the above expression [Eq. (3.5)] for the FB asymmetry by dividing by the total decay rate,

$$A(s^-, s^+, \hat{s}) = \frac{\bar{A}(s^-, s^+, \hat{s})}{d\Gamma/d\hat{s}}. \quad (3.5)$$

In analogy to the prescription given in Bensalem *et al.* [10] we can split this FB asymmetry into various polarization components,<sup>3</sup>

$$A(s^-, s^+) = A + A_i^- s_i^- + A_i^+ s_i^+ + A_{ij} s_i^+ s_j^- \quad (3.6)$$

where  $i, j = L, T, N$  are the longitudinal, transverse and normal components of the polarization. Using this definition we can write the single and double lepton polarized FB asymmetries. From Eq. (3.7) the single polarized lepton FB asymmetry can be written as

$$\begin{aligned} A_i^- &= A(s^-, s^+ = j) + A(s^-, s^+ = -j) \\ &\quad - A(s^- = -i, s^+ = j) - A(s^- = -i, s^+ = -j) \end{aligned} \quad (3.7)$$

$$\begin{aligned} A_i^+ &= A(s^- = j, s^+ = i) + A(s^- = -j, s^+ = i) \\ &\quad - A(s^- = j, s^+ = -i) - A(s^- = -j, s^+ = -i) \end{aligned} \quad (3.8)$$

and the doubly polarized FB asymmetry can be written as

$$\begin{aligned} A_{ij} &= A(s^- = i, s^+ = j) - A(s^- = i, s^+ = -j) \\ &\quad - A(s^- = -i, s^+ = j) + A(s^- = -i, s^+ = -j). \end{aligned} \quad (3.9)$$

<sup>3</sup>The convention followed is that the repeated index is summed over.

Using the above expressions of the FB asymmetries the results of the unpolarized FB asymmetry are evaluated to be

$$A = 2\hat{m}_\ell \lambda \sqrt{1 - \frac{4\hat{m}_\ell^2}{\hat{s}}} \frac{\text{Re}(A^*C)}{\Delta}. \quad (3.11)$$

From the expression given above and Eq. (2.8), we can see that the unpolarized FB asymmetry is proportional to  $C_{Q_1}$ . This point has been emphasized in many earlier works [5,18]. In the SM,  $C_{Q_1}$  is absent and hence for the decay modes  $B \rightarrow K(\pi)\ell^+\ell^-$  the FB asymmetry within the SM vanishes. However, in SUSY (and 2HDM) extensions of the SM there exists a nonvanishing value of  $C_{Q_1}$ , and hence a nonvanishing value of the FB asymmetry [5,18] is possible. Therefore a nonvanishing value of the FB asymmetry can be regarded as a clear signal of new physics beyond the SM.

The analytical results of the polarized FB asymmetries are

$$A_L^- = \frac{4\lambda^{1/2}\hat{m}_\ell}{\Delta_{up}} [\hat{m}_\ell(-1 + \hat{m}_K^2 + \hat{s})\text{Re}(A^*B) - \hat{m}_\ell\text{Re}(A^*D)] \quad (3.12)$$

$$A_N^- = 0 \quad (3.13)$$

$$A_T^- = \frac{4\hat{m}_\ell}{3\hat{s}\Delta} \sqrt{1 - \frac{4\hat{m}_\ell^2}{\hat{s}}} \lambda \text{Re}(A^*B) \quad (3.14)$$

$$A_L^+ = A_L^- \quad (3.15)$$

$$A_N^+ = 0 \quad (3.16)$$

$$A_T^+ = A_T^- \quad (3.17)$$

$$A_{LL} = A \quad (3.18)$$

$$A_{LN} = \frac{4\hat{m}_\ell}{3\sqrt{\hat{s}}\Delta} \sqrt{1 - \frac{4\hat{m}_\ell^2}{\hat{s}}} \lambda \text{Im}(A^*B) \quad (3.19)$$

$$A_{LT} = \frac{4\hat{m}_\ell}{3\sqrt{\hat{s}}} \sqrt{1 - \frac{4\hat{m}_\ell^2}{\hat{s}}} \lambda \frac{|A|^2}{\Delta} \quad (3.20)$$

$$A_{NL} = A_{LN} \quad (3.21)$$

$$A_{NN} = -A \quad (3.22)$$

$$A_{NT} = 4\hat{m}_\ell \lambda^{1/2} \frac{(1 - \hat{m}_K^2 - \hat{s})\hat{m}_\ell \text{Im}(A^*B) + \text{Im}(A^*D)}{\Delta} \quad (3.23)$$

$$A_{TL} = -A_{LT} \quad (3.24)$$

$$A_{TN} = A_{NT} \quad (3.25)$$

$$A_{TT} = A \quad (3.26)$$

TABLE I. The SM predictions for the integrated observables.

$\text{Br}(B \rightarrow K \tau^- \tau^+)$	$A$	$A_L^-$	$A_T^-$	$A_{LN}$	$A_{LT}$	$A_{NT}$
$1.17 \times 10^{-7}$	0	0.363	-0.097	0.023	0.187	.0847

where  $\Delta$  is given in Eq. (2.11) and  $A$  is the unpolarized FB asymmetry given in Eq. (3.11). We will discuss the above obtained expressions of the various FB asymmetries and present our numerical analysis of the same in the next section.

#### IV. NUMERICAL ANALYSIS, RESULTS AND CONCLUSION

In this section we shall present our numerical analysis of the observables whose analytical expressions were given in the previous section. We will also present the variation of all the observables with the dilepton invariant mass.

As it is experimentally more useful to have the average values of these quantities we shall present our results as the average values of these quantities, where we will define our averages as

$$\langle A \rangle \equiv \frac{\int_{(3.646+0.02)^2/m_B^2}^{(m_B-m_K)^2/m_B^2} A \frac{d\Gamma}{d\hat{s}} d\hat{s}}{\int_{(3.646+0.02)^2/m_B^2}^{(m_B-m_K)^2/m_B^2} \frac{d\Gamma}{d\hat{s}} d\hat{s}} \quad (4.1)$$

which means that in calculating the average we have taken the lower limit of integration to be above the first resonance.<sup>4</sup> The input parameters of our numerical analysis are defined in Appendix B, and our SM predictions of the integrated observables are given in Table I.

Before discussing our results we shall first elaborate on the models in which we have performed our numerical analysis. We have worked with the minimal supersymmetric extension of the standard model (MSSM), this being the simplest SUSY extension of the SM with the least number of parameters introduced [19]. But even in the MSSM we are required to introduce a large number of parameters, over and above the number of parameters in the SM. To ease out this problem and to reduce such a large number of parameters to a more manageable level, many models have been introduced such as the dilaton, moduli, minimal supergravity (MSUGRA) [20], AMSB (anomaly mediated SUSY breaking) [21] and the GMSB (gauge mediated SUSY breaking) [22] models. The generic feature of all these models is that they assume some sort of unification of the parameters of the MSSM at a higher scale. In the literature the MSUGRA model is also known as the CMSSM (constraint MSSM) [23]. We shall further assume that the soft SUSY breaking parameters are real. For our numerical analysis we will use two types of SUGRA (supergravity) models, namely MSUGRA and RSUGRA (relaxed SUGRA) which we de-

<sup>4</sup>The first resonance here means the resonance after the threshold of the decay, which is  $s \geq 4m_\tau^2$ .



scribe. In both these models it is believed that the SUSY breaking occurs in a hidden sector and is communicated to the visible sector only by gravitational-strength interactions. As such, soft breaking terms are assumed to be flavor blind (like gravitational interactions).

In the MSUGRA model along with the unification of the coupling constants  $g_{1,2,3}$  [of the U(1), SU(2) and SU(3) gauge theories], the other unification conditions are as follows: (i) unification of the gaugino masses ( $m_{1/2}$ ), (ii) unification of the scalar (sfermion and Higgs) masses ( $m_0$ ), and (iii) unification of the trilinear couplings ( $A$ ), all at the GUT scale. There are two other parameters. The first is the ratio of VEV (vacuum expectation value of two Higgs bosons), namely  $\tan \beta$ . The second arises in the process of evolving the soft SUSY breaking parameters from the GUT scale to the electroweak scale and then imposing the correct low energy electroweak symmetry breaking condition. This condition fixes the magnitude of  $\mu$  (the two Higgs coupling parameter), however, the sign still remains uncertain.<sup>5</sup> The sign of  $\mu$  thus also enters taken as a parameter.

Therefore, in all MSUGRA frameworks we have five parameters, namely

$$m_{1/2}, m_0, A, \tan \beta \text{ and } \text{sgn}(\mu) \quad (4.2)$$

Unification of all the scalars and also all the gauginos is not an essential requirement of SUGRA models. One can have models where either all the scalars do not have a universal mass at GUT scale or there is a nonuniversality of gaugino masses at the GUT scale. We shall also explore such a model, where we would relax the condition of the universality of the scalar masses at the GUT scale [24]. In the literature these models are known as nonminimal SUGRA models [24]. We will call such a model the relaxed SUGRA (RSUGRA) model. We will assume that the values of squarks and the Higgs sector scalar masses differ at the GUT scale. This shall introduce another parameter into the model. We will take this additional parameter as the mass of the pseudoscalar Higgs boson ( $m_A$ ). However for our numerical analysis we will consider only the region of the SUGRA parameter space which is consistent with the  $B \rightarrow X_s \gamma$  95% C.L. [1,25],

$$2 \times 10^{-4} \leq \text{Br}(B \rightarrow X_s \gamma) \leq 4.5 \times 10^{-4}.$$

We present our results for the various decay rates and asymmetry parameters considered in Figs. 1–21. The branching fractions of the decays considered are of course too low to be observed with the current luminosities of the  $B$  factories, but they will certainly be possible in the foreseeable future. As can be seen from the expressions of the polarized FB asymmetry, they are sensitive to the Wilson coefficients that arise only beyond the standard model, and thus a measurement of these would be one more test of physics beyond the SM.

<sup>5</sup>There are many conventions followed regarding the sign of  $\mu$ . Our convention is such that  $\mu$  appears in the chargino mass matrix with a positive sign [where the  $(g-2)_\mu$  giving the parameter  $\mu$  a negative sign is disfavored].

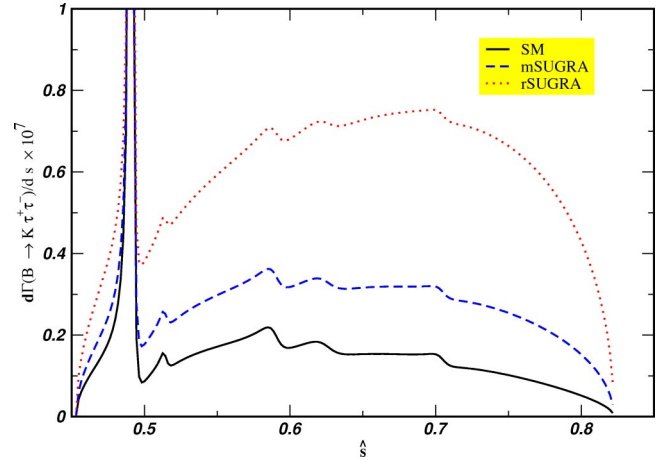


FIG. 1. The differential decay rate  $d\Gamma/d\hat{s}$  against  $\hat{s}$ . The parameters for the MSUGRA model are  $m_0=400$  GeV,  $m_{1/2}=500$  GeV,  $\tan \beta=40$  and  $A=0$ . The additional parameter for the RSUGRA models is taken to be  $m_A=345$  GeV.

We now turn to the uncertainties in the estimates we obtained, where the Wilson coefficients we used are in the NLL approximation and do not introduce significant uncertainties. The CKM parameters typically have uncertainties of the order of 10% [26] while the other SM parameters involved do not suffer from any large uncertainties. The SUSY parameters are input parameters. For a particular choice of parameters the major uncertainty in our results arises from the definition of the form factors used. They typically have a 15% uncertainty [27]. There are regions in the SUSY parameter space where SUSY effects exceed the kind of errors which are introduced by the form factor definition, for example the graph of the branching ratio of the mode concerned,  $B \rightarrow K \ell^- \ell^+$  in Figs. 1, 2, and 3. Of course, at the present level an uncertainty of 20% modifications arising from SUSY or any other physics beyond the SM, or levels less than this, would not be distinguishable from SM results. We, however, emphasize that the null polarization results which we have pointed out for some of the double polarization

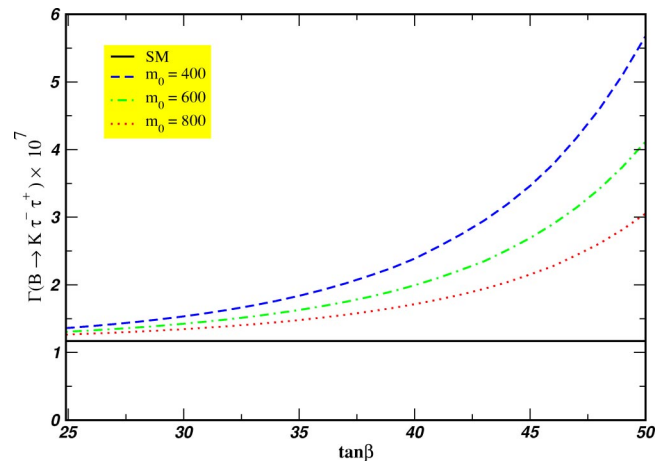


FIG. 2.  $\text{Br}(B \rightarrow K \tau^- \tau^+)$  variation with  $\tan \beta$  in MSUGRA for various values of  $m_0$ . Other model parameters are  $m_{1/2}=500$  GeV,  $A=0$ .

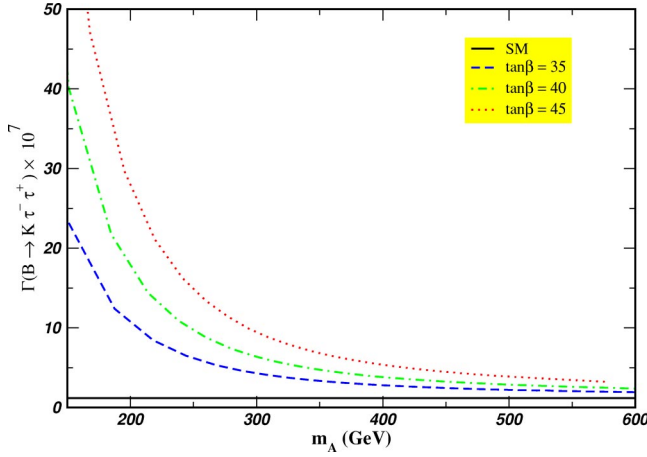


FIG. 3.  $\text{Br}(B \rightarrow K \tau^- \tau^+)$  variation with  $m_A$  in RSUGRA for various  $\tan \beta$  values. Other model parameters are  $m_0 = 500$  GeV,  $m_{1/2} = 500$  GeV,  $A = 0$ .

asymmetries (for a mixture of  $B_0$  and  $\bar{B}_0$ ) do not depend on the parameters. The experimental deviation of these from null values would indicate the presence of interactions beyond the one considered here, or may indicate the source of  $CP$  violation in the SUSY extension, for example a complex value of  $\mu$ .

Regarding our numerical analysis we have presented all the observables in Sec. III, where the plots are given for all the possible observables with the dilepton invariant mass ( $\hat{s}$ ), in Figs. 1, 4, 7, 10, 13, 16, and 19 for the SM, MSUGRA and RSUGRA models. We have also tested the sensitivity of the observables to various MSSM parameters. For this purpose we have also presented the results of the averaged values of the observables. For averaging we have used the procedure defined in Eq. (4.1). The plots of averaged observables with  $\tan \beta$  for various values of  $m_0$  are given in Figs. 2, 5, 8, 11, 14, 17, and 20 in the MSUGRA model. The other model parameters are given in respective figure captions. In the RSUGRA model we have plotted the averaged value of the observables as a function of pseudo-

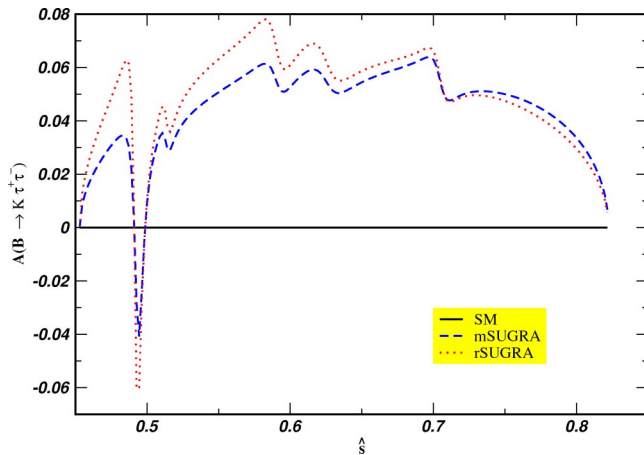


FIG. 4. Unpolarized FB asymmetry with dilepton invariant mass  $\hat{s}$ . Other parameters are of MSUGRA and RSUGRA.

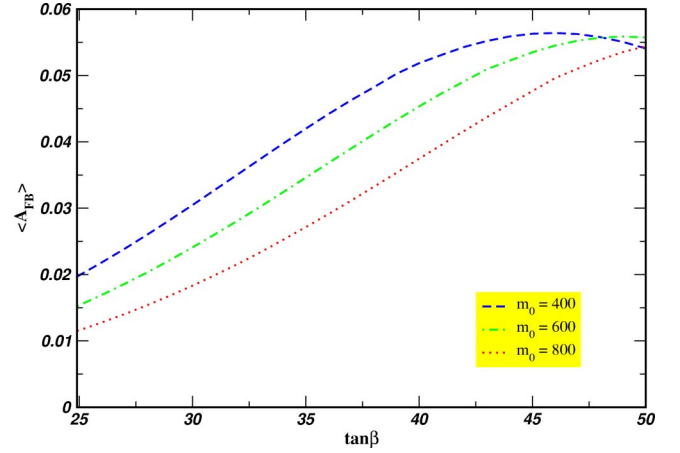


FIG. 5. Integrated FB asymmetry variation with  $\tan \beta$  in MSUGRA model. Other parameters are the same as given in Fig. 2.

scalar Higgs mass ( $m_A$ ) for various values of  $\tan \beta$  in Figs. 3, 6, 9, 12, 15, 18, and 21.

As can be seen from the plots all the observables are very sensitive to the various MSSM parameters, both in the MSUGRA and RSUGRA models, as may have been expected. However, these plots of the deviation of the observables from their respective SM values are more pronounced for the RSUGRA model than the MSUGRA model. This is essentially due to the additional parameter in the RSUGRA model, which is controlled by the pseudoscalar Higgs mass. The value of the new Wilson coefficients corresponding to scalar and pseudoscalar operators is directly proportional to  $\tan^3 \beta$  and  $m_A^2$ , and the lower the value of  $m_A$ , the higher the value of those Wilson coefficients. Similarly, the higher the  $\tan \beta$ , the higher the new Wilson coefficients. We effectively have for the RSUGRA model a low  $m_A$  and high  $\tan \beta$  region of MSSM parameter space also available which can generate large values of the new Wilson coefficients. This was not the case in the MSUGRA model.

In Fig. 2 we have plotted of the integrated branching ratio in the MSUGRA. The integrated branching ratio can, at best,

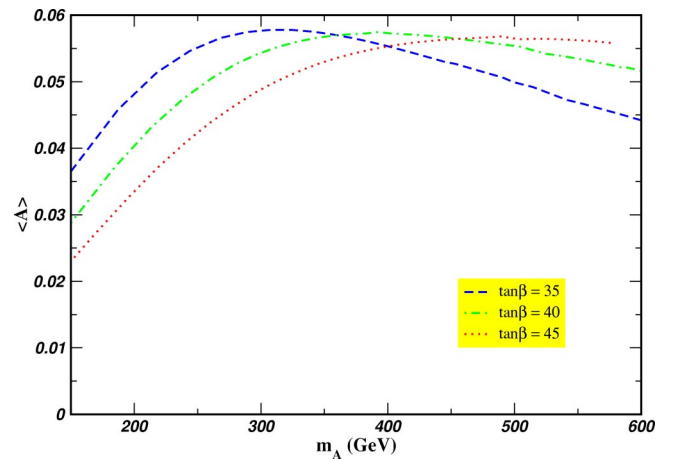


FIG. 6. Integrated FB asymmetry variation with  $m_A$  in RSUGRA model, with other model parameters as given in Fig. 1.

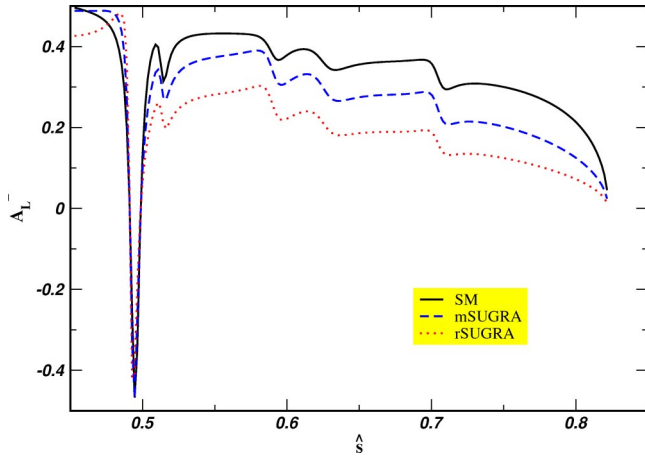


FIG. 7.  $A_L^-$  variation with  $\hat{s}$ . Other parameters of MSUGRA and RSUGRA model are the same as given in Fig. 1.

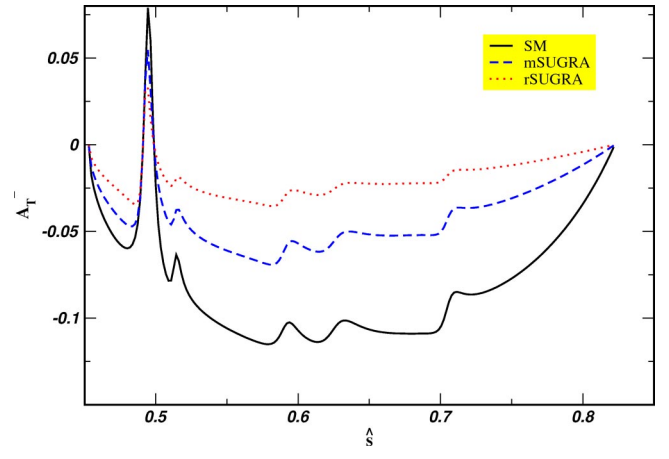


FIG. 10.  $A_T^-$  with  $\hat{s}$ . Other parameters are the same as given in Fig. 1.

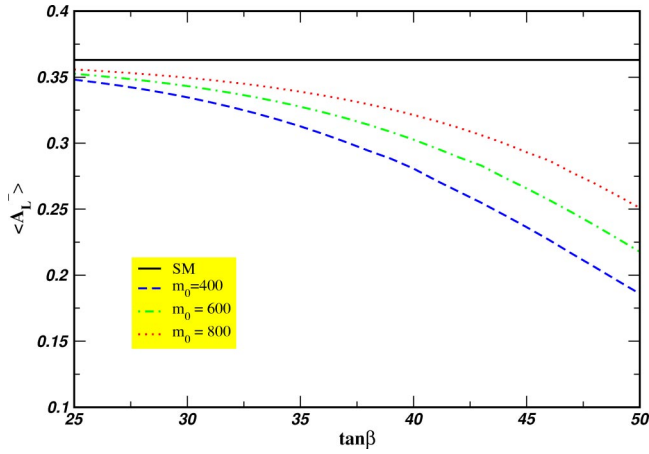


FIG. 8.  $\langle A_L^- \rangle$  variation with  $\tan \beta$  in MSUGRA model for various values of  $m_0$ . Other model parameters are as given in Fig. 3.

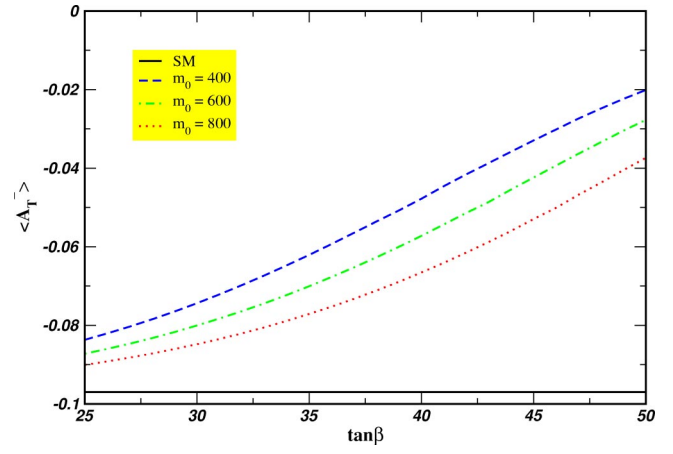


FIG. 11.  $\langle A_T^- \rangle$  with  $\tan \beta$  in MSUGRA, with other parameters the same as in Fig. 9.

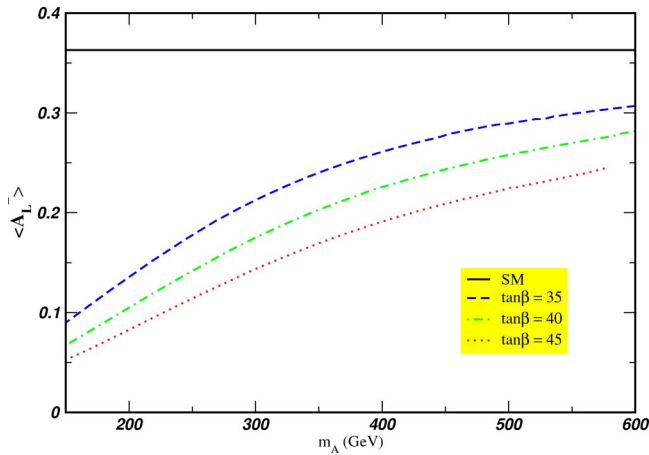


FIG. 9.  $\langle A_L^- \rangle$  variation with  $m_A$  in RSUGRA model for various values of  $\tan \beta$ . Other model parameters are as given in Fig. 3.

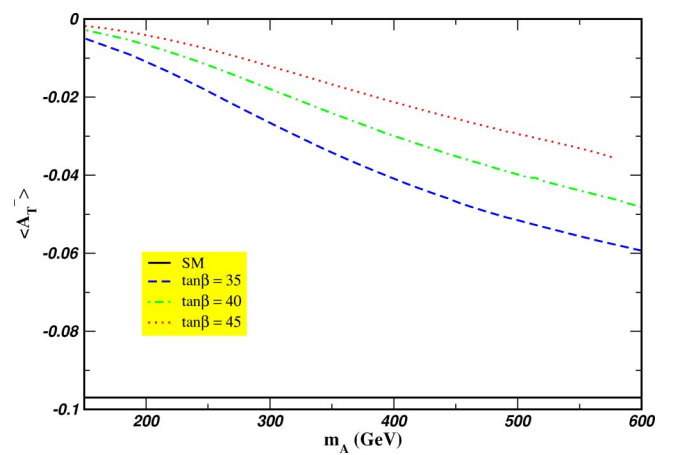


FIG. 12.  $\langle A_T^- \rangle$  with  $m_A$  in RSUGRA, with other parameters the same as in Fig. 10.

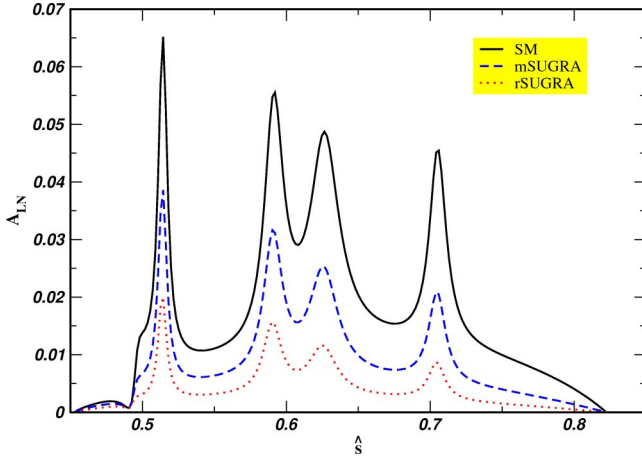


FIG. 13.  $A_{LN}$  with  $\hat{s}$ . Other parameters are the same as given in Fig. 1.

be 4 to 5 times the SM branching ratio for low values of  $m_{1/2}$  and very high values of  $\tan\beta$ . However, for the RSUGRA model there can be a much higher enhancement to the branching ratio (as given in Fig. 3) when compared to the SM value for low  $m_A$  and high  $\tan\beta$ . As can be seen from the other graphs of the averaged observables, the various observables show marked deviation from their respective SM values for a very wide region of the MSSM parameter space. This deviation cannot be explained solely on the basis of uncertainties in the form factors (which are at worst 15%). Thus such variations, if observed in future  $B$  factories, could be very useful in testing the underlying operator structure of the theory and in fixing the numerical value of the Wilson coefficients.

There is also a further aspect to our results in relation to  $CP$  asymmetry. Consider the FB asymmetry of the conjugate process  $b \rightarrow s \tau^+ \tau^-$ , where due to the smallness of the coupling of the  $b$ -quark with the  $u$ -quark the CKM-factor in all amplitudes involving the  $b \rightarrow s$  transition, like the present one, will essentially be an overall factor. In the version of supersymmetry that we have considered and the parameter

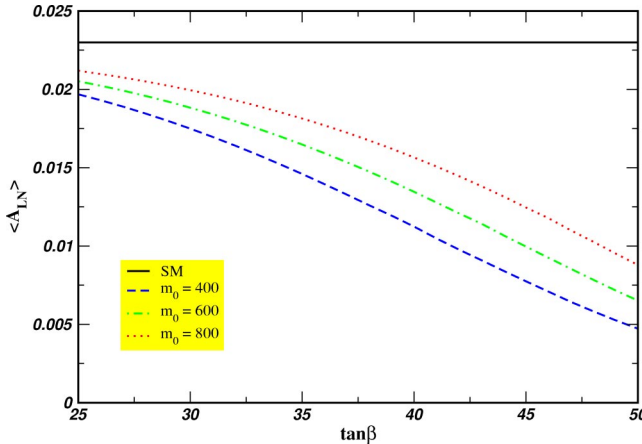


FIG. 14.  $\langle A_{LN} \rangle$  with  $\tan\beta$  in MSUGRA, with other parameters the same as in Fig. 9.

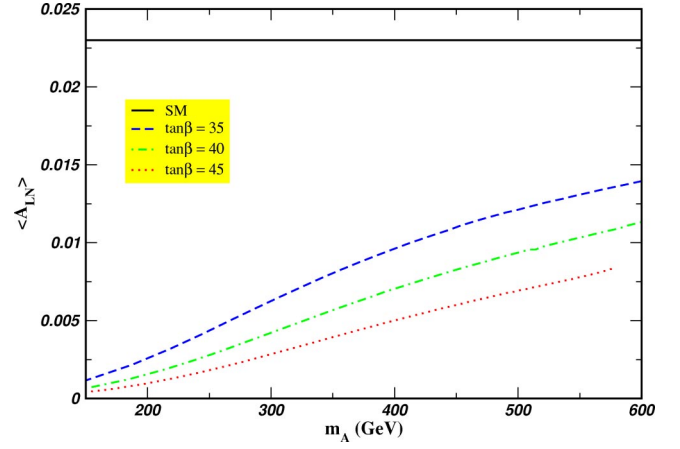


FIG. 15.  $\langle A_{LN} \rangle$  with  $m_A$  in RSUGRA model, with other parameters the same as in Fig. 10.

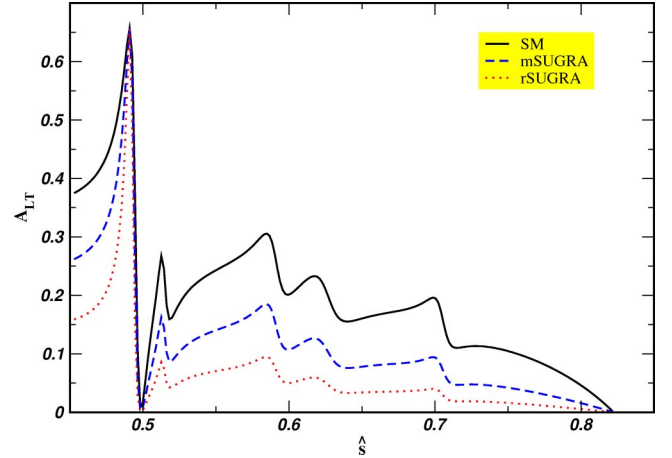


FIG. 16.  $A_{LT}$  with  $\hat{s}$ . Other parameters are the same as given in Fig. 1.

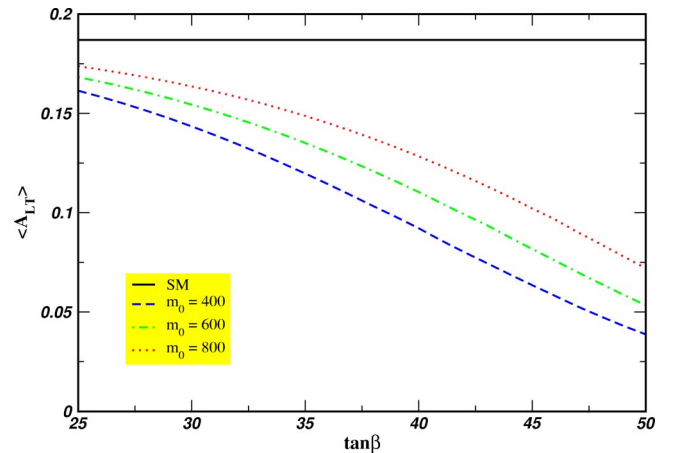


FIG. 17.  $\langle A_{LT} \rangle$  with  $\tan\beta$  in MSUGRA, with the other parameters being the same as in Fig. 9.



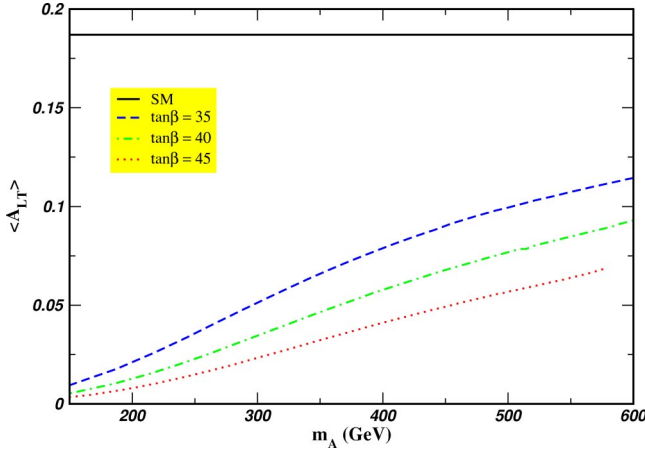


FIG. 18.  $\langle A_{LT} \rangle$  with  $m_A$ , with the other parameters the same as in Fig. 10.

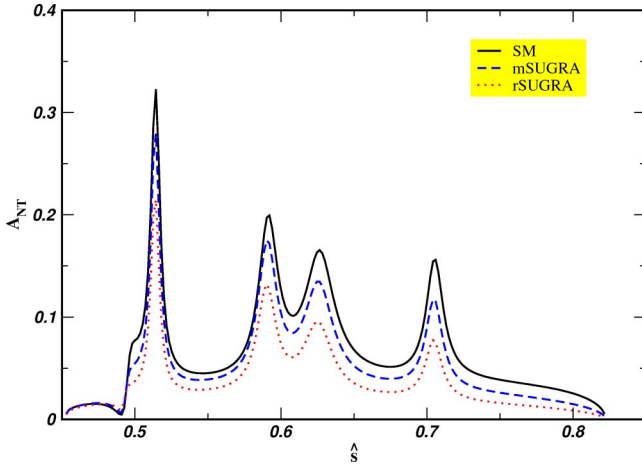


FIG. 19.  $A_{NT}$  with  $\hat{s}$ . Other parameters are the same as given in Fig. 1.

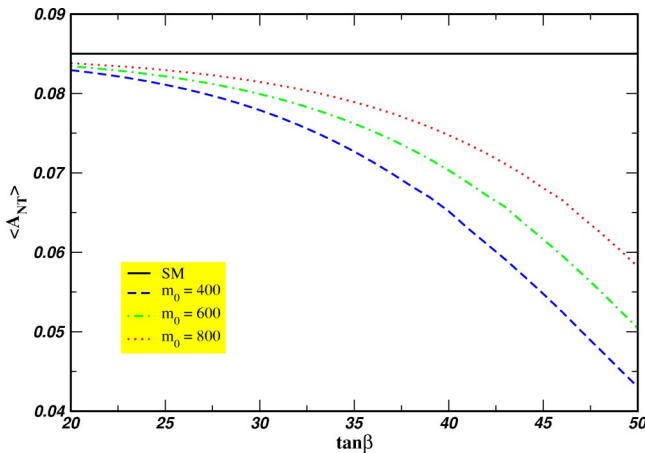


FIG. 20.  $\langle A_{NT} \rangle$  with  $\tan\beta$  in MSUGRA, with other parameters the same as in Fig. 9.

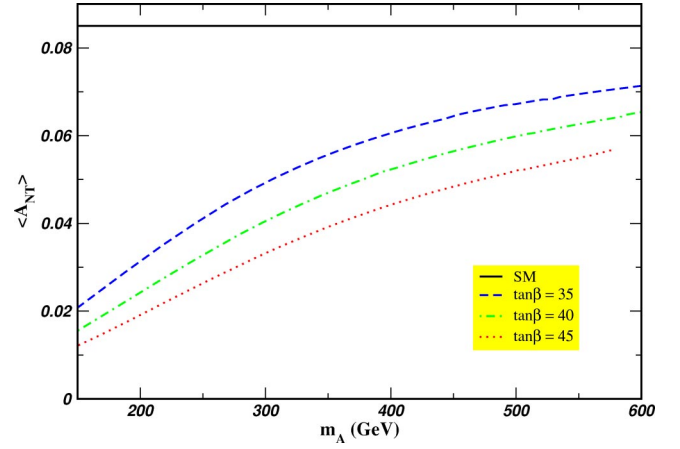


FIG. 21.  $\langle A_{NT} \rangle$  with  $m_A$  in RSUGRA model, with the other parameters the same as in Fig. 10.

space thereof, there are no extra  $CP$ -violating phases. Thus in calculating decay rates the phase will be washed away and we have in effect a  $CP$ -invariant theory. The asymmetries of the process  $b \rightarrow s \tau^- \tau^+$  and the conjugate process  $\bar{b} \rightarrow \bar{s} \tau^+ \tau^-$  are thus related. In fact the unpolarized FB asymmetry for the  $b \rightarrow s$  transition will vanish in an untagged ( $CP$  even) sample [28]. Defining the forward and backward directions as referring to the  $\tau^-$  and denoting the asymmetries of the conjugate process by  $\bar{A}$ , we get

$$\bar{A}_{ij} = p_{ij} A_{ji} \quad (4.3)$$

with the parity factor  $p_{ij}$  equaling  $-1$  for all  $i$ 's and  $j$ 's except for the combinations  $LN$  and  $NT$ . If we have an untagged sample containing an equal number of  $B$ 's and  $\bar{B}$ 's, then just as in the unpolarized sample, the asymmetries observed for the combinations  $LL$ ,  $NN$ ,  $TT$  as well as for  $(LT + TL)$  will vanish. However, for the combinations  $(LN + NL)$  and  $(NT + TN)$ , the asymmetries will add up. We thus have a situation in these two cases wherein a measurement of FB asymmetry for an untagged sample can lead to a meaningful non-null valued comparison between theory and experiment.

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## APPENDIX A: FORM FACTORS

The form factors for the  $B \rightarrow K$  transition are given in Ref. [8]:

$$\begin{aligned} & \langle K(p_K) | \bar{s} \gamma_\mu b | B(p_B) \rangle \\ &= f_+ \left[ (p_B + p_K)_\mu - \frac{m_B^2 - m_K^2}{q^2} q_\mu \right] + f_0 \frac{m_B^2 - m_K^2}{q^2} q_\mu \end{aligned} \quad (\text{A1})$$

$$\begin{aligned} & \langle K(p_K) | \bar{s} \sigma_{\mu\nu} q^\nu b | B(p_B) \rangle \\ &= i \frac{f_T}{m_B + m_K} [(p_B + p_K)_\mu q^2 - q_\mu (m_B^2 - m_K^2)] \end{aligned} \quad (\text{A2})$$

where  $q (= p_+ + p_-)$  is the sum of four momenta  $\ell^-$  and  $\ell^+$ , i.e. the momentum transfer;  $f_+$ ,  $f_0$  and  $f_T$  are the form factors. Multiplying Eq. (A1) by  $q^\mu$  and by using the equations of motion we get

$$\langle K(p_K) | \bar{s} b | B(p_B) \rangle = f_0 \frac{(m_B^2 - m_K^2)}{m_b} \quad (\text{A3})$$

where all the other matrix elements vanish.

For the form factors  $f_+$ ,  $f_0$  and  $f_T$  we will take the parametrization

TABLE II. Form factors for  $B \rightarrow K$  transition.

	$f_+$	$f_0$	$f_T$
$F(0)$	0.319	0.319	0.355
$c_1$	1.465	0.633	1.478
$c_2$	0.372	-0.095	0.373
$c_3$	0.782	0.591	0.700

$$F(\hat{s}) = F(0) \exp(c_1 \hat{s} + c_2 \hat{s}^2 + c_3 \hat{s}^3) \quad (\text{A4})$$

where the values of the parameters are given in Table II.

## APPENDIX B: INPUT PARAMETERS

$$m_B = 5.26 \text{ GeV}, \quad m_b = 4.8 \text{ GeV}, \quad m_c = 1.4 \text{ GeV}$$

$$m_\mu = 0.106 \text{ GeV}, \quad m_\tau = 1.77 \text{ GeV}, \quad m_w = 80.4 \text{ GeV},$$

$$m_z = 91.19 \text{ GeV}$$

$$V_{tb} V_{ts}^* = 0.0385, \quad \alpha = \frac{1}{129}, \quad m_K = 0.49 \text{ GeV},$$

$$\Gamma_B = 4.22 \times 10^{-13} \text{ GeV}$$

$$G_F = 1.17 \times 10^{-5} \text{ GeV}^{-2}.$$

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