Mixing and oscillations of neutral particles in quantum field theory

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We study the mixing of neutral particles in quantum field theory: the neutral boson field and Majorana field are treated in the case of mixing among two generations. We derive the orthogonality of flavor and mass representations and show how to consistently calculate oscillation formulas, which agree with previous results for charged fields and exhibit corrections with respect to the usual quantum mechanical expressions.

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The study of the mixing of fields of different masses in the context of quantum field theory (QFT) has recently produced very interesting and in some sense unexpected results [1–14]. The story began in 1995 when, in Ref. [1], the unitary inequivalence of the Hilbert spaces was proved for (fermion) fields with definite flavor on one side and those (free fields) with definite mass on the other. The proof was then generalized to any number of fermion generations [7] and to bosonic fields [2,5]. This result strikes of the common sense of quantum mechanics (QM), where one has only one Hilbert space at hand: the inconsistencies that arise there have generated much controversy and it has even been claimed that it is impossible to construct a Hilbert space for flavor states [15] (see, however, Ref. [6] for a criticism of that argument).

In fact, not only the flavor Hilbert space can be consistently defined [1], but it also provides a tool for the calculation of flavor oscillation formulas in QFT [3,8–14,16–18], exhibiting corrections with respect to the usual QM ones [19,20].

From a general point of view, the above results show that mixing is an "example of nonperturbative physics which can be exactly solved," as stated in Ref. [13]. Indeed, the flavor Hilbert space is a space for particles which are not on shell and this situation is analogous to what one encounters when quantizing fields at finite temperature [21] or in a curved background [22].

In the derivation of the oscillation formulas by use of flavor Hilbert space, a central role is played by the flavor charges [9] and indeed it was found that these operators satisfy very specific physical requirements [6,8]. However, these charges vanish identically in the case of neutral fields and this is the main reason why the study of field mixing has been limited up to now to only the case of charged (complex) fields. We fill here this gap by providing a consistent treatment of both neutral bosons and Majorana fermions. To keep the discussion transparent, we limit ourselves to the case of two generations.

Apart from an explicit quantization of the neutral mixed fields, the main result of this paper is the study of the momentum operator (and of the energy-momentum tensor) for those fields. We show how to define it in a consistent way and by its use we then derive the oscillation formulas, which match those for charged fields. We also comment on its relevance for the study of charged mixed fields, where, in the case when *CP* violation is present, the charges present a problematic interpretation which is still not completely clarified [10,11,14].

The paper is organized as follows: first we discuss the mixing of neutral (spin-0) bosonic fields. Then, we treat Majorana fields, and finally we discuss some general consequences of our results and draw conclusions.

We consider here the simple case of mixing of two spin-0 neutral boson fields. We follow the case of charged fields as discussed already in Refs. [2,5,12] and introduce the mixing relations as

$$\phi_A(x)=\cos\theta\,\phi_1(x)+\sin\theta\,\phi_2(x),$$
$$\phi_B(x)=-\sin\theta\,\phi_1(x)+\cos\theta\,\phi_2(x), \tag{1}$$

and similar ones for the conjugate momenta $\pi=\partial_0\phi$. We denoted the mixed fields with suffixes A and B. The free fields ϕ_1, ϕ_2 can be quantized in the usual way ($x_0\equiv t$):

$$\phi_j(x)=\int\frac{d^3\mathbf{k}}{(2\pi)^{3/2}}\frac{1}{\sqrt{2\omega_{k,j}}}\left(a_{\mathbf{k},j}e^{-i\omega_{k,j}t}+a^\dagger_{-\mathbf{k},j}e^{i\omega_{k,j}t}\right)e^{i\mathbf{k}\cdot\mathbf{x}},\quad j=1,2, \tag{2}$$

with canonical commutation relations. We now recast Eq. (1) into the form $[\sigma,j=(A,1),(B,2)]$:

$$\phi_\sigma(x)=G_\theta^{-1}(t)\,\phi_j(x)\,G_\theta(t),$$
$$G_\theta(t)=\exp[\theta(S_+(t)-S_-(t))], \tag{3}$$

and similar ones for $\pi_\sigma(x)$. As for charged bosons [12], the mixing generator G_θ is an element of SU(2) with

$$S_+(t)\equiv-i\int d^3\mathbf{x}\,\pi_1(x)\,\phi_2(x), \tag{5}$$

$$S_-(t)\equiv-i\int d^3\mathbf{x}\,\pi_2(x)\,\phi_1(x), \tag{6}$$

$$S_3 \equiv \frac{-i}{2}\int d^3\mathbf{x}[\pi_1(x)\phi_1(x)-\pi_2(x)\phi_2(x)], \quad (7)$$

The flavor fields can be thus expanded as

$$\phi_\sigma(x)=\int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}}\frac{1}{\sqrt{2\omega_{k,j}}}[a_{\mathbf{k},\sigma}(t)e^{-i\omega_{k,j}t} + a^\dagger_{-\mathbf{k},\sigma}(t)e^{i\omega_{k,j}t}]e^{i\mathbf{k}\cdot\mathbf{x}}, \quad (8)$$

with $\sigma,j=(A,1),(B,2)$ and

$$a_{\mathbf{k},A}(t)=\cos\theta a_{\mathbf{k},1}+\sin\theta(\hat{U}^*_{\mathbf{k}}(t)a_{\mathbf{k},2}+\hat{V}_{\mathbf{k}}(t)a^\dagger_{-\mathbf{k},2}), \quad (9)$$

$$a_{\mathbf{k},B}(t)=\cos\theta a_{\mathbf{k},2}-\sin\theta(\hat{U}_{\mathbf{k}}(t)a_{\mathbf{k},1}-\hat{V}_{\mathbf{k}}(t)a^\dagger_{-\mathbf{k},1}). \quad (10)$$

where $\hat{U}_{\mathbf{k}}(t)$ and $\hat{V}_{\mathbf{k}}(t)$ are Bogoliubov coefficients

$$\hat{U}_{\mathbf{k}}(t)\equiv|\hat{U}_{\mathbf{k}}|e^{i(\omega_{k,2}-\omega_{k,1})t}, \quad \hat{V}_{\mathbf{k}}(t)\equiv|\hat{V}_{\mathbf{k}}|e^{i(\omega_{k,1}+\omega_{k,2})t},$$

$$|\hat{V}_{\mathbf{k}}|\equiv\frac{1}{2}\left(\sqrt{\frac{\omega_{k,1}}{\omega_{k,2}}}-\sqrt{\frac{\omega_{k,2}}{\omega_{k,1}}}\right), \quad |\hat{U}_{\mathbf{k}}|^2-|\hat{V}_{\mathbf{k}}|^2=1. \quad (11)$$

We now consider the action of the generator of the mixing transformations on the vacuum $|0\rangle_{1,2}$ for the fields $\phi_i(x)$: $a_{\mathbf{k},i}|0\rangle_{1,2}=0$, $i=1,2$. The mixing generator induces an SU(2) coherent state structure on such state [23]:

$$|0(\theta,t)\rangle_{A,B}\equiv G^{-1}_\theta(t)|0\rangle_{1,2}. \quad (12)$$

We will refer to the state $|0(\theta,t)\rangle_{A,B}$ as to the *flavor vacuum* for bosons. We have ${}_{A,B}\langle 0(\theta,t)|0(\theta,t)\rangle_{A,B}=1$. For future convenience, we define $|0(t)\rangle_{A,B}\equiv|0(\theta,t)\rangle_{A,B}$ and $|0\rangle_{A,B}\equiv|0(\theta,t=0)\rangle_{A,B}$. A crucial point is that the flavor and mass vacua are orthogonal in the infinite volume limit. We indeed have (see also [12]): ${}_{1,2}\langle 0|0(t)\rangle_{A,B}=\Pi_k f^k_0(\theta)$, for any t with $f^k_0(\theta)\equiv(1+\sin^2\theta|\hat{V}_k|^2)^{-1}$. In the infinite volume limit, we obtain

$$\lim_{V\to\infty}{}_{1,2}\langle 0|0(t)\rangle_{A,B}=\lim_{V\to\infty}\exp\left(\frac{V}{(2\pi)^3}\int d^3k\ln f^{\mathbf{k}}_0(\theta)\right)=0\ . \quad (13)$$

We have that $\ln f^{\mathbf{k}}_0(\theta)$ is indeed negative for any values of $\mathbf{k}$, θ and m_1,m_2 (note that $0\leqslant\theta\leqslant\pi/4$). Observe that the orthogonality disappears when $\theta=0$ and/or $m_1=m_2$. We define the state for a mixed particle with flavor A and momentum $\mathbf{k}$ as

$$|a_{\mathbf{k},A}(t)\rangle\equiv a^\dagger_{\mathbf{k},A}(t)|0(t)\rangle_{A,B}=G^{-1}_\theta(t)a^\dagger_{\mathbf{k},1}|0\rangle_{1,2}. \quad (14)$$

In the following we work in the Heisenberg picture, and flavor states will be taken at reference time $t=0$ (including the flavor vacuum); we also define $|a_{\mathbf{k},A}\rangle\equiv|a_{\mathbf{k},A}(0)\rangle$.

Let us now consider the (nonvanishing) commutators of the flavor ladder operators at different times and observe that the following quantity is constant in time:

$$\sum_{\sigma=A,B}|[a_{\mathbf{k},\sigma}(t),a^\dagger_{\mathbf{k},A}(t')]|^2-|[a^\dagger_{-\mathbf{k},\sigma}(t),a^\dagger_{\mathbf{k},A}(t')]|^2=1. \quad (15)$$

In Ref. [12], the equation that corresponds to Eq. (15) for charged fields was consistently interpreted as expressing the conservation of total charge. In the present case we are dealing with a neutral field and thus the charge operator vanishes identically. Nevertheless, the commutators in Eq. (15) are well defined and are the neutral-field counterparts of those for charged fields. Thus we look for a physical interpretation of such oscillating quantities.

Let us consider the momentum operator, defined as the diagonal space part of the energy-momentum tensor [24]: $P^j\equiv\int d^3\mathbf{x}\Theta^{0j}(x)$, with $\Theta^{\mu\nu}\equiv\partial^\mu\phi\partial^\nu\phi-g^{\mu\nu}[\frac{1}{2}(\partial\phi)^2-\frac{1}{2}m^2\phi^2]$. For the free fields ϕ_j we have

$$\mathbf{P}_j=\int d^3\mathbf{k}\frac{\mathbf{k}}{2}(a^\dagger_{\mathbf{k},j}a_{\mathbf{k},j}-a^\dagger_{-\mathbf{k},j}a_{-\mathbf{k},j}), \quad (16)$$

with $j=1,2$. In a similar way we can define the momentum operator for the mixed fields:

$$\mathbf{P}_\sigma(t)=\int d^3\mathbf{k}\frac{\mathbf{k}}{2}[a^\dagger_{\mathbf{k},\sigma}(t)a_{\mathbf{k},\sigma}(t)-a^\dagger_{-\mathbf{k},\sigma}(t)a_{-\mathbf{k},\sigma}(t)], \quad (17)$$

where $\sigma=A,B$. The two operators are obviously related: $\mathbf{P}_\sigma(t)=G^{-1}_\theta(t)\mathbf{P}_jG_\theta(t)$. Note that the total momentum is conserved in time since it commutes with the generator of mixing transformations (at any time):

$$\mathbf{P}_A(t)+\mathbf{P}_B(t)=\mathbf{P}_1+\mathbf{P}_2\equiv\mathbf{P}, \ [\mathbf{P}, G_\theta(t)]=0, \ [\mathbf{P},H]=0. \quad (18)$$

Thus in the mixing of neutral fields, the momentum operator plays an analogous role to that of the charge for complex fields [12].

We now consider the expectation values of the momentum operator for flavor fields on the flavor state $|a_{\mathbf{k},A}\rangle$. This is an eigenstate of $\mathbf{P}_A(t)$ at time $t=0$: $\mathbf{P}_A(0)|a_{\mathbf{k},A}\rangle=\mathbf{k}|a_{\mathbf{k},A}\rangle$, which follows from $\mathbf{P}_1|a_{\mathbf{k},1}\rangle=\mathbf{k}|a_{\mathbf{k},1}\rangle$ by application of $G^{-1}_\theta(0)$. At time $t\neq0$, the expectation value of the momentum (normalized to the initial value) gives ($\sigma=A,B$):

$$\mathcal{P}^A_\sigma(t)\equiv\frac{\langle a_{\mathbf{k},A}|\mathbf{P}_\sigma(t)|a_{\mathbf{k},A}\rangle}{\langle a_{\mathbf{k},A}|\mathbf{P}_\sigma(0)|a_{\mathbf{k},A}\rangle} =|[a_{\mathbf{k},\sigma}(t),a^\dagger_{\mathbf{k},A}(0)]|^2-|[a^\dagger_{-\mathbf{k},\sigma}(t),a^\dagger_{\mathbf{k},A}(0)]|^2. \quad (19)$$

One can check that the (flavor) vacuum expectation value of $\mathbf{P}_\sigma(t)$ does vanish at all times: ${}_{A,B}\langle 0|\mathbf{P}_\sigma(t)|0\rangle_{A,B}=0$ which can be understood by realizing that the flavor vacuum $|0\rangle_{A,B}$ is a condensate of pairs with zero total momentum. We finally obtain

$$\mathcal{P}^A_{\mathbf{k},B}(t)=\sin^2(2\theta)\left[|\hat{U}_\mathbf{k}|^2\sin^2\left(\frac{\omega_{k,2}-\omega_{k,1}}{2}t\right)\right.$$
$$\left.-|\hat{V}_\mathbf{k}|^2\sin^2\left(\frac{\omega_{k,2}+\omega_{k,1}}{2}t\right)\right], \qquad (20)$$

with $\mathcal{P}^A_{\mathbf{k},A}(t)=1-\mathcal{P}^A_{\mathbf{k},B}(t)$, in complete agreement with the charged field case [12].

Now we consider the case of mixing of two Majorana fermion fields. A Majorana fermion field satisfies the Dirac equation $(i\partial\!\!\!/-m)\psi=0$ and the self-conjugation relation $\psi=\psi^c$ with $\psi^c(x)\equiv\gamma_0\mathcal{C}\psi^*(x)$ and the charge-conjugation operator $\mathcal{C}$ is defined as satisfying the relations $\mathcal{C}^{-1}\gamma_\mu\mathcal{C}=-\gamma_\mu^T$, $\mathcal{C}^\dagger=\mathcal{C}^{-1}$, and $\mathcal{C}^T=-\mathcal{C}$. The mixing relations are

$$\nu_e(x)=\cos\theta\nu_1(x)+\sin\theta\nu_2(x),$$
$$\nu_\mu(x)=-\sin\theta\nu_1(x)+\cos\theta\nu_2(x), \qquad (21)$$

where ν_e, ν_μ are the neutrino fields with definite flavors. ν_1, ν_2 are the (free) neutrino fields with definite masses m_1, m_2, respectively. θ is the mixing angle. The quantization of the free fields is given by [25]

$$\nu_j(x)=\sum_{r=1,2}\int\frac{d^3\mathbf{k}}{(2\pi)^{3/2}}[u^r_{\mathbf{k},j}(t)\alpha^r_{\mathbf{k},j}$$
$$+v^r_{-\mathbf{k},j}(t)\alpha^{r\dagger}_{-\mathbf{k},j}]e^{i\mathbf{k}\cdot\mathbf{x}}, \quad j=1,2, \qquad (22)$$

where $u^r_{\mathbf{k},j}(t)=e^{-i\omega_{k,j}t}u^r_{\mathbf{k},j}$ and $v^r_{\mathbf{k},j}(t)=e^{i\omega_{k,j}t}v^r_{\mathbf{k},j}$. In order for the above Majorana condition to be satisfied the four spinors must also satisfy the conditions $v^s_{\mathbf{p},j}=\gamma_0\mathcal{C}(u^s_{\mathbf{p},j})^*$ and $u^s_{\mathbf{p},j}=\gamma_0\mathcal{C}(v^s_{\mathbf{p},j})^*$. The equal time anticommutation relations are $\{\nu^\alpha_i(x),\nu^{\beta\dagger}_j(y)\}_{t=t'}=\delta^3(\mathbf{x}-\mathbf{y})\delta_{\alpha\beta}\delta_{ij}$ and $\{\nu^\alpha_i(x),\nu^\beta_j(y)\}_{t=t'}=\delta^3(\mathbf{x}-\mathbf{y})(\gamma_0\mathcal{C})^{\alpha\beta}\delta_{ij}$ with $\alpha,\beta=1,\dots,4$ and $\{\alpha^r_{\mathbf{k},i},\alpha^{s\dagger}_{\mathbf{q},j}\}=\delta^3(\mathbf{k}-\mathbf{q})\delta_{rs}\delta_{ij}$ with $i,j=1,2$. All other anticommutators are zero. The orthonormality and completeness relations are the same as in Ref. [1].

We can recast Eqs. (21) into the following form:

$$\nu^\alpha_\sigma(x)=G^{-1}_\theta(t)\nu^\alpha_j(x)G_\theta(t),$$
$$G_\theta(t)=\exp\left[\frac{\theta}{2}\int d^3\mathbf{x}[\nu_1^\dagger(x)\nu_2(x)-\nu_2^\dagger(x)\nu_1(x)]\right], \qquad (23)$$

with $\sigma,j=(e,1),(\mu,2)$. The flavor fields can be thus expanded as

$$\nu_\sigma(x)=\sum_{r=1,2}\int\frac{d^3\mathbf{k}}{(2\pi)^{3/2}}[u^r_{\mathbf{k},j}(t)\alpha^r_{\mathbf{k},\sigma}(t)$$
$$+v^r_{-\mathbf{k},j}(t)\alpha^{r\dagger}_{-\mathbf{k},\sigma}(t)]e^{i\mathbf{k}\cdot\mathbf{x}}, \qquad (24)$$

with $\sigma,j=(e,1),(\mu,2)$ and [for $\mathbf{k}=(0,0,|\mathbf{k}|)$]

$$\alpha^r_{\mathbf{k},e}(t)=\cos\theta\alpha^r_{\mathbf{k},1}+\sin\theta(U^*_\mathbf{k}(t)\alpha^r_{\mathbf{k},2}+\epsilon^r V_\mathbf{k}(t)\alpha^{r\dagger}_{-\mathbf{k},2}), \quad (25)$$

$$\alpha^r_{\mathbf{k},\mu}(t)=\cos\theta\alpha^r_{\mathbf{k},2}-\sin\theta(U_\mathbf{k}(t)\alpha^r_{\mathbf{k},1}-\epsilon^r V_\mathbf{k}(t)\alpha^{r\dagger}_{-\mathbf{k},1}), \quad (26)$$

where $U_\mathbf{k}(t)\equiv|U_\mathbf{k}|e^{i(\omega_{k,2}-\omega_{k,1})t}$ and $V_\mathbf{k}(t)\equiv|V_\mathbf{k}|e^{i(\omega_{k,2}+\omega_{k,1})t}$, with $|U_k|^2+|V_k|^2=1$ and

$$|V_\mathbf{k}|=\frac{(\omega_{k,1}+m_1)-(\omega_{k,2}+m_2)}{2\sqrt{\omega_{k,1}\omega_{k,2}(\omega_{k,1}+m_1)(\omega_{k,2}+m_2)}}|\mathbf{k}|, \qquad (27)$$

are the Bogoliubov coefficients.

As for the neutral boson case we now consider the action of the generator of the mixing transformations on the vacuum $|0\rangle_{1,2}$. The flavor vacuum is then defined as $|0(\theta,t)\rangle_{e,\mu}\equiv G^{-1}_\theta(t)|0\rangle_{1,2}$ and the state for a mixed particle with definite flavor, spin, and momentum as $|\alpha^r_{\mathbf{k},\sigma}(t)\rangle\equiv\alpha^{r\dagger}_{\mathbf{k},\sigma}(t)|0(t)\rangle_{e,\mu}$. The anticommutators of the flavor ladder operators at different times satisfy the following relation (notice the difference in the signs with respect to the boson case):

$$\sum_{\sigma=e,\mu}|\{\alpha^r_{\mathbf{k},\sigma}(t),\alpha^{r\dagger}_{\mathbf{k},e}(t')\}|^2+|\{\alpha^{r\dagger}_{-\mathbf{k},\sigma}(t),\alpha^{r\dagger}_{\mathbf{k},e}(t')\}|^2=1. \qquad (28)$$

The momentum operator is now $P^j\equiv\int d^3\mathbf{x}\mathcal{J}^{0j}(x)$, where the energy-momentum tensor for the fermion field is $\mathcal{J}^{\mu\nu}\equiv(i/2)\overline{\psi}\gamma^\nu\partial_\mu\psi$. For the free fields ν_j we have

$$\mathbf{P}_j=\sum_{r=1,2}\int d^3\mathbf{k}\frac{\mathbf{k}}{2}(\alpha^{r\dagger}_{\mathbf{k},j}\alpha^r_{\mathbf{k},j}-\alpha^{r\dagger}_{-\mathbf{k},j}\alpha^r_{-\mathbf{k},j}), \qquad (29)$$

where $j=1,2$. For mixed fields,

$$\mathbf{P}_\sigma(t)=G^{-1}_\theta(t)\mathbf{P}_jG_\theta(t), \quad (\sigma,j)=(e,1),(\mu,2) \qquad (30)$$

We have $\Sigma_\sigma\mathbf{P}_\sigma(t)=\Sigma_j\mathbf{P}_j=\mathbf{P}$ and $[\mathbf{P},G_\theta(t)]=0$, $[\mathbf{P},H]=0$. We now consider the expectation values on the flavor state $|\alpha^r_{\mathbf{k},e}\rangle\equiv|\alpha^r_{\mathbf{k},e}(0)\rangle$. At time $t=0$, this is an eigenstate of the momentum operator $\mathbf{P}_e(0)$: $\mathbf{P}_e(0)|\alpha^r_{\mathbf{k},e}\rangle=\mathbf{k}|\alpha^r_{\mathbf{k},e}\rangle$. At $t\neq0$ the expectation value for the momentum (normalized to initial value) gives

$$\mathcal{P}^e_{\mathbf{k},\sigma}(t)\equiv\frac{\langle\alpha^r_{\mathbf{k},e}|\mathbf{P}_\sigma(t)|\alpha^r_{\mathbf{k},e}\rangle}{\langle\alpha^r_{\mathbf{k},e}|\mathbf{P}_\sigma(0)|\alpha^r_{\mathbf{k},e}\rangle}$$
$$=|\{\alpha^r_{\mathbf{k},\sigma}(t),\alpha^{r\dagger}_{\mathbf{k},e}(0)\}|^2+|\{\alpha^{r\dagger}_{-\mathbf{k},\sigma}(t),\alpha^{r\dagger}_{\mathbf{k},e}(0)\}|^2, \quad (31)$$

where $\sigma=e,\mu$. This is the same form one obtains for the expectation values of the flavor charges in the case of Dirac fields [3]. The flavor vacuum expectation value of the momentum operator $\mathbf{P}_\sigma(t)$ vanishes at all times: ${}_{e,\mu}\langle0|\mathbf{P}_\sigma(t)|0\rangle_{e,\mu}=0$ with $\sigma=e,\mu$. We thus obtain

$$\mathcal{P}^e_{\mathbf{k},\mu}(t)=\sin^22\theta\left[|U_\mathbf{k}|^2\sin^2\left(\frac{\omega_{k,2}-\omega_{k,1}}{2}t\right)\right.$$
$$\left.+|V_\mathbf{k}|^2\sin^2\left(\frac{\omega_{k,2}+\omega_{k,1}}{2}t\right)\right], \qquad (32)$$

with $\mathcal{P}^e_{\mathbf{k},e}(t)=1-\mathcal{P}^e_{\mathbf{k},\mu}(t)$, in complete agreement with the Dirac field case [3].

In this paper, we have studied in detail the mixing of neutral particles in the context of quantum field theory. We considered the mixing among two generations, both in the case of neutral scalar fields and for Majorana fields. Our analysis confirms previous results on the mixing of charged fields, and indeed we show that also for neutral fields the Hilbert spaces for definite flavor and for definite masses are orthogonal in the infinite volume limit.

The main result of this paper is, however, the calculation of oscillation formulas, which we obtained by use of the momentum operator. Our results confirm the oscillation formulas already obtained in the case of charged fields by use of the flavor charges, and it is also revealed to be useful in the case of three-flavor mixing, where the presence of the *CP* violating phase introduces ambiguities in the treatment based on flavor charges [14].

Let us comment on this point: for (mixed) Dirac fields, the momentum operator is ($\sigma=e,\mu,\tau$).

$$\mathbf{P}_\sigma(t)=\int d^3\mathbf{k}\sum_r \frac{\mathbf{k}}{2}[\alpha^{r\dagger}_{\mathbf{k},\sigma}(t)\alpha^r_{\mathbf{k},\sigma}(t)-\alpha^{r\dagger}_{-\mathbf{k},\sigma}(t)\alpha^r_{-\mathbf{k},\sigma}(t)+\beta^{r\dagger}_{\mathbf{k},\sigma}(t)\beta^r_{\mathbf{k},\sigma}(t)-\beta^{r\dagger}_{-\mathbf{k},\sigma}(t)\beta^r_{-\mathbf{k},\sigma}(t)],\quad \sigma=e,\mu,\tau. \tag{33}$$

This operator can be used for the calculation of the oscillation formulas for Dirac neutrinos: although in this case the charge operator is available, it has emerged that, in the presence of *CP* violating, there are complications in the identification of the observables and indeed the matter is still object of discussion [10,11,14]. The main problem there is that the flavor charges at time $t\neq 0$ do not annihilate the flavor vacuum, ${}_f\langle 0|Q_\sigma(t)|0\rangle_f\neq 0$. This expectation value needs then to be subtracted by hand in order to get the correct oscillation formulas [14].

However, we see how the use of the momentum operator confirms the results of Ref. [14], without presenting any ambiguity. We have indeed ${}_f\langle 0|\mathbf{P}_\sigma(t)|0\rangle_f=0$ and

$$\frac{\langle\nu_\rho|\mathbf{P}_\sigma(t)|\nu_\rho\rangle}{\langle\nu_\rho|\mathbf{P}_\sigma(0)|\nu_\rho\rangle}=|\{\alpha^r_{\mathbf{k},\sigma}(t),\alpha^{r\dagger}_{\mathbf{k},\rho}(0)\}|^2+|\{\beta^{r\dagger}_{-\mathbf{k},\sigma}(t),\alpha^{r\dagger}_{\mathbf{k},\rho}(0)\}|^2, \tag{34}$$

with $\sigma,\rho=e,\mu,\tau$ and $|\nu_\rho\rangle\equiv\alpha^{r\dagger}_{\mathbf{k},\rho}(0)|0\rangle_f$. This follows from the relations

$$\begin{aligned}{}_f\langle 0|\alpha^{r\dagger}_{\mathbf{k},\sigma}(t)\alpha^r_{\mathbf{k},\sigma}(t)|0\rangle_f&={}_f\langle 0|\alpha^{r\dagger}_{-\mathbf{k},\sigma}(t)\alpha^r_{-\mathbf{k},\sigma}(t)|0\rangle_f,\\ {}_f\langle 0|\beta^{r\dagger}_{\mathbf{k},\sigma}(t)\beta^r_{\mathbf{k},\sigma}(t)|0\rangle_f&={}_f\langle 0|\beta^{r\dagger}_{-\mathbf{k},\sigma}(t)\beta^r_{-\mathbf{k},\sigma}(t)|0\rangle_f,\end{aligned} \tag{35}$$

which are valid even in presence of *CP* violation, when ${}_f\langle 0|\alpha^{r\dagger}_{\mathbf{k},\sigma}(t)\alpha^r_{\mathbf{k},\sigma}(t)|0\rangle_f\neq{}_f\langle 0|\beta^{r\dagger}_{-\mathbf{k},\sigma}(t)\beta^r_{-\mathbf{k},\sigma}(t)|0\rangle_f$.

These results seem to suggest that a redefinition of the flavor charge operators is necessary in the presence of *CP* violation and further study along this direction is in progress.

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