Chiral dynamics from AdS space

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We study the low energy dynamics of pions in a gravity dual of chiral symmetry breaking. The string theory construction consists of a probe D7 brane in the Constable-Myers nonsupersymmetric background, which has been shown to describe chiral symmetry breaking in the pattern of QCD. We expand the D7 brane's Dirac-Born-Infeld action for fluctuations that correspond to the Goldstone mode and show that they take the form of a nonlinear chiral Lagrangian. We numerically compute the quark condensate, pion decay constant, and higher order Gasser-Leutwyler coefficients. We find their form is consistent with naive dimensional analysis estimates. We also explore the gauging of the quark's chiral symmetries and the vector meson spectrum.

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I. INTRODUCTION

The dynamics and phenomenology of QCD are dominated by quarks. In particular the vast majority of known hadronic states can be identified as having constituent quarks and the low energy dynamics is controlled by the chiral symmetry breaking quark condensate. The discovery of the AdS conformal field theory (CFT) correspondence [1–3] has raised the hope of providing a weakly coupled gravitational description of the strong coupling phase of QCD. Understanding fundamental representation quarks in this setting must therefore be a priority.

Recently a simple mechanism for including quarks in the AdS/CFT correspondence has been found by Karch and Katz [4]. A probe D7 brane is added to the original D3 brane construction of the AdS/CFT correspondence. The new "37" open strings generate quarks in the field theory on the D3 brane world volume. The world volume theory has $\mathcal{N}=2$ supersymmetry. Karch and Katz identified the additional "77" open strings on the D7 world volume as playing a holographically dual role to the gauge invariant quark operators of the gauge theory. Treating the D7 as a probe corresponds to quenching in the gauge theory. The dynamics of quarks and their bound states have been studied in a number of supersymmetric gauge field backgrounds using these techniques [5–13].

If we wish to study chiral symmetry breaking in the pattern of QCD we must look at a nonsupersymmetric gauge theory since supersymmetry forbids such a quark condensate. A number of gravitational duals of nonsupersymmetric backgrounds exist ([14–18] and [19–25]). The simplest cases involve the deformation of the original AdS/CFT correspondence by the inclusion of relevant operators in the field theory [26], which corresponds to switching on bulk supergravity fields. In these cases the UV of the gauge theory returns to $\mathcal{N}=4$ super-Yang-Mills and the operator identification between the two dual theories is cleanly understood. Such theories do not have total decoupling of the superpartners that are given mass since the theory is strongly coupled in the UV—these extra states have masses of order the strong interaction scale Λ. Simply breaking supersymmetry is sufficient to allow a quark condensate though. Nonsupersymmetric deformations [19–25] of the Klebanov-Strassler [27] and Maldecena-Nunez [28] gravity duals also exist but here the operator matching, even in the UV, is less clear.

The first study of chiral symmetry breaking in this formalism was made in [7]. The deformed $\mathrm{AdS}_5\times S^5$ geometry of Constable and Myers [18], which corresponds to the addition of an R-chargeless dimension 4 operator such as $\mathrm{Tr}\,F^{\mu\nu}F_{\mu\nu}$ to the $\mathcal{N}=4$ theory, was used. The existence of a condensate and massless pions in the limit where the quark mass went to zero were identified. In this paper we will make further study of that case. The Constable-Myers geometry has a singularity in the interior, the precise significance of which is unclear; the singularity might correspond to the presence of some expanded D-brane set up in the interior and signal the strong interaction scale Λ. It turns out that the core of the geometry is repulsive to the D7 brane probe when quarks are included and this is what triggers the chiral symmetry breaking in the model. Pleasingly this repulsion also makes sure that the D7 branes avoid the central singularity, so for the purposes of this study we can set aside study of the singularity.

Chiral symmetry breaking by the same mechanism has also been studied in [8]. They use a geometry around D4 branes wrapped on a circle which describes a (3+1)-dimensional Yang-Mills-like theory in the IR. This geometry has no interior singularity but the core is again repulsive to D7 brane probes triggering chiral symmetry breaking. In this case though the UV of the theory becomes strongly coupled and wishes to become an M5 brane construction. The universality of the IR mechanism is encouraging and provides support for further study in the Constable-Myers background.

Here we will return to that Constable-Myers configuration [7,18] and examine chiral symmetry breaking and its consequences in more detail. First we refine the numerical analysis of [7] by solving the Euler-Lagrange equation, describing how the D7 brane lies in the geometry, starting with the appropriate regular infrared boundary conditions. This allows us to identify the regular physical flows without tuning. We stress the geometrical description of chiral symmetry breaking provided by the setup where the repulsion of the interior geometry forces the D7 brane to lie in a symmetry

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breaking configuration. For small quark mass the vacuum energy of the configuration is proportional to the quark mass as expected in the chiral Lagrangian formalism [29,30] (which we will review in the next section). It is therefore possible to extract the quark condensate which we show matches with the value obtained in [7] by looking at the UV boundary conditions on the flows.

Next we move on to study fluctuations of the D7 about the vacuum configuration, that describe the Goldstone mode, or pion fields. We show that the Lagrangian terms match those expected in the chiral Lagrangian and then compute the couplings. In particular the pion mass is shown numerically to have a linear dependence on the square root of the quark mass for low quark mass. It is then possible to compute the pion decay constant which we show has the correct dependence on the number of colors N and has a numerical suppression factor relative to the strong coupling scale consistent with that seen in QCD and naive arguments [29–31]. In the chiral limit we then look at terms in the low energy Lagrangian involving four pion fields and estimate their size. We again find a match to naive estimates [31,32].

A natural next step is to include multiple D7 probes and study the resulting theory with N_f quark flavors via the non-Abelian DBI action [33]. In fact the quarks couple to the adjoint scalar field in the $\mathcal{N}=2$ UV gauge theory via a superpotential term $\tilde{Q}AQ$ and enhancing the number of quark flavors does not therefore enhance the chiral flavor group. The adjoint scalar though is massive on the scale Λ and we might expect an accidental symmetry in the IR. This phenomenon has been discussed in the context of [8]. The additional fields in the non-Abelian DBI action are also massless but have interaction terms that are not Goldstone-like—these are just the remnants of the usual commutator interactions of the scalar fields on a brane. As a test of the Goldstone-like nature of these fields, we compute the pion decay constant via Lagrangian terms that are not present in the Abelian case, neglecting the commutator interactions. We find excellent numerical agreement with our previous value showing that these fields are rather Goldstone-like.

Finally we study the gauge field on the world volume of the D7 which is dual to weakly gauging the vector U(1) baryon number symmetry and also the vector meson spectrum. We have been unable to identify the fields associated with the axial vector mesons though, which, for example, stops us from testing vector meson dominance in the Weinberg sum rules [36]. Nevertheless we compare the vector meson spectrum to that of the $\mathcal{N}=2$ theory provided by a pure AdS background.

II. CHIRAL DYNAMICS

QCD with N_f massless quarks has a chiral $SU(N_f)_L \times SU(N_f)_R$ global symmetry. When asymptotic freedom drives the coupling strong it is believed that this symmetry is broken by a quark bilinear condensate to the vector $SU(N_f)$ subgroup. The symmetry breaking produces N_f^2-1 Goldstone bosons (N_f^2 at large N_c where instanton effects are suppressed) which are associated with the pion multiplet in nature. Since the quarks have small current masses the pions are only pseudo-Goldstone fields. The very low energy dynamics only involves these Goldstone fields and can be completely described by a theory that realizes the broken symmetry nonlinearly [29,30]. Such a phenomenological theory is called a chiral Lagrangian. We introduce fields Π^a [with T^a the generators of the broken $SU(N_f)$ group normalized such that $\mathrm{Tr}\, T^aT^b=\frac{1}{2}\delta^{ab}$]

$$U=\exp(2i\Pi^a T^a/f_\Pi), \quad U\to L^\dagger UR \tag{1}$$

where f_Π is the pion decay constant and where the chiral symmetry transformation properties are shown. Remembering that the quark mass matrix transforms as $M\to L^\dagger MR$ we can then write a Lagrangian at leading order in both a derivative expansion and an expansion in M/f_Π,

$$\mathcal{L}=\frac{f_\Pi^2}{4}\mathrm{Tr}\,\partial^\mu U\partial_\mu U^\dagger+\nu^3\mathrm{Tr}\,MU^\dagger+\nu^3\mathrm{Tr}\,UM^\dagger+\cdots. \tag{2}$$

Expanding U to find the leading terms for the pion fields gives

$$\mathcal{L}=2N_f\nu^3 m+\frac{1}{2}(\partial^\mu\Pi^a)^2-\frac{1}{2}\frac{4\nu^3 m}{f_\Pi^2}\Pi^{a2}. \tag{3}$$

The couplings f_Π, etc. must be found phenomenologically in the low energy theory but are in principle predictions of the full high energy QCD theory. Since M is a source for the quark bilinear condensate, the coupling ν is related to the value of the quark condensate

$$\nu^3=\frac{1}{2}\langle\bar{q}q\rangle. \tag{4}$$

Note the pion mass can then be written as the Gell-Mann-Oakes-Renner relation $m_\Pi^2=2m_q\langle\bar{q}q\rangle/f_\pi^2$ [34].

The interaction terms in the low energy theory at next order in the chiral expansion have been written down by Gasser and Leutwyler [32] and can be parametrized as

$$\begin{aligned}\mathcal{L}=&L_1\mathrm{tr}(D^\mu UD_\mu U^\dagger)^2+L_2\mathrm{tr}[(D^\mu UD^\nu U^\dagger)(D_\mu UD_\nu U^\dagger)]+L_3\mathrm{tr}(D^\mu UD_\mu U^\dagger D^\nu UD_\nu U^\dagger)+L_4\mathrm{tr}(D^\mu UD_\mu U^\dagger)\mathrm{tr}(M^\dagger U\\&+MU^\dagger)+L_5\mathrm{tr}(D^\mu UD_\mu U^\dagger)(M^\dagger U+MU^\dagger)+L_6(M^\dagger U+MU^\dagger)^2+L_7\mathrm{tr}(M^\dagger U-MU^\dagger)+L_8\mathrm{tr}(M^\dagger UM^\dagger U+MU^\dagger MU^\dagger)\\&+iL_9\mathrm{tr}(F^R_{\mu\nu}D^\mu UD^\nu U^\dagger+F^L_{\mu\nu}D^\mu UD^\nu U^\dagger)+L_{10}\mathrm{tr}(U^\dagger F^R_{\mu\nu}UF^{L\mu\nu})+L_{11}\mathrm{tr}(D^2UD^2U^\dagger)+L_{12}\mathrm{tr}(M^\dagger D^2U+MD^2U^\dagger).\end{aligned} \tag{5}$$

We will be interested in showing that the pions of our gravity construction conform to this structure. We will also estimate some of these coefficients below using the AdS/CFT correspondence method.

A. Naive dimensional analysis

A simple set of rules for estimating the size of chiral Lagrangian couplings has been devised [31]. In QCD there are two scales—Λ the strong coupling scale generated by the running coupling, and the pion decay constant, f_π, which is approximately

$$f_\Pi^2 \sim \frac{N}{(4\pi)^2}\Lambda. \tag{6}$$

Naive dimensional analysis says that one should give all chiral Lagrangian terms a common coefficient of $\Lambda^2 f_\Pi^2$ with any occurrences of M or D^μ being dimensionally balanced by a factor of Λ. Note that the pion fields enter in U dimensionally balanced by f_Π. For example the L_i coefficients are predicted to be of order $1/16\pi^2$ using these rules which reasonably matches their physical values.

One of our goals in this paper is to test this naive power counting in the strongly coupled gauge theory for which we have a gravitational dual. We will find reasonable agreement below.

III. THE BRANE CONSTRUCTION AND GRAVITY DUAL

AdS/CFT correspondence type duals are obtained by equating the physics on a stack of N coincident, flat D3 branes and the supergravity background dynamically generated by the D3 branes' tension. We will consider the nonsupersymmetric deformed AdS geometry originally constructed in [18]. This geometry is dual to the $\mathcal{N}=4$ super-Yang-Mills theory

$$\mathcal{L} = \frac{1}{g_{YM}^2}\left[\frac{1}{4}\mathrm{Tr} F^{\mu\nu}F_{\mu\nu} + \cdots\right] \tag{7}$$

(with $g_{YM}^2 = 2\pi g_s$) deformed by the presence of a vacuum expectation value for an R-singlet operator with dimension four (such as $\mathrm{Tr}\, F^{\mu\nu}F_{\mu\nu}$). The supergravity background has a dilaton and S^5 volume factor depending on the radial direction. The geometry has a naked singularity in the interior which we loosely expect to correspond to the presence of the central stack of D3 branes. $\mathrm{Tr}\, F^2$ is not a modulus of the gauge theory and so the Constable-Myers background does not represent a vacuum of the theory. Nevertheless one could study the gauge theory with the operator present. This would correspond to ''holding'' the vacuum up the side of a potential well. If one ''let go'' the theory would oscillate about the minimum but one could imagine pinning it with a source term. Here we simply want to investigate whether some strongly coupled gauge configuration without supersymmetric constraints will cause chiral symmetry breaking in the quark sector, for which this setup suffices. In terms of the brane construction we would expect there to be some excited D3 configuration that sits at the center of the geometry and resolves the singularity. These branes would naturally decay back to a supersymmetric configuration in time if not held by a source, but again that excited configuration should still exist in principle.

The geometry in *Einstein frame* is given by

$$ds^2 = H^{-1/2}\left(\frac{w^4+b^4}{w^4-b^4}\right)^{\delta/4} dx_4^2 + H^{1/2}\left(\frac{w^4+b^4}{w^4-b^4}\right)^{(2-\delta)/4}\frac{w^4-b^4}{w^4}\sum_{i=1}^{6} dw_i^2, \tag{8}$$

where

$$H = \left(\frac{w^4+b^4}{w^4-b^4}\right)^{\delta} - 1 \tag{9}$$

and the dilaton and four-form become

$$e^{2\phi} = e^{2\phi_0}\left(\frac{w^4+b^4}{w^4-b^4}\right)^{\Delta},$$

$$C_{(4)} = -\frac{1}{4}H^{-1}dt\wedge dx\wedge dy\wedge dz. \tag{10}$$

There are formally two free parameters, R and b, since

$$\delta = \frac{R^4}{2b^4}, \quad \Delta^2 = 10 - \delta^2. \tag{11}$$

As usual in the AdS/CFT correspondence the w directions have the conformal scaling properties of energy scale in the field theory. Thus b is the only object that breaks the conformal (and super) symmetry of the gauge theory. We define an associated mass scale

$$\Lambda_b = \frac{b}{2\pi\alpha'} \tag{12}$$

which is the mass of a string of length b. Since Λ_b is the scale of conformal symmetry breaking and the theory is strongly coupled at that scale we expect the dynamical strong coupling scale of the theory, Λ, to lie close to Λ_b. We will hence forth loosely associate the two. In fact we are not interested in changing the scale Λ_b since it is the only mass scale so we can set $b=1$ below. First, though, let us consider the R dependence of the solution.

The parameter R determines $g_{YM}^2 N$ in the field theory as usual in the correspondence ($R^2 = \sqrt{4\pi g_s N}\alpha'$). We find it easiest to track the R dependence by writing w and b in units of R so that $\delta = 1/2b^4$. This means that our fundamental energy scale Λ_b scales with R so we should express masses as a ratio of

$$\Lambda_b = \frac{Rb}{2\pi\alpha'}. \tag{13}$$

Now we can set $b=1$ and the metric we will use is

$$ds^2=H^{-1/2}\left(\frac{w^4+1}{w^4-1}\right)^{\delta/4}dx_4^2 + R^2H^{1/2}\left(\frac{w^4+1}{w^4-1}\right)^{(2-\delta)/4}\frac{w^4-1}{w^4}\sum_{i=1}^{6}dw_i^2, \quad (14)$$

where

$$H=\left(\frac{w^4+1}{w^4-1}\right)^{\delta}-1, \quad e^{2\phi}=e^{2\phi_0}\left(\frac{w^4+1}{w^4-1}\right)^{\Delta}. \quad (15)$$

We will now introduce one flavor of quark via a D7 brane probe in the geometry. The D7 brane lies in the x_4 directions and w_1-w_4 (it is convenient to define a coordinate ρ such that $\Sigma_{i=1}^4 dw_i^2=d\rho^2+\rho^2 d\Omega_3^2$). This configuration in pure AdS preserves $\mathcal{N}=2$ supersymmetry on the D3 world volume so it corresponds to introducing a quark hypermultiplet Q and $\tilde{Q}$. $\mathcal{N}=2$ supersymmetry will ensure there is a superpotential coupling to one of the three adjoint chiral multiplets A of the original $\mathcal{N}=4$ theory ($W=\tilde{Q}AQ$). In the nonsupersymmetric field theory we expect the scalar fields to gain masses of order the supersymmetry breaking scale. The remaining fermionic terms in the gauge theory are of the form

$$\mathcal{L}=\frac{1}{g_{YM}^2}[\bar{q}\not{D}q+\cdots] \quad (16)$$

where one should note that g_s enters the normalization as for the gauge fields [this is not the standard normalization for comparison with, e.g., Eq. (4) above].

The Dirac-Born-Infeld action for the probe is

$$S_{D7}=-\frac{1}{(2\pi)^7\alpha'^4 g_s}\int d^8\xi e^{\phi}\sqrt{-\det(P[G_{ab}])} \quad (17)$$

where P indicates the pullback of the space-time metric onto the D7 world volume. This action will determine how the D7 lies in the remaining w_5-w_6 directions. We will use a complex coordinate in this plane,

$$\Phi=w_6+iw_5=\sigma e^{i\theta}. \quad (18)$$

Asymptotically where the geometry returns to AdS the resulting Euler-Lagrange equation is

$$\frac{d}{d\rho}\left[\rho^3\frac{d\Phi}{d\rho}\right]=0 \quad (19)$$

and has solutions

$$\Phi=m+\frac{c}{\rho^2}. \quad (20)$$

The two integration constants correspond to a mass and vacuum expectation value (VEV) for the quark bilinear $\bar{q}q$ with

$$m_q=\frac{mR}{2\pi\alpha'}, \quad \bar{q}q=\frac{cR^3}{(2\pi\alpha')^3}. \quad (21)$$

Note that we are writing Φ in units of R and hence it has the same N scaling as the parameter b. These measures of the quark mass and condensate when expressed as multiples of Λ_b do not scale with N.

In the massless limit where the D7 brane lives at $\Phi=0$ there is a U(1) symmetry acting in the Φ plane. Clearly this corresponds to an angular rotation on $\bar{q}q$ and is hence the $U(1)_A$ symmetry of the quarks. In fact this symmetry is also part of the isometries of the space transverse to the central D3 brane stack and is thus part of the $SO(6)_R$ symmetry of the gauge background. This reflects the presence of the UV superpotential term $W=\tilde{Q}AQ$ which mixes the axial and R symmetries. The A field is hopefully somewhat massive in this configuration (of order Λ_b) but the symmetry is nevertheless this mixture.

IV. CHIRAL SYMMETRY BREAKING

Let us begin by considering the massless quark limit where there is a good $U(1)_A$ symmetry. Asymptotically in the UV the D7 brane lies at $\Phi=0$. The equation of motion that determines how it lies in the interior is given by

$$\frac{d}{d\rho}\left[\frac{e^{\phi}\mathcal{G}(\rho,\Phi)}{\sqrt{1+|\partial_\rho\Phi|^2}}(\partial_\rho\Phi)\right] - \sqrt{1+|\partial_\rho\Phi|^2}\frac{d}{d\bar{\Phi}}[e^{\phi}\mathcal{G}(\rho,\Phi)]=0, \quad (22)$$

where

$$\mathcal{G}(\rho,\Phi)=\rho^3\frac{[(\rho^2+|\Phi|^2)^2+1][(\rho^2+|\Phi|^2)^2-1]}{(\rho^2+|\Phi|^2)^4}. \quad (23)$$

The final term in the equation of motion is a "potential"-like term that is evaluated to be

$$\frac{d}{d\bar{\Phi}}[e^{\phi}\mathcal{G}(\rho,\Phi)]=\frac{4\rho^3\Phi}{(\rho^2+|\Phi|^2)^5}\left(\frac{[(\rho^2+|\Phi|^2)^2+1]}{[(\rho^2+|\Phi|^2)^2-1]}\right)^{\Delta/2} \times[2-\Delta(\rho^2+|\Phi|^2)^2]. \quad (24)$$

The equation of motion has an explicit $\Phi\to e^{i\alpha}\Phi$ symmetry. In the massless case this is the $U(1)_A$ symmetry on the quarks. The solutions to the equation of motion in the interior simplify to the D7 brane sitting at a fixed angle in the plane, θ, with the radial behavior given by setting $\Phi=\sigma$ real in the equation of motion.

There are two regular solutions of the equation of motion with UV boundary condition $\sigma=c/\rho^2$, shown in Fig. 1. They have $c=\pm 1.86$ showing that the solution indeed prefers the formation of a chiral symmetry breaking condensate. Note that in the IR (small ρ) these solutions are $\sigma=$const, so they can also be found by numerically solving up from the IR and

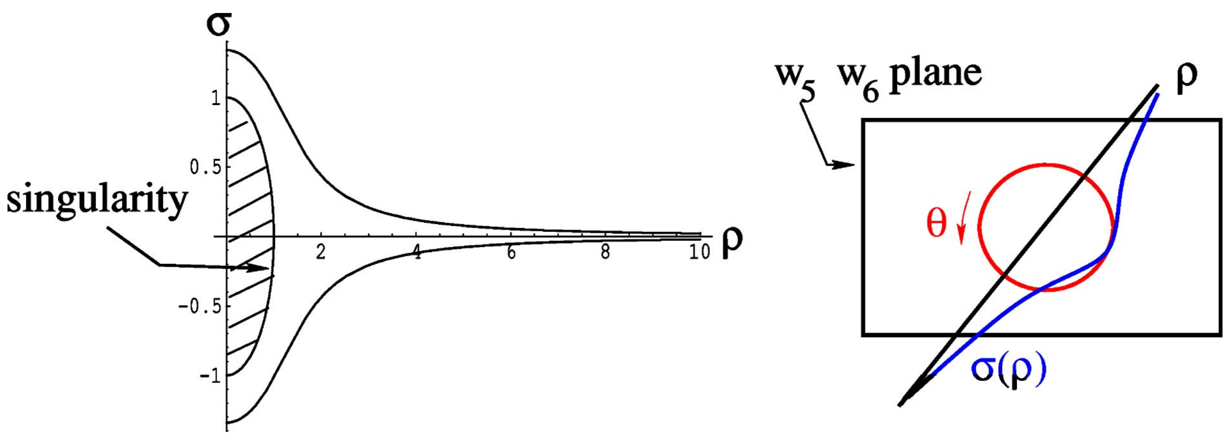

FIG. 1. Solutions of Eq. (22) for the position of the D7 brane showing chiral symmetry breaking in the massless limit and a sketch of the D7s position in the space.

matching to the UV boundary conditions. This is the simplest way to find solutions since any IR boundary condition of this sort produces a regular flow and it is then not necessary to tune onto the regular flows. The two solutions are just two opposing points on a circle in Φ verifying that there is indeed a set of solutions with the same radial behavior at each value of Φ. This circle of degenerate solutions is the vacuum manifold.

It is interesting to see how the chiral symmetry breaking manifests itself geometrically. The solution for the position of the D7 brane is sketched in Fig. 1. Asymptotically it lies at the origin of the plane but in the interior the D7 brane is repelled by the core of the geometry forcing it out into the plane where it breaks the U(1) symmetry.

Massive case

It is straightforward to also introduce a mass into the UV boundary conditions for Φ. Let us first look at solutions of the form $\theta=0$ and $\sigma=m+c/\rho^2$ in the UV. These will turn out to describe the true vacuum of the theory. The regular solutions have σ tend to a constant in the IR so are easily found by flowing up from the IR. The mass and condensate can then be read from the solution in the UV. We plot a few of these solutions in Fig. 2. This procedure also allows us to determine the mass dependence of the condensate and we plot that also in Fig. 2.

It is also interesting to study the vacua with a real mass term but with a phase on the condensate—these will correspond to the vacua around the circle in θ in the $m=0$ limit and we expect their energy to be lifted relative to the true vacuum when the mass is nonzero. To study these we use boundary conditions on the UV fields

$$w_5=m+\frac{c}{\rho^2}\cos\theta, \quad w_6=\frac{c}{\rho^2}\sin\theta. \tag{25}$$

In general we are looking for a regular solution for both w_5, w_6 and this is very hard to achieve numerically. As an example of evidence for the existence of these solutions though we show in Fig. 3 some solutions for w_5, w_6 as a function of ρ for the case $m=0.1$, $\theta=90°$. The solutions are plotted with asymptotic UV boundary conditions with two different values of the condensate parameter c. Both w_5, w_6 fields change behavior in this range suggesting there may indeed be a regular solution in between. We have not been able to pin down the solution to greater accuracy than this though.

More easily we can study the two solutions of the equation of motion with Φ taken real. These correspond to the

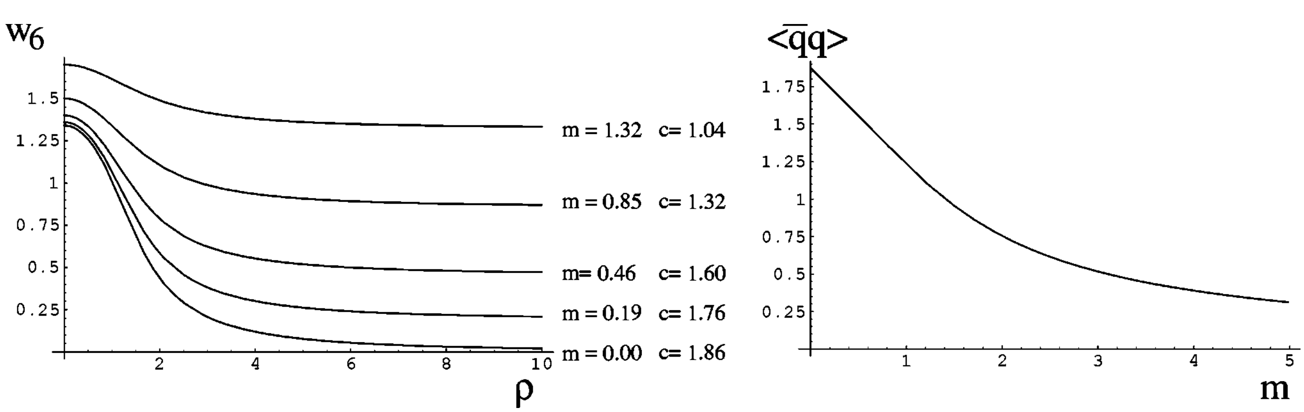

FIG. 2. Solutions for the w_6 flow when $w_5=0$ showing the dependence of the condensate on the quark mass.

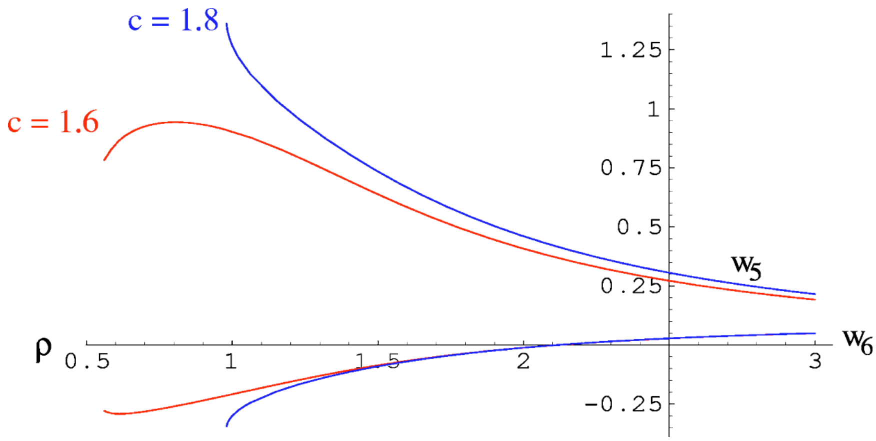

FIG. 3. w_5, w_6 flows for $m = 0.1$, $\theta = 90°$ showing the possibility of a regular flow between these values of c.

cases of $\pi = 0°$ and 180°. The two solutions have the same value of m in the UV but opposite c. Example solutions for a mass of 0.46 are plotted in Fig. 4.

We can now begin to determine the chiral Lagrangian parameters predicted by this model. We can calculate the vacuum energy as a function of the quark mass for the positive and negative condensate solutions. The vacuum energy is given by the DBI action

$$S_{D7} = -\frac{R^4}{(2\pi)^7 \alpha'^4 g_s} \int d^8\xi e^{\phi} \mathcal{G}(\rho,\Phi) \sqrt{1 + (\partial_\rho \Phi)^2}. \tag{26}$$

The angular integral over the S^3 which the D7 wraps gives $2\pi^2$. The resulting four dimensional cosmological term, Ω^4, should be normalized by Λ_b^4,

$$\frac{\Omega^4}{\Lambda_b^4} = \frac{1}{2\pi g_s} \mathcal{I}_0, \quad \mathcal{I}_0 = \frac{1}{2} \int d\rho e^{\phi} \mathcal{G}(\rho,\Phi) \sqrt{1 + (\partial_\rho \Phi)^2} \tag{27}$$

where the integral $\mathcal{I}_0$ is over the solutions described above. Note that the factor of g_s shows us that the vacuum energy scales as N in the large N limit with $g_s N$ kept constant. This is consistent with expectations for the vacuum energy contributions from fundamental quarks.

Asymptotically $\Omega^4 \simeq \rho_{UV}^4$ indicating the expected UV divergence of the vacuum energy. This will be present for all our configurations no matter the quark mass or condensate.

We are interested in the corrections due to the low energy chiral dynamics [those in Eq. (3)] so subtract the massless result for Ω^4 from the vacuum energy for each configuration. We show numerical results for Ω^4/Λ_b^4 in Fig. 5.

The positive condensate solutions are energetically favorable. This is because the potential for the quark condensate has a tilted wine bottle shape of the form shown in Fig. 6. There we sketch a potential of the form $V = \alpha|\Phi|^4 - \beta^2|\Phi|^2 + m\,\mathrm{Re}(\Phi)$—as m increases the potential tilts giving a single true vacuum. In this simple model there is a critical value of m where the central "hump" disappears and there is a single unique vacuum. We have numerically looked for such a solution in the D7 brane case. As the boundary condition on $m \to 1.5$ the solution for the metastable vacuum (of the type shown in Fig. 4) has a singular derivative as it passes through the w_6 axis and we lose numerical control. This may well be an indication of the absence of such solutions for larger m corresponding to the loss of the hump in the naive model.

Finally we can use the vacuum energy to compute ν^3 in Eq. (3) and to provide an alternative identification of the quark condensate in the model. The chiral Lagrangian predicts that the vacuum energy will be given by

$$\Omega^4 = 2\nu^3 m = \langle \bar{q}q \rangle m. \tag{28}$$

Before extracting $\langle \bar{q}q \rangle$ in this way we must normalize the quark fields in the standard way appropriate for these equations. This removes the factor of $1/2\pi g_s$ in Eq. (27) and we

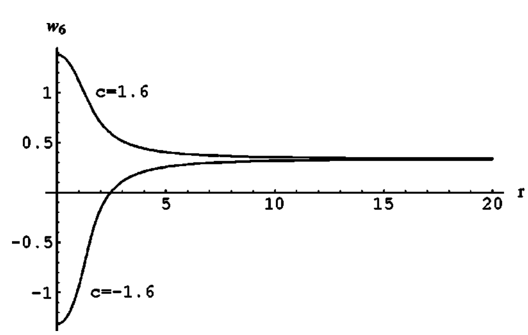

FIG. 4. w_6 flows for m=0.46 with positive and negative condensate solutions.

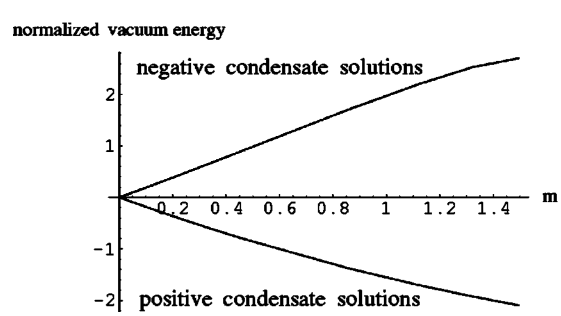

FIG. 5. Vacuum energy, with that at $m = 0$ subtracted, for $\theta = 0°$ and 180° showing a lower energy for the case where the mass and condensate are both positive.

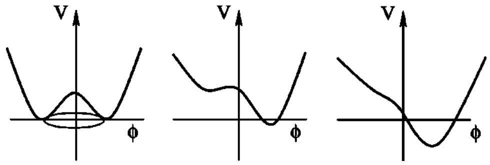

FIG. 6. Sketch of a simple Higgs potential with an explicit symmetry breaking term $m\,\mathrm{Re}(\Phi)$, showing the spontaneous symmetry breaking potential as m increases.

then extract the condensate from the slope of the vacuum energy vs mass plot. We find numerically

$$\langle \bar{q}q\rangle = 1.86 \qquad (29)$$

which almost precisely matches the value of c in the $m\to 0$ limit above. The fact that c corresponds to the condensate of the canonically normalized fields explains why it does not scale with N. This identification confirms c is the condensate or equally the form of the Gell-Mann-Oakes-Renner relation.

V. PIONS AND THEIR INTERACTIONS

We will now turn to studying the Goldstone bosons of the chiral symmetry breaking we have observed above. So far we have considered the dynamical breaking of the $U(1)_A$ symmetry of a single flavor of quark. The Goldstone boson will therefore be the analogue of the η' in QCD, which becomes degenerate with the QCD pions in the limits where the quark masses vanish and at large N where the anomaly is suppressed. We will loosely refer to our state as a pion. We will shortly consider extending this construction to non-Abelian axial symmetries.

A. The Pion Mass and f_π

The pion in the one flavor case will be a bound state of a quark and an antiquark and hence will be described by the D7 world volume field Φ. The Goldstone fluctuation corresponds to oscillations of the field along the vacuum manifold—in the brane language this will correspond to fluctuations of the brane in the direction of the possible set of solutions for its background position. In the chiral limit this corresponds to fluctuations in the angular θ direction in the w_5-w_6 plane. To leading order, if we have a background configuration for the D7 brane described by a particular real solution of Eq. (22), σ_0, we can look at small fluctuations in the θ angular direction

$$\theta(\rho,x) = f(\rho)\sin kx. \qquad (30)$$

We look for solutions where $k^2 = M^2$. The action for these fluctuations to quadratic order is given by expanding the DBI action (note that the metric is independent of the angle θ)

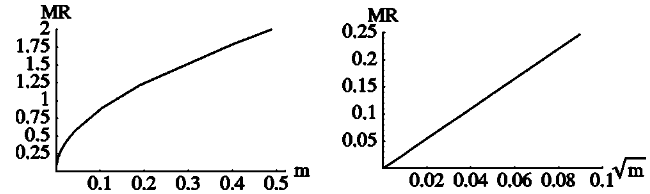

FIG. 7. Quark mass versus meson mass showing massless pion at $m=0$. Also shown is the square root behavior of the relationship.

$$S = \frac{2\pi^2}{(2\pi)^7\alpha'^4 g_s}\int d\rho d^4x e^{\phi} R^4 \mathcal{G}\sqrt{1+(\partial_\rho\sigma_0)^2} \times\left[1+\frac{1}{2}\frac{g^{\rho\rho}g_{\theta\theta}(\partial_\rho\theta)^2}{[1+(\partial_\rho\sigma_0)^2]}+\frac{1}{2}\frac{g^{\mu\nu}g_{\theta\theta}(\partial_\mu\theta)(\partial_\nu\theta)}{[1+(\partial_\rho\sigma_0)^2]}\right]. \qquad (31)$$

Note that the DBI action only produces terms with derivatives of θ—however, we can obtain potential terms in the pion field when those derivatives are ρ derivatives that act on $f(\rho)$. We find the resulting equation of motion for f,

$$\frac{d}{d\rho}\left[\frac{e^{\phi}\mathcal{G}}{\sqrt{1+(\partial_\rho\sigma_0)^2}}\sigma_0^2(\partial_\rho f)\right]+\frac{R^2M^2e^{\phi}\mathcal{G}}{\sqrt{1+(\partial_\rho\sigma_0)^2}} \times H\left(\frac{(\rho^2+\sigma_0^2)^2+1}{(\rho^2+\sigma_0^2)^2-1}\right)^{(1-\delta)/2}\frac{(\rho^2+\sigma_0^2)^2-1}{(\rho^2+\sigma_0^2)^2}\sigma_0^2 f = 0. \qquad (32)$$

This equation can be numerically solved for MR as a function of m using the UV boundary condition $f = 1/\rho^2$ (reflecting the fact that the pion has the UV scaling dimension of $\bar{q}q$) and seeking regular solutions for f. We plot the result (in which m determines the σ_0 flow) in Fig. 7. We indeed find a massless pion at $m=0$ in accordance with Goldstone's theorem. Note that in the case where the pion mass vanishes the equation of motion is just the first term above and when substituted back into Eq. (31), after integration by parts, explicitly makes the $(\partial_\rho\theta)^2$ term vanish. In the chiral limit if one worked to higher order all higher order terms involving $(\partial_\rho\theta)^n$ would also vanish directly. This demonstrates that the vacuum manifold is a truly flat direction. Below we will only look at terms at higher order involving the spatial derivatives ∂_μ.

In Fig. 7 we also plot MR vs $\sqrt{m}$ for small m—there is a good linear fit matching the expectations for the pion mass dependence on m in the chiral Lagrangian (3). We will write

$$MR = \kappa\sqrt{m} \qquad (33)$$

where we have numerically determined $\kappa = 2.75$.

Note that the appropriate four dimensional, low energy action can be found by writing $\theta = 2\pi\alpha' f(\rho)\Pi(x)$ and substituting the equation of motion for $f(\rho)$ back into the action (31), after integrating by parts. This gives

$$\mathcal{L}=\frac{R^6}{16\pi^3\alpha' g_s}\mathcal{I}_1\left[\frac{1}{2}(\partial^\mu\Pi)^2-\frac{1}{2}M^2\Pi^2+\cdots\right] \tag{34}$$

where

$$\mathcal{I}_1=\int d\rho f(\rho)^2\frac{e^\phi\mathcal{G}}{\sqrt{[1+(\partial_\rho\sigma_0)^2]}} \times H\left(\frac{(\rho^2+\sigma_0^2)^2+1}{(\rho^2+\sigma_0^2)^2-1}\right)^{(1-\delta)/2}\frac{(\rho^2+\sigma_0^2)^2-1}{(\rho^2+\sigma_0^2)^2}\sigma_0^2. \tag{35}$$

The coefficient at the front of Eq. (34) should be absorbed into the normalization of Π to make the kinetic term canonical. We must be careful to include this normalization below. Note also that $\mathcal{I}_1$ is only defined up to the normalization of f which is free in the linearized equation of motion—this factor will cancel when Π is canonically normalized in all quantities below.

The chiral Lagrangian predicts that

$$M_\pi^2=\frac{4\nu^3 m}{f_\pi^2}. \tag{36}$$

Since we have extracted ν^3 from the vacuum energy above we can now determine f_π. With the supersymmetric normalization of fields we found numerically

$$\frac{\nu^3}{\Lambda_b^3}=\frac{1}{4\pi g_s}c. \tag{37}$$

Writing our result for the pion mass in units of Λ_b gives

$$\frac{M^2}{\Lambda_b^2}=\kappa^2\frac{m_q}{\Lambda_b}\frac{\pi}{g_s N} \tag{38}$$

and hence we find for f_π

$$\frac{f_\pi^2}{\Lambda_b^2}=\frac{N}{\pi^2\kappa^2}c=0.246\frac{N}{\pi^2} \tag{39}$$

which has the expected dependence on N and is suppressed relative to Λ_b by roughly a factor of $4\pi^2$. This seems to match well with the naive estimate of Sec. II.

B. Higher order interactions

It is also interesting to study higher order interaction terms such as those in Eq. (5). To do this we can expand the DBI action beyond linearized order in θ and use the linearized solution for $f(\rho)$.

In the chiral limit the identification of the pion field with fluctuations of the brane in the θ direction is rigorous. However, when there is a quark mass present the vacuum will be distorted from a true circle centered on the origin in Φ and this procedure does not correctly identify the pseudo-Goldstone boson (the angular direction will have some of the massive ''Higgs''-like mode mixed in). In principle to find the pseudo-Goldstone boson in the massive case one would need to know how to separate the function $\sigma_0(r)$ into pieces that represent the running of the mass and the condensate separately. The pseudo-Goldstone boson would correspond to angular fluctuations about just the condensate piece. In other words simply changing the phase on σ_0 changes the phase on the mass, which one does not want to do. We have not been able to resolve this so we will simply study the massless chiral limit here.

As mentioned above, in the chiral limit any terms in the action involving $(\partial_\rho\theta)^n$ vanish by the equations of motion so we will neglect those here [in fact we have checked that they explicitly vanish when evaluated on our solution for $f(\rho)$ in this limit]. Now we can study the Gasser-Leutwyler coefficients L_1, L_2, L_3. In the chiral Lagrangian, these appear in the form

$$L_1\mathrm{tr}(\partial^\mu U\partial_\mu U^\dagger)^2+L_2\mathrm{tr}[(\partial^\mu U\partial^\nu U^\dagger)(\partial_\mu U\partial_\nu U^\dagger)] +L_3\mathrm{tr}(\partial^\mu U\partial_\mu U^\dagger\partial^\nu U\partial_\nu U^\dagger). \tag{40}$$

Since we only have an Abelian U(1) symmetry breaking pattern, we cannot differentiate between these terms. When U is expanded, each term has a factor of $(\partial^\mu\Pi)^4$ so we cannot extract any information about L_1, L_2, or L_3 separately. It is still interesting to extract the coefficient of the $(\partial^\mu\Pi)^4$ term. This term in the action is

$$\mathcal{L}=\frac{2\pi^2R^4}{(2\pi)^7 g_s\alpha'^4}\int d\rho e^\phi\mathcal{G}\sqrt{1+(\partial_\rho\sigma_0)^2} \times\left[-\frac{1}{4}\frac{(g_{\theta\theta}g^{\mu\nu})^2}{[1+(\partial_\rho\sigma_0)^2]}(\partial_\mu\theta)^4\right]. \tag{41}$$

Rewriting this in terms of the field Π gives

$$\mathcal{L}=-\frac{R^8}{16\pi g_s}\mathcal{I}_2(\eta^{\mu\nu}\partial_\mu\Pi\partial_\nu\Pi)^2 \tag{42}$$

where

$$\mathcal{I}_2=\int d\rho e^\phi\frac{\mathcal{G}f(\rho)^4\sigma_0^4}{[1+(\partial_\rho\sigma_0)^2]^{3/2}} \times\left[H^2\left(\frac{\omega^4+1}{\omega^4-1}\right)^{1/2}\left(\frac{\omega^4-1}{\omega^4}\right)^2\right]. \tag{43}$$

We must remember to normalize the pion field so the kinetic term in Eq. (34) is canonically normalized, from which we find the coefficient of the $(\partial_\mu\Pi)^4$ term (which we loosely write as $4L/f_\pi^4$) given by

$$\frac{L}{f_\pi^4}=\frac{16\pi^5\alpha'^4 g_s}{R^4}\frac{\mathcal{I}_2}{\mathcal{I}_1^2} \tag{44}$$

or using the expression above for f_π

$$L=\frac{(g_{YM}^2N)Nc^2}{\kappa^4 2\pi^4}\frac{\mathcal{I}_2}{\mathcal{I}_1^2}. \qquad (45)$$

This has the same large N scaling (scaling as f_Π^2) as the naive dimensional analysis estimate. Numerically we find $\mathcal{I}_2/\mathcal{I}_1^2=0.29$. If we take $g_{YM}^2N=(4\pi)^2$ which seems a reasonable strong coupling value then we find $L\simeq 0.016N$. This is about a factor of 2–3 bigger than $N/16\pi^2=0.006N$ and hence broadly consistent with naive dimensional analysis.

C. Non-Abelian case

Naively we can introduce extra flavors of quarks by including additional probe D7 branes and using the non-Abelian DBI action [33]. As mentioned in the Introduction we do not generate a U(N) axial symmetry because of the superpotential term $\tilde{Q}AQ$ in the UV theory. The adjoint scalar though is expected to have a mass of order Λ and we might expect the theory to have an accidental chiral symmetry. The non-Abelian DBI action requires us to treat the scalar field Φ as a matrix which for the pions will require us to write

$$\Phi=\sigma_0 e^{i\Pi^a T^a}, \qquad (46)$$

where T^a are the broken generators of the Lie group. With N_f D7 branes this is $U(N_f)$. The DBI action acquires a trace over the flavor indices. As pointed out in [8] if this were the only change then we would find N_f^2 copies of the action we had above and hence N_f^2 massless pions. The DBI action also contains terms though that generate the usual commutator scalar interactions (tr$[\phi_i,\phi_j]^2$)—however, these do not generate a mass for the pions. The explicit chiral symmetry breaking appears therefore to be quite weak in its effects. To pursue this issue further we can study whether the interactions of the pions, neglecting the commutator terms, coincide with expectations from chiral symmetry.

For simplicity we will work in the case with just two flavors and look at just the interactions of two pions, Π_1 and Π_2. This means that we should write

$$\Phi=\sigma_0\exp\left[i(2\pi\alpha')f(\rho)\times\begin{pmatrix}0 & \Pi_1(x)+I\Pi_2(x)\\ \Pi_1(x)-I\Pi_2(x) & 0\end{pmatrix}\right]. \qquad (47)$$

Note that in the chiral limit we can drop all terms in the DBI action involving ρ derivatives since they will cancel by the equation of motion as discussed above. The resulting action then takes the form of a chiral Lagrangian with spatial derivative acting on the field U. We find the Lagrangian for the two pion fields takes the form

$$\begin{aligned}\mathcal{L}=&\mathcal{A}\{[\partial_\mu\Pi_1(x)]^2+[\partial_\mu\Pi_2(x)]^2\}\\&+\frac{1}{3}\mathcal{B}\{2\Pi_1(x)\Pi_2(x)[\partial_\mu\Pi_1(x)\partial^\mu\Pi_2(x)]\\&-\Pi_1^2[\partial_\mu\Pi_2(x)]^2+\Pi_2^2[\partial_\mu\Pi_1(x)]^2\},\end{aligned} \qquad (48)$$

where

$$\mathcal{A}=\frac{2\pi^2R^4}{(2\pi)^7\alpha'^4g_s}(2\pi\alpha'R)^2\mathcal{I}_1$$

$$\mathcal{B}=\frac{2\pi^2R^4}{(2\pi)^7\alpha'^4g_s}(2\pi\alpha')^4R^2\mathcal{I}_3, \qquad (49)$$

and

$$\mathcal{I}_3=\int e^\phi\mathcal{G}H\left(\frac{w^4+1}{w^4-1}\right)^{1/4}\frac{w^4-1}{w^4}\frac{\sigma_0^2f^4}{\sqrt{[1+(\partial_\rho\sigma_0)^2]}}. \qquad (50)$$

Performing the equivalent expansion in the chiral Lagrangian to compare these terms we find

$$\begin{aligned}\mathcal{L}_\chi=&\frac{1}{2}\{[\partial_\mu\Pi_1(x)]^2+[\partial_\mu\Pi_2(x)]^2\}\\&+\frac{1}{3f_\Pi^2}\{\Pi_1(x)\Pi_2(x)[\partial_\mu\Pi_1(x)\partial^\mu\Pi_2(x)]\}\\&-\frac{1}{6f_\Pi^2}\{\Pi_1(x)^2[\partial_\mu\Pi_2(x)]^2+\Pi_2(x)^2[\partial_\mu\Pi_1(x)]^2\}.\end{aligned} \qquad (51)$$

Note that the relative sizes of the two new interaction terms match between the DBI and chiral Lagrangians (in both cases because they result from the expansion of the same exponential form). The two new interaction terms provide an additional opportunity to calculate f_Π. Again, we canonically normalize the kinetic term and compare with the chiral Lagrangian,

$$\begin{aligned}\frac{f_\pi^2}{\Lambda_b^2}=&\frac{\mathcal{I}_1^2}{\mathcal{I}_3}\frac{4AR^2}{6}\frac{(2\pi\alpha')^2}{R^2}\\=&\frac{\mathcal{I}_1^2}{\mathcal{I}_3}\frac{N}{6\pi^2}=0.246\frac{N}{\pi^2}.\end{aligned} \qquad (52)$$

Within our numerical accuracy this is the same answer as that calculated in the Abelian case from the mass term which gave the value $\simeq N/4\pi^2$. The extra massless bosons indeed seem to behave as Goldstone fields up to the commutator interactions.

VI. VECTOR MESONS AND WEAKLY GAUGED CHIRAL SYMMETRIES

There is one additional bosonic field in the low energy DBI action of the D7 brane—the gauge field partner of the scalar, Φ, discussed above. The field strength tensor of this gauge field enters in the standard way as $2\pi\alpha' F^{ab}$ in the square root. The leading Lagrangian for this field, in a background configuration σ_0, is

$$\mathcal{L}=\frac{2\pi^2R^4}{(2\pi)^7\alpha'^4 g_s}\int d\rho e^{\phi}\mathcal{G}\left[\sqrt{1+(\partial_\rho\sigma_0)^2}H\left(\frac{(\rho^2+\sigma_0^2)^2-1}{(\rho^2+\sigma_0^2)^2+1}\right)^{1/4}(2\pi\alpha')^2\frac{1}{4}F^{\mu\nu}F_{\mu\nu}\right. \tag{53}$$

$$\left.+\frac{1}{2R^2}\frac{1}{\sqrt{1+(\partial_\rho\sigma_0)^2}}\left(\frac{(\rho^2+\sigma_0^2)^2-1}{(\rho^2+\sigma_0^2)^2+1}\right)^{1/2}\frac{(\rho^2+\sigma_0^2)^2}{(\rho^2+\sigma_0^2)^2-1}F^{\mu\rho}F_{\mu\rho}\right], \tag{54}$$

where μ and ν run over the space-time indices. There is an additional term that could be added onto the DBI action. This is the Wess-Zumino term which gives the coupling of the four-form $C^{(4)}$ to the gauge fields. We do not include this because when we calculate the equations of motion for the gauge fields, this term is only relevant for the gauge fields with a vector index on the S^3. We will only be interested in the states that carry no SO(4) R-charge where a,b take values in the four dimensional space of the gauge theory. We write the gauge field as

$$A^{\mu}=g(\rho)\sin(kx)\epsilon^{\mu}. \tag{55}$$

The equations of motion for the gauge field are given by

$$e^{\phi}\mathcal{G}\sqrt{1+(\partial_\rho\sigma_0)^2}M^2R^2g(\rho)H\left(\frac{w^4-1}{w^4+1}\right)^{1/4}+\partial_\rho\left(\frac{e^{\phi}\mathcal{G}}{\sqrt{1+(\partial_\rho\sigma_0)^2}}\partial_\rho g(\rho)\frac{w^4}{\sqrt{(w^4+1)(w^4-1)}}\right)=0. \tag{56}$$

There is clearly a solution with $M=0$ and $g(\rho)=$const. This corresponds to introducing a background gauge field associated with U(1) baryon number in the field theory. Note that asymptotically in the UV the Lagrangian for this field is

$$\mathcal{L}\simeq\frac{N}{4\pi^2}\log\frac{\Lambda_{UV}}{\Lambda_b}\frac{1}{4}F^{\mu\nu}F_{\mu\nu} \tag{57}$$

which reflects the logarithmic running of the flavor gauge coupling.

There is a second asymptotic solution of the equation of motion where $g(r)\sim 1/\rho^2$ and these solutions correspond to the vector meson spectrum associated with the operator $\bar{q}\gamma^{\mu}q$. By seeking smooth solutions of the equation of motion we can determine the vector meson mass spectrum. The results of the analysis are shown in Table I compared to those calculated in the pure AdS background. The spectrum (of singlets on the S_3) in the pure AdS case is given by [6]

$$M_v^2R^2=4(n+1)(n+2). \tag{58}$$

There are a number of interesting questions one might ask with regard to the vector mesons and weakly gauged chiral symmetries. For example in models of dynamical symmetry breaking such as technicolor the value of the parameter L_{10} in Eq. (5) is related to the phenomenological S parameter which is well measured by the precision electroweak data. This is a UV finite quantity which in this model would be related to the difference between the coefficient of the vector F^2 term calculated above and the equivalent term for the axial gauge field. Unfortunately the DBI action has no field that corresponds to the axial gauge field. Presumably equally the operators $\bar{q}\gamma^{\mu}\gamma_5 q$ are described by a string mode not present in the DBI action, so it is therefore not possible for us to estimate this parameter. Similarly it would have been nice to have tested the Weinberg sum rules [35] such as, assuming vector meson dominance,

$$f_{\Pi}^2=\frac{g_V^2}{M_V^2}-\frac{g_A^2}{M_A^2}, \tag{59}$$

where $g_{V/A}$ are the couplings of the lightest vector and axial vector mesons to their respective currents. We have no description of the axial vector mesons though. It would be productive to understand this sector in the future.

TABLE I. Vector meson spectrum comparing CM and pure AdS backgrounds.

n	AdS case	CM case
0	2.83	2.16
1	4.90	4.85
2	6.93	7.05
3	8.94	9.20
4	11.0	11.3
5	13.0	13.5
6	15.0	15.6
7	17.0	17.7
8	19.0	19.9

These issues have been highlighted in [36] where a toy model of large N QCD was proposed inspired by the general structure of the AdS/CFT correspondence. That model has a flavor gauge field living in a (deconstructed) finite fifth dimension with symmetry breaking boundary conditions. They interpret A^5 as the pion field and the KK modes of the flavor gauge field as the vector and axial vector mesons. This is distinct from our scenario where the pion fields are described by an additional scalar field and the flavor gauge field only describes vector mesons—the two pictures do not support each other therefore.

VII. SUMMARY

We have analyzed chiral symmetry breaking via the gravitational dual construction of D7 brane probes in the Constable-Myers nonsupersymmetric geometry. The ground state of the D7 brane indeed describes chiral symmetry breaking and a vacuum manifold with massless pions in the chiral limit. We have shown that these pion fields are described by a chiral Lagrangian in the infrared as expected. We have computed the couplings of the low energy Lagrangian and shown that they match expectations from naive dimensional analysis.

One might have expected significant deviations in the magnitudes of parameters because the gauge coupling of the model remains strong in the UV (as in for example a ''walking'' gauge theory [37]). However, the UV of the theory has an approximate $\mathcal{N}=4$ supersymmetry that forbids a chiral condensate so that above our symmetry breaking scale Λ_b the condensate decreases rapidly in spite of the strong coupling. The chiral symmetry breaking behavior of this model is therefore rather pleasingly analogous to that of QCD.

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