

Electric fields at the quark surface of strange stars in the color-flavor locked phase

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It is shown that extremely strong electric fields may be generated at the surface of strange quark matter in the color-flavor locked phase because of the surface effects. Some properties of strange stars made of this matter are briefly discussed.

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I. INTRODUCTION

Strange quark matter (SQM) that consists of deconfined u , d , and s quarks may be the absolute ground state of the strong interaction, i.e., absolutely stable with respect to ^{56}Fe [1,2]. If SQM is approximated as noninteracting quarks, chemical equilibrium with respect to the weak interaction together with the relatively large mass of the s quark imply that the s quarks are less abundant than the other quarks, and electrons are required in SQM to neutralize the electric charge of the quarks. The electron density at vanishing pressure is $\sim 10^{-4}$ of the quark density [3,4]. The electrons, being bound to SQM by the electromagnetic interaction alone, are able to move freely across the SQM surface, but clearly cannot move to infinity because of the bulk electrostatic attraction to the quarks. The distribution of electrons extends several hundred fermis above the SQM surface, and an enormous electric field $E \approx 5 \times 10^{17} \text{ V cm}^{-1}$ is generated in the surface electron layer to prevent the electrons from escaping to infinity, counterbalancing the degeneracy and thermal pressure [3–5].

Strange stars made entirely of SQM have long been proposed as an alternative to neutron stars (e.g., [1,3]). The electric field at the surface of strange stars drastically affects their observational appearance. The point is that this field is a few 10 times higher than the critical field $E_{\text{cr}} \approx 1.3 \times 10^{16} \text{ V cm}^{-1}$ at which vacuum is unstable to creation of e^+e^- pairs. Therefore, a hot strange star with a bare SQM surface may be a powerful source of e^+e^- pairs which are created in the surface electric field and flow away from the star [6–8]. The thermal luminosity of such a star in pairs depends on the surface temperature T_S and may be as high as $\sim 10^{51} \text{ ergs s}^{-1}$ at the moment of formation when T_S may be up to $\sim 10^{11} \text{ K}$. The luminosity of a young bare strange star in pairs may remain high enough ($\geq 10^{36} \text{ ergs s}^{-1}$) for $\sim 10^3 \text{ yr}$ [9].

The surface electric field may be also responsible for existence of a crust of “normal” matter (ions and electrons) at the SQM surface of a strange star [3]. The ions in the inner layer are supported against the gravitational attraction to the underlying strange star by the electric field.

It is becoming widely accepted that because of an attractive interaction between quarks in some specific

channels, the ground state of SQM is a color superconductor [10]. At asymptotic densities ($\gg n_0$), this superconductor is likely to be in the color-flavor locked (CFL) phase in which quarks of all three flavors and three colors are paired in a single condensate, where $n_0 \approx 0.16 \text{ fm}^{-3}$ is the normal nuclear density. Unfortunately, at intermediate densities ($\sim 2n_0$) that are relevant to the surface layers of strange stars, the QCD phase of SQM is uncertain. In this low density regime, the SQM may be not only in the CFL phase, but also in the “two color-flavor superconductor” (2SC) phase in which only u and d quarks of two color are paired in a single condensate, while the ones of third color and the s quarks of all three colors are unpaired. However, it was recently argued that the density range where the 2SC phase may exist is small, if any [11]. In the CFL and 2SC phases, the Cooper pairs are made of quarks with equal and opposite momenta. Another possibility is a crystalline color superconductor (CCS), which involves pairing between quarks whose momenta do not add to zero [12]. In the 2SC and CCS phases, electrons are present, and the electron density is more or less the same as in the unpaired SQM. Therefore, superconducting strange stars with the SQM surface layers in one of these phases are expected to be rather similar to strange stars that are made of noninteracting, unpaired quarks. It is now commonly accepted that the CFL phase of SQM in bulk consists of equal numbers of u , d , and s quarks and is electrically neutral in the absence of any electrons [13]. At first sight, an extremely strong electric field is absent at the surface of SQM in the CFL phase. In turn, this could result in qualitatively changes of many properties of strange stars (such as absence of both the intense emission of e^+e^- pairs and normal-matter crusts) if the strange star surface is in the CFL phase. In this paper, we show that this is not the fact, and supercritical electric fields may be generated at the CFL phase surface because of the surface effects.

II. ELECTRIC FIELDS AT THE CFL SURFACE

The modification of the density of quark states near the boundary of SQM gives rise to rather large positive surface tension for massive quarks [14]. Other sources of surface tension are probably smaller [2], and we ignore them here.

The reason for the electrical neutrality of the CFL phase in bulk is that BCS-like pairing minimizes the energy if the quark Fermi momenta are equal. In turn, for equal Fermi momenta, the numbers of u , d , s quarks are equal, and the electric charge of the quarks is zero. The properties of the CFL phase with taking into account the surface effects were discussed by Madsen [15], and it was shown that the number of massive quarks is reduced near the boundary relative to the number of massless quarks at fixed Fermi momenta. The change in number of quarks of flavor i per unit area is [15]

$$n_{i,s} = -\frac{3}{4\pi} p_{F,i}^2 \left[\frac{1}{2} + \frac{\lambda_i}{\pi} - \frac{1}{\pi} (1 + \lambda_i^2) \tan^{-1}(\lambda_i^{-1}) \right], \quad (1)$$

where $i = \{u, d, s\}$, $p_{F,i}$ is the Fermi momentum of quarks of flavor i , $\lambda_i = m_i/p_{F,i}$, and m_i is the rest mass of quarks of flavor i . The value of $n_{i,s}$ is always negative, approaching zero for $\lambda_i \rightarrow 0$ (massless quarks). The rest masses of u and d quarks are very small, and their densities are not modified significantly by the surface. Thus, the only appreciable contribution to the surface corrections arises from the s quarks, i.e., surface effects are highly flavor dependent. Because of surface depletion of s quarks thin layers at the surface of the CFL phase are no longer electrically neutral as in bulk. The charge per unit area is positive and equals

$$\sigma = -\frac{1}{3} e n_{s,s}. \quad (2)$$

The thickness of the charged layer at the surface of SQM in the CFL phase is of order of 1 fm, which is a typical strong interaction length scale.

Electrons are required to neutralize electrically the charged layer of SQM. The thickness of the electron distribution is about two order more than the thickness of the SQM charged layer (see [3,4] and below), and therefore, we assume that the last is infinitesimal. In this case, the density of electrons n_e and the electrostatic potential V are symmetric to the SQM charged layer, i.e., $n_e(-z) = n_e(z)$ and $V(-z) = V(z)$, where z is a space coordinate measuring height above the SQM surface. In turn, the electric field $E = -dV/dz$ is directed from the SQM charged layer, and $E(-z) = -E(z)$. The strength of the electric field at the SQM surface ($|z| \rightarrow 0$) is $E_0 = [E(+0) - E(-0)]/2 = 2\pi\sigma$, where σ is given by Eq. (2). Taking $m_s \simeq 150$ MeV and $p_{F,s} \simeq 300$ MeV as typical parameters of SQM, from Eqs. (1) and (2) we have $E_0 \simeq 2.7 \times 10^{18} \text{ V cm}^{-1} \simeq 200 E_{\text{cr}}$ that is ~ 5 times larger than the surface electric field calculated for SQM in the unpaired phase neglecting the surface effects (e.g., [3–5]). We hope to deal with these effects for the unpaired, 2SC and CCS phases elsewhere.

In a simple Thomas-Fermi model, the distribution of the electrostatic potential $V(z, T_S)$ near the SQM surface with the temperature T_S is described by Poisson's equation

(e.g., [3,4,16])

$$\frac{d^2 V}{dz^2} = \frac{4\alpha}{3\pi} (e^2 V^3 + \pi^2 T_S^2 V), \quad (3)$$

where α is the fine-structure constant. The boundary conditions for Eq. (3) are

$$dV/dz = \mp 2\pi\sigma \quad (4)$$

at the external ($z = +0$) and internal ($z = -0$) sides of the SQM surface, respectively, and $V \rightarrow 0$ as $z \rightarrow \pm\infty$. The first integral of Eq. (3), which satisfies the boundary condition at $z \rightarrow \pm\infty$, is

$$\frac{dV}{dz} = \mp \left(\frac{2\alpha}{3\pi} \right)^{1/2} (e^2 V^4 + 2\pi^2 T_S^2 V^2)^{1/2}. \quad (5)$$

where the sign $-$ or $+$ has to be taken at $z > 0$ or $z < 0$, respectively. Using the boundary condition (4) and Eqs. (2) and (5), we have the electrostatic potential at the SQM surface

$$V(0, T_S) = V(0, 0) \{ [1 + (T_S/T_*)^4]^{1/2} - (T_S/T_*)^2 \}^{1/2}, \quad (6)$$

where

$$V(0, 0) = \left(\frac{2\pi^3 n_{s,s}^2}{3\alpha} \right)^{1/4} \quad \text{and} \quad T_* = \frac{eV(0, 0)}{\pi}. \quad (7)$$

or numerically

$$V(0, 0) \simeq 3.6 \times 10^7 \text{ V} \quad \text{and} \quad T_* \simeq 11.6 \text{ MeV} \quad (8)$$

for $m_s \simeq 150$ MeV and $p_{F,s} \simeq 300$ MeV.

In the case of rather low temperatures ($T_S \ll T_*$), from Eq. (5) the electrostatic potential is

$$V(z, 0) = \left(\frac{3\pi}{2\alpha} \right)^{1/2} \frac{1}{e(|z| + z_0)}, \quad (9)$$

where $z_0 = (3\pi/2\alpha)^{1/2} [eV(0, 0)]^{-1} \simeq 2 \times 10^2 \text{ fm}$ is the typical thickness of the surface electron layer with a strong electric field. In this layer, the number density of electrons is (e.g., [4,16])

$$n_e(z, 0) = \frac{e^3 V^3(z, 0)}{3\pi^2} = \frac{1}{3\pi^2} \left(\frac{3\pi}{2\alpha} \right)^{3/2} \frac{1}{(|z| + z_0)^3}. \quad (10)$$

Hence, at the surface of SQM in the CFL phase a Coulomb barrier with an extremely strong electric field may be present because of the surface effects in spite of that this phase in bulk consists of equal numbers of u , d , and s quarks and is electrically neutral in the absence of any electrons. In this case the electric field is directed from the SQM surface and $E(-z) = -E(z)$. It is worth noting that the pairing energy contribution has been neglected by Madsen [15] in the derivation of Eq. (1). However, since the pairing energy is rather small compared with the other contributions to the energy, we think that this approximation does not affect the conclusions in

this paper at least when the pairing energy is not extremely high.

III. ASTROPHYSICAL CONSEQUENCES

A strange star at the moment of its formation may be very hot, and the rate of neutrino-induced mass ejection from the stellar surface may be very high [17]. In this case, in a few second after the star formation the normal-matter crust is blown away, and the SQM surface is nearly (or completely) bare [7]. We have shown that a Coulomb barrier with the electrostatic potential of $\sim 3.6 \times 10^7$ V may be present at the bare SQM surface of a strange star in the CFL phase. Such a barrier may be a powerful source of e^+e^- pairs created in its extremely strong electric field [6]. The strange star luminosity in pairs remains high enough ($\geq 10^{36}$ ergs s $^{-1}$) as long as the surface temperature is higher than $\sim 6 \times 10^8$ K [7]. Below this temperature, $T_S < 6 \times 10^8$ K, nonequilibrium quark-quark [16] and electron-electron [18] bremsstrahlung radiation dominates in the thermal emission from the surface of a bare strange star, i.e., the pair production by the Coulomb barrier is not significant.

At the surface of a strange star in the CFL phase a massive normal-matter crust may form by accretion of

matter onto the star [3,19]. For the Coulomb barrier at the surface of such a star the electrostatic potential of electrons is $eV(0,0) \approx 36$ MeV that is more than the electron chemical potential (~ 25 MeV) at which neutron drip occurs [20]. Therefore, the maximum density of the crust is limited by neutron drip and is about 4.3×10^{11} g cm $^{-3}$ [3]. At this density free neutrons begin to drip out of the most stable nucleus, ^{118}Kr ($Z = 36$). Being electrically charge neutral, the neutrons can gravitate toward the star's quark core where they are converted into SQM. For a strange star in the CFL phase the maximum mass of the crust is $\sim 10^{-5}M_\odot$. A massive ($\Delta M \sim 10^{-5}M_\odot$) crust of normal-matter completely obscures the star's SQM core, and in the observed mass range ($1 < M/M_\odot < 2$) it is difficult to discriminate between neutron stars and strange stars with such crusts [21–23] (however cf. [24,25]).

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